Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science

6.977 Ultrafast Optics

Spring 2005

Problem Set 7

Issued: March 31, 2005. **Due:** 11am, April 7, 2005.
Reminder: April 12 is the term paper proposal due.

Problem 7.1: Active Mode-Locking with a Phase Modulator

The master equation for active mode-locking by a phase modulator is given by the following expression

$$T_R \frac{\partial A}{\partial T} = \left[g - l + D_g \frac{\partial^2}{\partial t^2} - jM\cos(\omega_M t) \right] A \tag{1}$$

(a) Use the Gaussian pulse analysis as applied to Problem 6.2 to show that two steady-state pulse solutions in form of chirped Gaussians are possible in this system. What is the steady-state pulse width and chirp?

(b) Show that a Gaussian pulse with a chirp larger than 1 becomes shorter when propagating through the gain medium with finite bandwidth.

(c) If we add positive dispersion to the system, does this favor one of the two solutions?

Problem 7.2: Actively Mode-Locked Laser with Flat and Finite Gain Profile

We want to investigate how the gain profile influences the pulse shape in a mode-locked laser. Let's assume that the gain profile is flat over a half-width Ω_g with a value g that saturates with the intra-cavity power according to $g = g_0/(1 + P/P_L)$. Beyond that bandwidth the gain is $-\infty$, i.e. outside the finite gain bandwidth there is infinite loss, which models the finite mirror bandwidth of a laser

$$g(\omega) = \begin{cases} g, & \text{for } |\omega| < \Omega_g \\ -\infty, & \text{elsewhere} \end{cases} \tag{2}$$

To be able to find an analytic solution we consider an actively mode-locked laser with the gain (2) and the loss modulator treated in parabolic approximation with loss curvature M_s.

(a) Give the master equation for the mode-locked laser in the frequency domain under the above assumptions.

(b) Find the pulse shapes that reproduce itself after each round-trip and the corresponding saturated gain levels to support these pulses.

(c) Plot the eigensolutions found in (b) (using a math package you are familiar with) and give a physical interpretation.

(d) What is the steady-state solution of this laser? Is it stable?

(e) What is the time bandwidth product of the steady-state solution?