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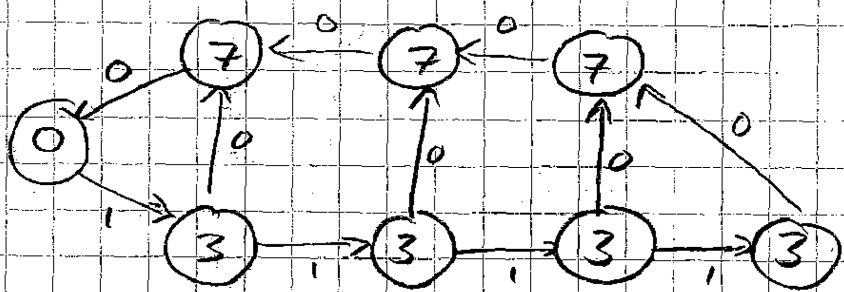
L8.1

Clock period

Circuit $G = (V, E, d, w)$
- V = comb elements
- E = interconnections
- d = propagation delay of comb elem
- w = latch (register) count on edge.

Ex.



Def. Path weight $p = v_0 \xrightarrow{e_0} v_1 \xrightarrow{e_1} \cdots \xrightarrow{e_{k-1}} v_k$ is

$$w(p) = \sum_{i=0}^{k-1} w(e_i)$$

Path delay

$$d(p) = \sum_{i=0}^{k} d(v_i)$$

《Note fencepost》

Legality constraints

D1. $d(v) \geq 0 \quad \forall v \in V$
W1. $w(e) \geq 0 \quad \forall e \in E$
W2. In any directed cycle p of G, $\exists\, e \in p$ $\ni w(e) \geq 1$.

W2: Synchronous circuits – no unclocked state.

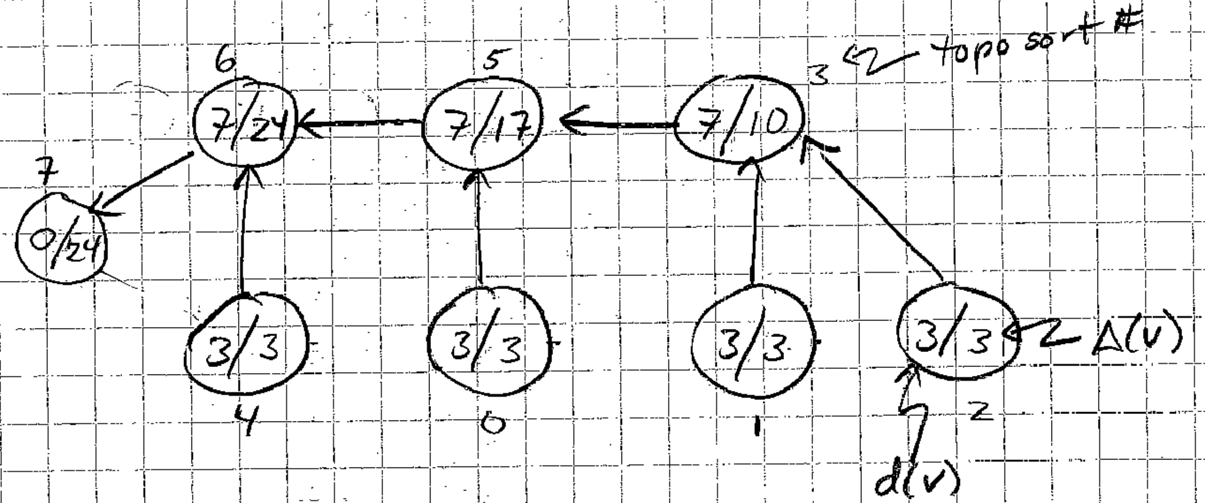
Def. The clock period of G is

$$\Phi(G) = \max_p \{d(p) : w(p) = 0\}. \quad (\text{Ex. } \Phi(G) = 24)$$

Computing clock period

1. Let G_0 be subgraph of G with only 0-wt edges. G_0 is acyclic (by W2).
2. Topologically sort vertices of G_0.
3. Scan through vertices in topo sort order.
   a. If no incoming edge to v, set $\Delta(v) = d(v)$.
   b. Otherwise, $\Delta(v) = d(v) + \max\limits_{u \to v \text{ in } G_0} \{\Delta(u)\}$
4. $\Phi(G) = \max\limits_{v \in V} \Delta(v)$ «Longest path in acyclic graph»

Ex.

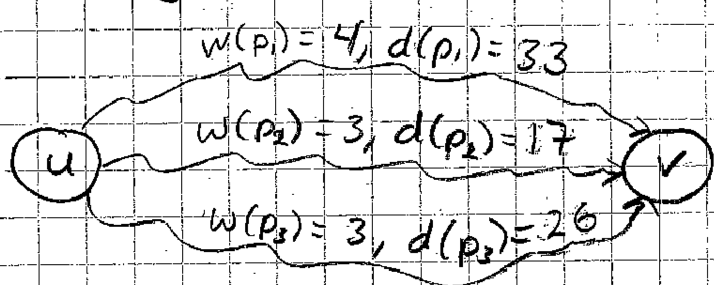


Running time: $\Theta(V+E) = \Theta(E)$ if G connected.

Retiming to minimize clock period

Recall: For path $u \overset{p}{\rightsquigarrow} v$,
$w_r(p) = w(p) - r(u) + r(v)$.

What paths might realize $\Phi(G_r)$?



Retiming reweights each path from u to v by same amount.

- Ignore p_1, since $w_r(p_1) \geq 1$ $\forall$ legal retimings.
- Ignore p_2, since $w_r(p_2) = 0 \Rightarrow w_r(p_3) = 0$ and $d(p_3) > d(p_2)$.

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Define $W(u,v) = \min\{w(p) : u \overset{p}{\rightsquigarrow} v\}$.

$u \overset{p}{\rightsquigarrow} v$ is a <u>critical path</u> if $w(p) = W(u,v)$.

Define $D(u,v) = \max\{d(p) : u \overset{p}{\rightsquigarrow} v \text{ and } w(p) = W(u,v)\}$.

<u>Lemma 1.</u> For any $c > 0$, $\Phi(G) \le c$ iff whenever $D(u,v) > c$, we have $W(u,v) \ge 1$.

Proof. ($\Rightarrow$) Sup. $\Phi(G) \le c$, and let $u, v \in V$ \$ $D(u,v) > c$.
Thus, $\exists\ u \overset{p}{\rightsquigarrow} v$ \$ $d(p) \ge c$ and $w(p) = W(u,v)$.
Must have $W(u,v) \ge 1$, or else p would be a 0-wt path with $d(p) > c$. Contradiction.

($\Leftarrow$) Sup. $\forall u, v \in V$, $D(u,v) > c \Rightarrow W(u,v) \ge 1$.
Let $u \overset{p}{\rightsquigarrow} v$ be 0-wt path. Then, $W(u,v) = w(p) = 0$, which implies $d(p) \le D(u,v) \le c$. $\boxtimes$

<u>Computing W and D</u>

1. Create graph with edge weights of $u \overset{e}{\rightarrow} v$ of $\langle w(e), -d(u)\rangle$
2. All-pairs sh. paths (Floyd-Washall $O(V^3)$
   Johnson + Fib heaps $O(VE + V^2 \lg V)$)
3. For weight $\langle x, y\rangle$ betw. u and v,
   $W(u,v) = x$
   $D(u,v) = d(v) - y$