

6.977 Ultrafast Optics

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Chapter 1

Introduction

1.1 Course Mission

- Generation of ultrashort pulses: Nano-, Pico-, Femto-, Attosecond Pulses
- Propagation of ultrashort pulses
- Linear and nonlinear effects.
- Applications in high precision measurements, nonlinear optics, optical signal processing, optical communications, x-ray generation,....

1.2 Pulse Characteristics

Most often, there is not an isolated pulse, but rather a pulse train.

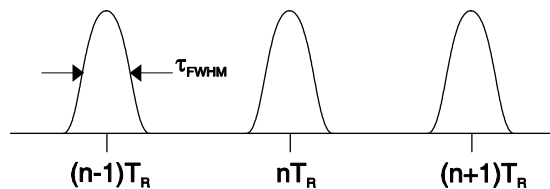


Figure 1.1: Periodic pulse train

T_R : pulse repetition time

W : pulse energy

$P_{ave} = W/T_R$: average power

τ_{FWHM} is the Full Width at Half Maximum of the intensity envelope of the pulse in the time domain.

The peak power is given by

$$P_p = \frac{W}{\tau_{FWHM}} = P_{ave} \frac{T_R}{\tau_{FWHM}}, \quad (1.1)$$

and the peak electric field is given by

$$E_p = \sqrt{2Z_{F_0} \frac{P_p}{A_{eff}}}. \quad (1.2)$$

A_{eff} is the beam cross-section and $Z_{F_0} = 377 \Omega$ is the free space impedance.

Time scales:

1 ns	\sim	30 cm	(high-speed electronics, GHz)
1 ps	\sim	300 μm	
1 fs	\sim	300 nm	
1 as = 10^{-18} s	\sim	0.3 nm = 3 \AA	(typ-lattice constant in metal)

The shortest pulses generated to date are about 4 – 5 fs at 800 nm ($\lambda/c = 2.7$ fs), less than two optical cycles and 250 as at 25 nm. For few-cycle pulses, the electric field becomes important, not only the intensity!

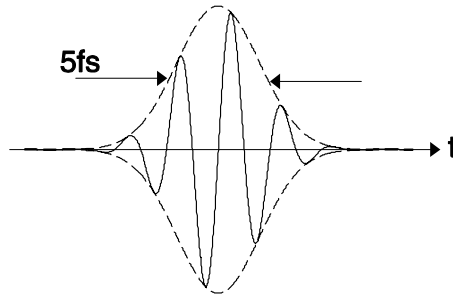


Figure 1.2: Electric field waveform of a 5 fs pulse at a center wavelength of 800 nm. The electric field depends on the carrier-envelope phase.

average power:

$$P_{ave} \sim 1W, \text{ up to } 100 \text{ W in progress.}$$

$$kW \text{ possible, not yet pulsed}$$

repetition rates:

$$T_R^{-1} = f_R = \text{mHz} - 100 \text{ GHz}$$

pulse energy:

$$W = 1\text{pJ} - 1\text{kJ}$$

pulse width:

$$\tau_{\text{FWHM}} = \begin{array}{ll} 5 \text{ fs} - 50 \text{ ps}, & \text{modelocked} \\ 30 \text{ ps} - 100 \text{ ns}, & \text{Q-switched} \end{array}$$

peak power:

$$P_p = \frac{1 \text{ kJ}}{1 \text{ ps}} \sim 1 \text{ PW},$$

obtained with Nd:glass (LLNL - USA, [1][2][3]).

For a typical lab pulse, the peak power is

$$P_p = \frac{10 \text{ nJ}}{10 \text{ fs}} \sim 1 \text{ MW}$$

peak field of typical lab pulse:

$$E_p = \sqrt{2 \times 377 \times \frac{10^6 \times 10^{12}}{\pi \times (1.5)^2} \frac{\text{V}}{\text{m}}} \approx 10^{10} \frac{\text{V}}{\text{m}} = \frac{10 \text{ V}}{\text{nm}}$$

1.3 Applications

- High time resolution: Ultrafast Spectroscopy, tracing of ultrafast physical processes in condensed matter (see Fig. 1.3), chemical reactions, physical and biological processes, influence chemical reactions with femtosecond pulses: Femto-Chemistry (Noble Prize, 2000 to A. Zewail), high speed electric circuit testing and sampling of electrical signals, see Fig. 1.4.

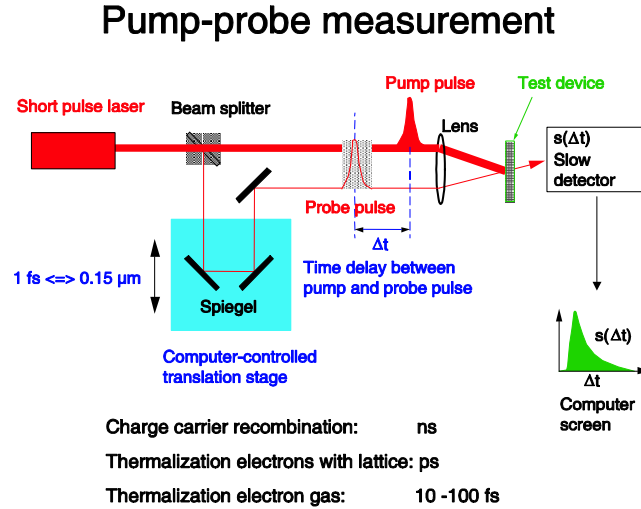


Figure 1.3: Pump-probe setup to extract time constants relevant for the carrier dynamics in semiconductors.

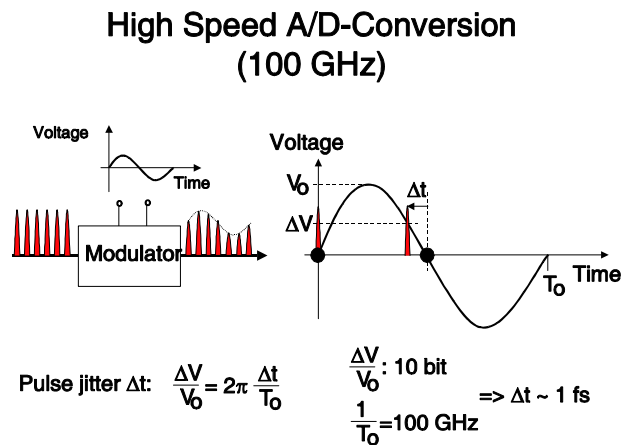


Figure 1.4: High speed A/D conversion with a high repetition rate pico- or femtosecond laser.

- High spatial resolution: $c\tau_{FWHM}$; optical imaging, e.g. optical coherence tomography, see Figs. 1.5-1.8).

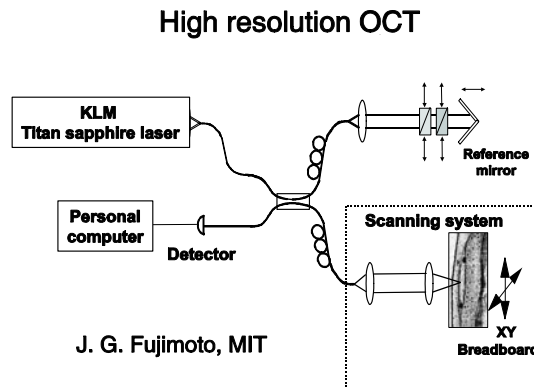


Figure 1.5: Setup for optical coherence tomography.
 Courtesy of James Fujimoto. Used with permission.

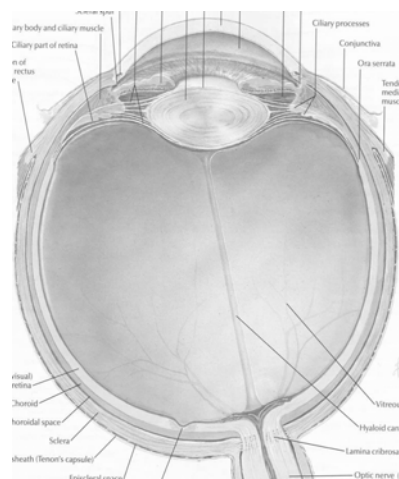


Figure 1.6: Cross section through the human eye.
 Courtesy of James Fujimoto. Used with permission.

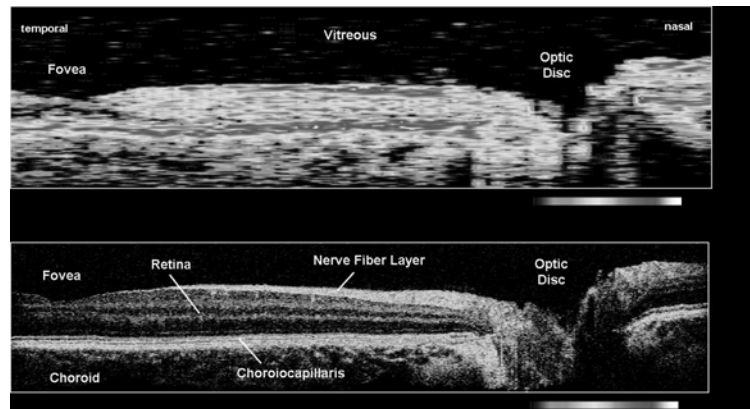


Figure 1.7: Comparison of retinal images taken with a superluminescence diode (top) versus a broadband Ti:sapphire laser (below).

Courtesy of James Fujimoto. Used with permission.

- Imaging through strongly scattering media:

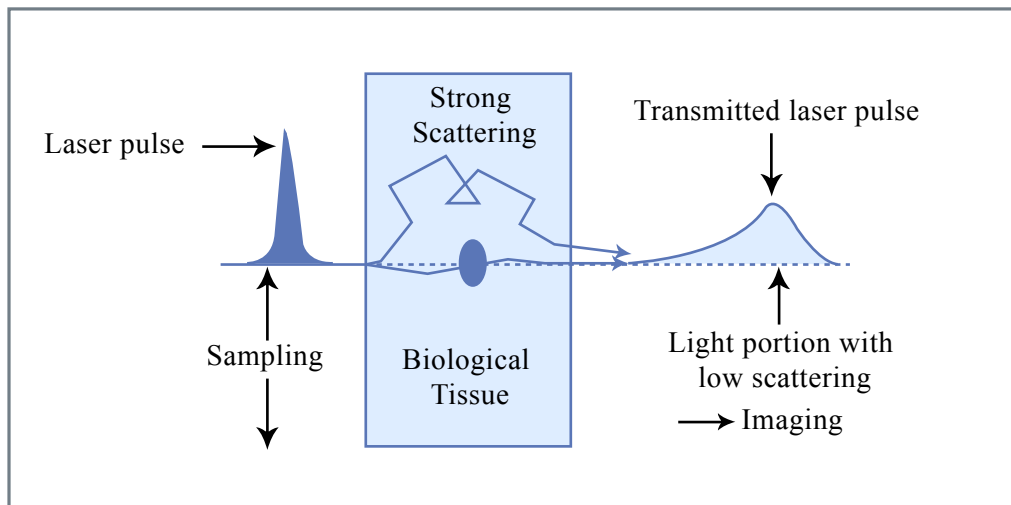


Figure 1.8: Imaging of the directly transmitted photons results in an un-blurred picture. Substitution for x-ray imaging; however, transmission is very low.

Figure by MIT OCW.

- High bandwidth: massive WDM - optical communications, many channels from one source or massive TDM, high bit-rate stream of short pulses.

- High intensities: Large intensities at low average power \Rightarrow Nonlinear frequency conversion, laser material processing, surgery, high intensity physics: x-ray generation, particle acceleration, ...

1.4 Review of Laser Essentials

Linear and ring cavities:

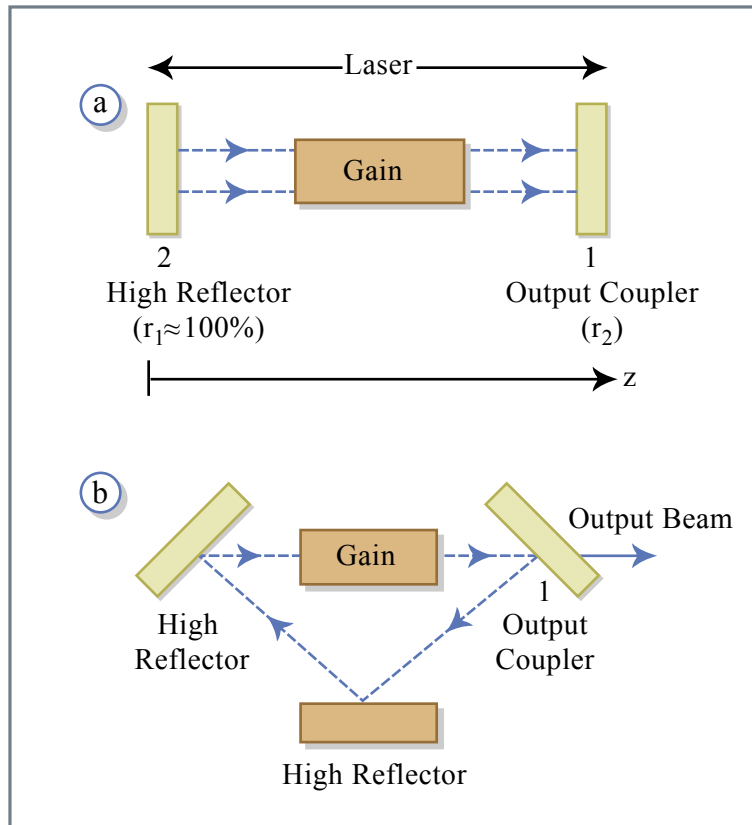


Figure 1.9: Possible cavity configurations. (a) Schematic of a linear cavity laser. (b) Schematic of a ring laser. [1]

Figure by MIT OCW.

Steady-state operation: Electric field must repeat itself after one roundtrip. Consider a monochromatic, linearly polarized field

$$E(z, t) = \Re \{ E_0 e^{j(\omega t - kz)} \}, \quad (1.3)$$

where

$$k = \frac{\omega}{c}n \quad (1.4)$$

is the propagation constant in a medium with refractive index n .

Consider linear resonator in Fig. 1.9a. Propagation from (1) to (2) is determined by $n = n' + jn''$ (complex refractive index), with the electric field given by

$$E = \Re \left\{ E_0 e^{\frac{\omega}{c}n''_g \ell_g} e^{j\omega t} e^{-j\frac{\omega}{c}(n'_g \ell_g + \ell_a)} \right\}, \quad (1.5)$$

where n_g is the complex refractive index of the gain medium (outside the gain medium $n = 1$ is assumed), ℓ_g is the length of the gain medium, ℓ_a is the outside gain medium, and $\ell = n_g \ell_g + \ell_a$ is the optical path length in the resonator.

Propagation back to (1), i.e. one full roundtrip results in

$$E = \Re \left\{ r_1 r_2 e^{2\frac{\omega}{c}n''_g \ell_g} E_0 e^{j\omega t - j2\frac{\omega}{c}\ell} \right\} \Rightarrow r_1 r_2 e^{2\frac{\omega}{c}n''_g \ell_g} = 1, \quad (1.6)$$

i.e. the gain equals the loss, and furthermore, we obtain the phase condition

$$\frac{2\omega\ell}{c} = 2m\pi. \quad (1.7)$$

The phase condition determines the resonance frequencies, i.e.

$$\omega_m = \frac{m\pi c}{\ell} \quad (1.8)$$

and

$$f_m = \frac{mc}{2\ell}. \quad (1.9)$$

The mode spacing of the longitudinal modes is

$$\Delta f = f_m - f_{m-1} = \frac{c}{2\ell} \quad (1.10)$$

(only true if there is no dispersion, i.e. $n \neq n(\omega)$). Assume frequency independent cavity loss and bell shaped gain (see Fig. 1.10).

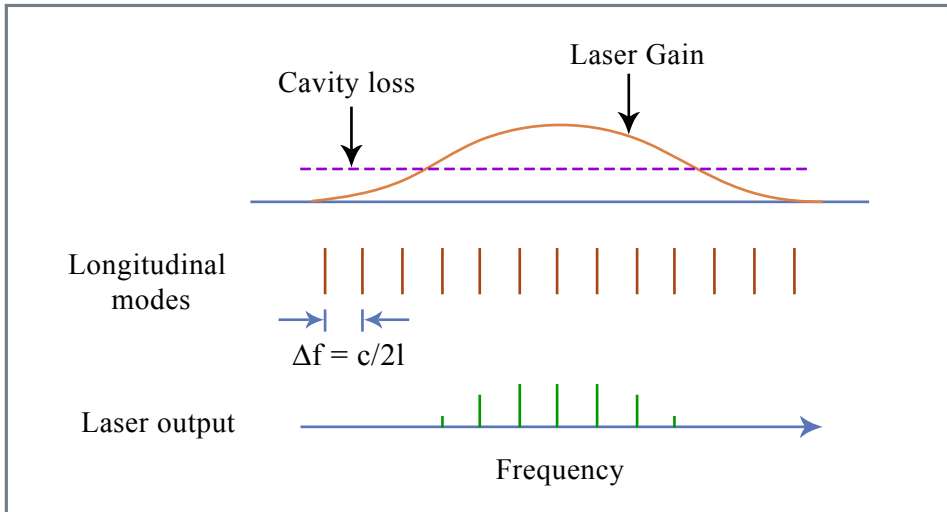


Figure 1.10: Laser gain and cavity loss spectra, longitudinal mode location, and laser output for multimode laser operation.

Figure by MIT OCW.

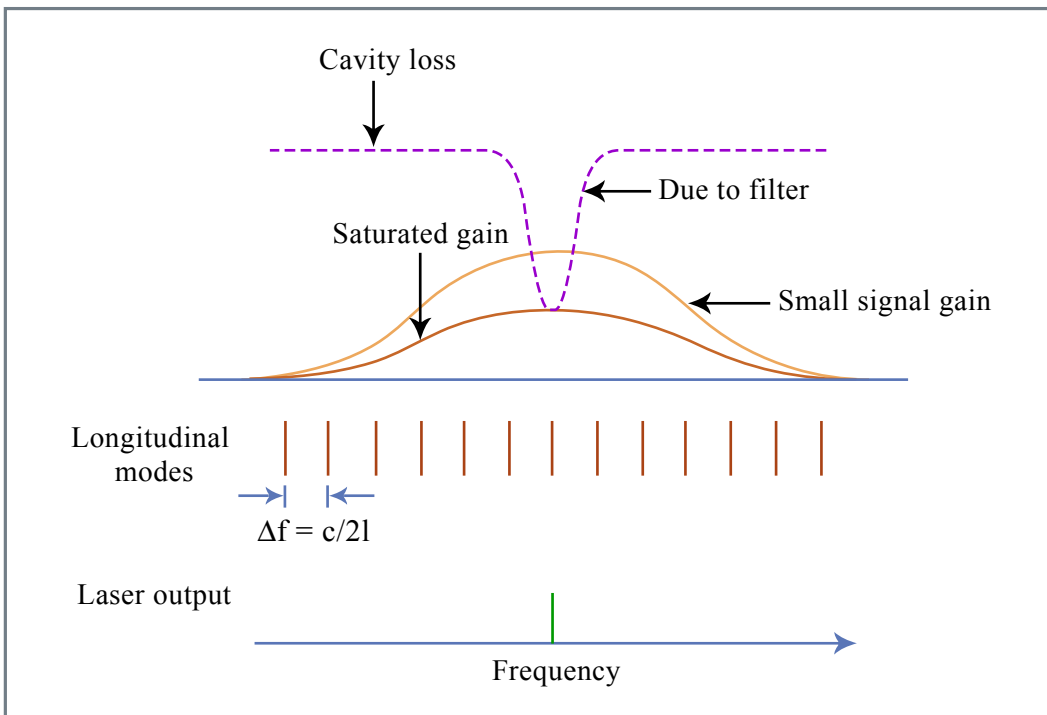


Figure 1.11: Gain and loss spectra, longitudinal mode locations, and laser output for single mode laser operation.

Figure by MIT OCW.

To assure single frequency operation use filter (etalon); distinguish between homogeneously and inhomogeneously broadened gain media, effects of spectral hole burning! Distinguish between small signal gain g_0 per roundtrip,

i.e. gain for laser intensity $I \rightarrow 0$, and large signal gain, most often given by

$$g = \frac{g_0}{1 + \frac{I}{I_{\text{sat}}}}, \quad (1.11)$$

where I_{sat} is the saturation intensity. Gain saturation is responsible for the steady state gain (see Fig. 1.11), and homogeneously broadened gain is assumed.

To generate short pulses, i.e. shorter than the cavity roundtrip time, we wish to have many longitudinal modes running in steady state. For a multimode laser the laser field is given by

$$E(z, t) = \Re \left[\sum_m \hat{E}_m e^{j(\omega_m t - k_m z + \phi_m)} \right], \quad (1.12a)$$

$$\omega_m = \omega_0 + m\Delta\omega = \omega_0 + \frac{m\pi c}{\ell}, \quad (1.12b)$$

$$k_m = \frac{\omega_m}{c}, \quad (1.12c)$$

where the symbol $\hat{}$ denotes a frequency domain quantity. Equation (1.12a) can be rewritten as

$$E(z, t) = \Re \left\{ e^{j\omega_0(t-z/c)} \sum_m \hat{E}_m e^{j(m\Delta\omega(t-z/c) + \phi_m)} \right\} \quad (1.13a)$$

$$= \Re [A(t - z/c) e^{j\omega_0(t-z/c)}] \quad (1.13b)$$

with the complex envelope

$$A\left(t - \frac{z}{c}\right) = \sum_m E_m e^{j(m\Delta\omega(t-z/c) + \phi_m)} = \text{complex envelope (slowly varying)}. \quad (1.14)$$

$e^{j\omega_0(t-z/c)}$ is the carrier wave (fast oscillation). Both carrier and envelope travel with the same speed (no dispersion assumed). The envelope function is periodic with period

$$T = \frac{2\pi}{\Delta\omega} = \frac{2\ell}{c} = \frac{L}{c}. \quad (1.15)$$

L is the roundtrip length (optical)!

Examples:

Examples:

We assume N modes with equal amplitudes $E_m = E_0$ and equal phases $\phi_m = 0$, and thus the envelope is given by

$$A(z, t) = E_0 \sum_{m=-(N-1)/2}^{(N-1)/2} e^{j(m\Delta\omega(t-z/c))}. \quad (1.16)$$

With

$$\sum_{m=0}^{q-1} a^m = \frac{1 - a^q}{1 - a}, \quad (1.17)$$

we obtain

$$A(z, t) = E_0 \frac{\sin \left[\frac{N\Delta\omega}{2} \left(t - \frac{z}{c} \right) \right]}{\sin \left[\frac{\Delta\omega}{2} \left(t - \frac{z}{c} \right) \right]}. \quad (1.18)$$

The laser intensity I is proportional to $E(z, t)^2$, averaged over one optical cycle: $I \sim |A(z, t)|^2$. At $z = 0$, we obtain

$$I(t) \sim |E_0|^2 \frac{\sin^2 \left(\frac{N\Delta\omega t}{2} \right)}{\sin^2 \left(\frac{\Delta\omega t}{2} \right)}. \quad (1.19)$$

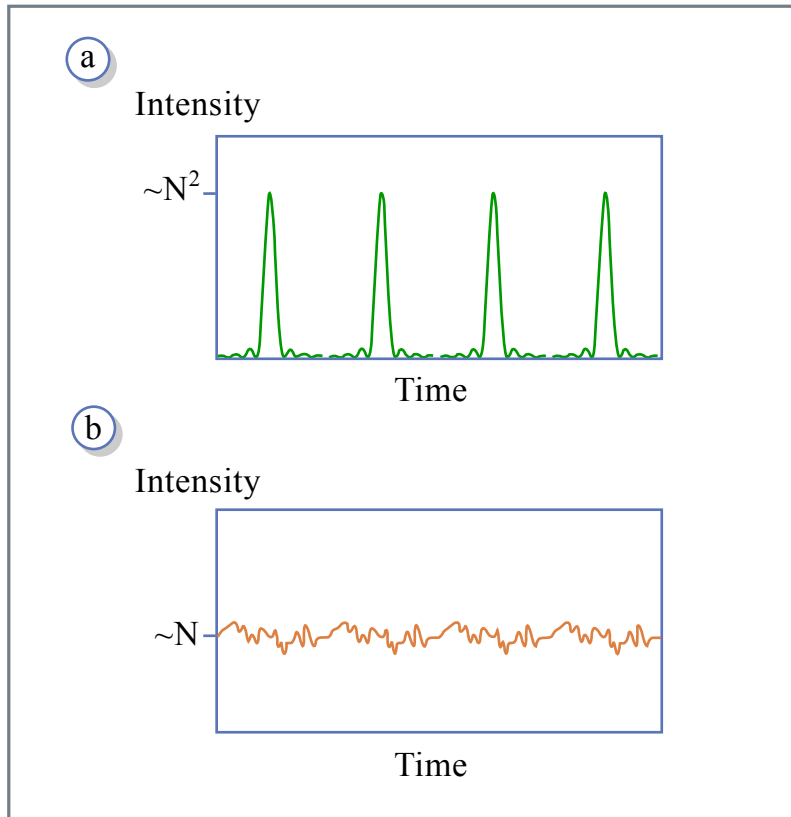


Figure 1.12: (a) mode-locked laser output with constant mode phase. (b) Laser output with randomly phased modes.

(a) Periodic pulses given by Eq. (1.19), period $T = 1/\Delta f = L/c$

- pulse duration

$$\Delta t = \frac{2\pi}{N\Delta\omega} = \frac{1}{N\Delta f} \quad (1.20)$$

- peak intensity $\sim N^2|E_0|^2$
- average intensity $\sim N|E_0|^2 \Rightarrow$ peak intensity is enhanced by a factor N .

(b) If phases of modes are not locked, i.e. ϕ_m random sequence

- Intensity fluctuates randomly about average value ($\sim N|E_0|^2$), same as modelocked case
- correlation time is $\Delta t_c \approx \frac{1}{N\Delta f}$
- Fluctuations are still periodic with period $T = 1/\Delta f$.

In a usual multimode laser, ϕ_m varies over t .

1.5 History

1960: First laser, ruby, Maiman [4].

1961: Proposal for Q-switching, Hellwarth [5].

1963: First indications of mode locking in ruby lasers, Guers and Mueller [6],[7], Statz and Tang [8]. on He-Ne lasers.

1964: Activemodelocking (HeNe, Ar, etc.), DiDomenico [9], [10] and Yariv [11].

1966: Passive modelocking with saturable dye absorber in ruby by A. J. Dellaria, Mocker and Collins [12].

1966: Dye laser, F. P. Schäfer, et al. [13].

1968: mode-locking (Q-Switching) of dye-lasers, Schmidt, Schäfer [14].

1972: cw-passive modelocking of dye laser, Ippen, Shank, Dienes [15].

1972: Analytic theories on active modelocking [21, 22].

1974: Sub-ps-pulses, Shank, Ippen [16].

1975: Theories for passive modelocking with slow [1], [24] and fast saturable absorbers [25] predicted hyperbolic secant pulse.

1981: Colliding-pulse mode-locked laser (CPM), [17].

1982: Pulse compression [20].

1984: Soliton Laser, Mollenauer, [26].

1985: Chirped pulse amplification, Strickland and Morou, [27].

1986: Ti:sapphire (solid-state laser), P. F. Moulton [28].

1987: 6 fs at 600 nm, external compression, Fork et al. [18, 19].

1988: Additive Pulse Modelocking (APM), [29, 30, 31].

1991: Kerr-lens modelocking, Spence et al. [32, 33, 34, 35, 36].

1993: Stretched pulse laser, Tamura et al [37].

1994: Chirped mirrors, Szipoecs et al. [38, 39]

1997: Double-chirped mirrors, Kaertner et al.[40]

2001: 5 fs, sub-two cycle pulses, octave spanning, Ell et al.[42]

2001: 250 as by High-Harmonic Generation, Krausz et al.[43]

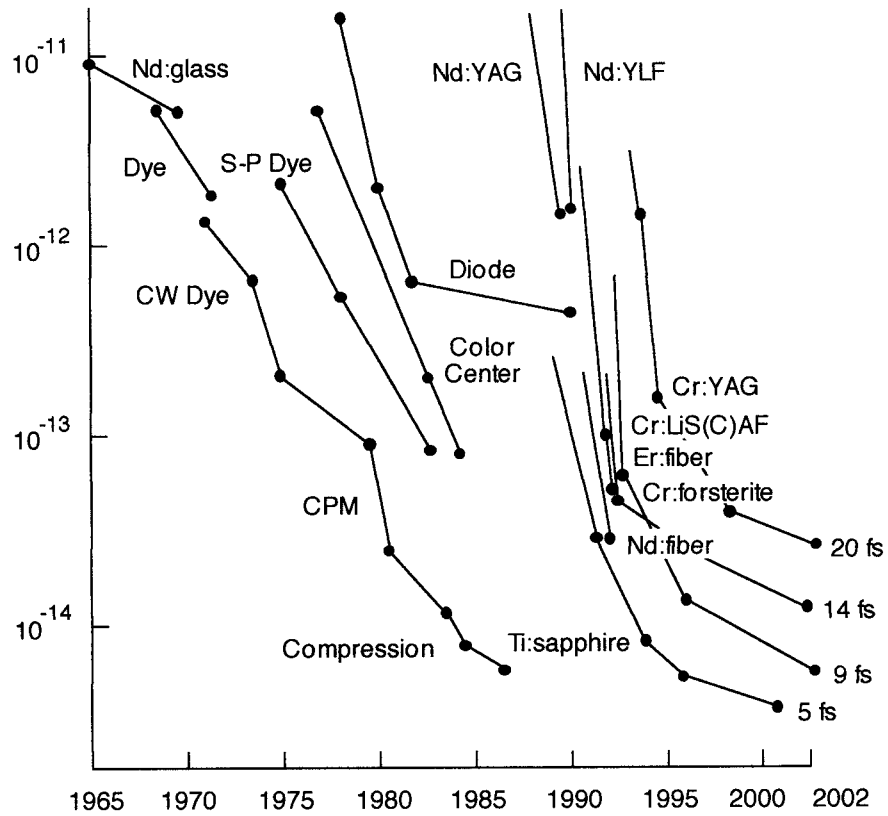


Figure 1.13: Pulse width of different laser systems by year.

Courtesy of Erich Ippen. Used with permission.

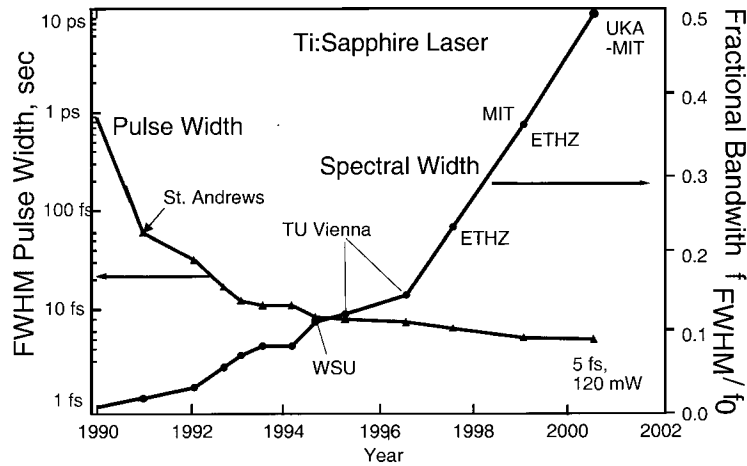


Figure 1.14: Pulse width of Ti:sapphire lasers by year.

1.6 Laser Materials

Laser Material	Absorption Wavelength	Average Emission λ	Band Width	Pulse Width
Nd:YAG	808 nm	1064 nm	0.45 nm	~ 6 ps
Nd:YLF	797 nm	1047 nm	1.3 nm	~ 3 ps
Nd:LSB	808 nm	1062 nm	4 nm	~ 1.6 ps
Nd:YVO ₄	808 nm	1064 nm	2 nm	~ 4.6 ps
Nd:fiber	804 nm	1053 nm	22-28 nm	~ 33 fs
Nd:glass	804 nm	1053 nm	22-28 nm	~ 60 fs
Yb:YAG	940, 968 nm	1030 nm	6 nm	~ 300 fs
Yb:glass	975 nm	1030 nm	30 nm	~ 90 fs
Ti:Al ₂ O ₃	480-540 nm	796 nm	200 nm	~ 5 fs
Cr ⁴⁺ :Mg ₂ SiO ₄ :	900-1100 nm	1260 nm	200 nm	~ 14 fs
Cr ⁴⁺ :YAG	900-1100 nm	1430 nm	180 nm	~ 19 fs

Transition metals: (Cr³⁺, Ti³⁺, Ni²⁺, CO²⁺, etc.) (outer *3d*-electrons)
 → broadband

Rare earth: (Nd³⁺, Tm³⁺, Ho³⁺, Er³⁺, etc.) (shielded *4f*-electrons)
 → narrow band.

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