6.852: Distributed Algorithms
Fall, 2009

Class 18

Today's plan

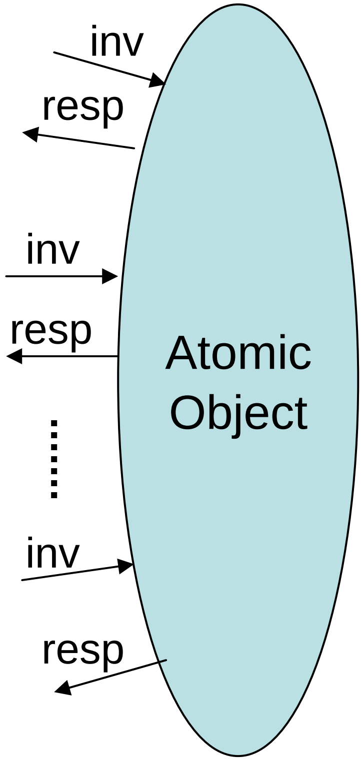
- Atomic objects:
  - Atomic snapshots of shared memory: Snapshot atomic objects.
  - Read/write atomic objects
- Reading: Sections 13.3-13.4
- Next:
  - Wait-free synchronization.
  - Reading:
    - [Herlihy, Wait-free synchronization]
    - [Attiya, Welch, Chapter 15]

Well, that was the plan for next time, but:

- We have an amended plan: Move classes 21 and 22 before 19 and 20.
- So really, next time:
  - Shared-memory multiprocessor computation
  - Techniques for implementing concurrent objects:
    - Coarse-grained mutual exclusion
    - Locking techniques
    - Lock-free algorithms
- Reading:
  - [Herlihy, Shavit] Chapter 9

Last time

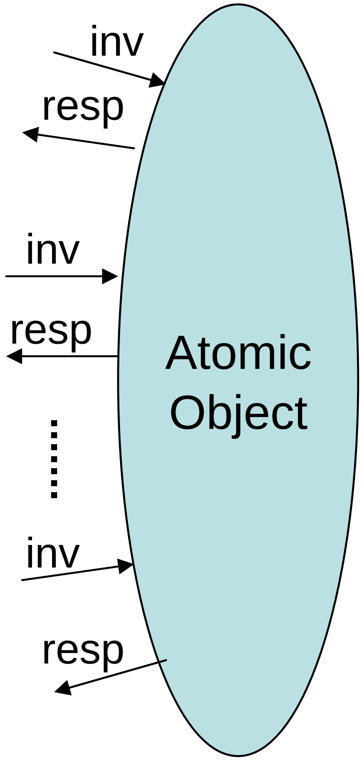
- Defined Atomic Objects.
- Atomic object of a given type is similar to an ordinary shared variable of that type, but it allows concurrent accesses by different processes.
- Still looks “as if” operations occur one at a time, sequentially, in some order consistent with order of invocations and responses.
- Correctness conditions:
  - Well-formedness, atomicity.
  - Fault-tolerance conditions:
    - Wait-free termination
    - f-failure termination



Atomic sequences

- Suppose β is any well-formed sequence of invocations and responses. Then β is atomic provided that one can
  - Insert serialization points for all complete operations.
  - Select a subset Φ of incomplete operations.
  - For each operation in Φ, insert a serialization point somewhere after the invocation, and make up a response.

In such a way that moving all matched invocations and their responses to the serialization points yields a trace of the variable type.



Canonical atomic object automaton

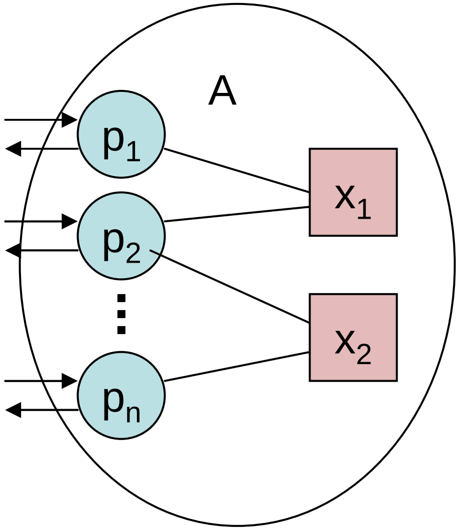
- Canonical object automaton keeps internal copy of the variable, plus delay buffers for invocations and responses.
- 3 kinds of steps:
  - Invoke: Invocation arrives, gets put into in-buffer.
  - Perform: Invoked operation gets performed on the internal copy of the variable, response gets put into resp-buffer.
  - Respond: Response returned to user.
- Perform step corresponds to serialization point.

Canonical atomic object automaton

- Equivalent to the original specification for a wait-free atomic object, in a precise sense.
- Can be used to prove correctness of algorithms that implement atomic objects, e.g., using simulation relations.
- Theorem 1: Every fair trace of the canonical automaton (with well-formed U) satisfies the properties that define a wait-free atomic object.
- Theorem 2: Every trace allowed by a wait-free atomic object (with well-formed U) is a fair trace of the canonical automaton.

Atomic objects vs. shared variables

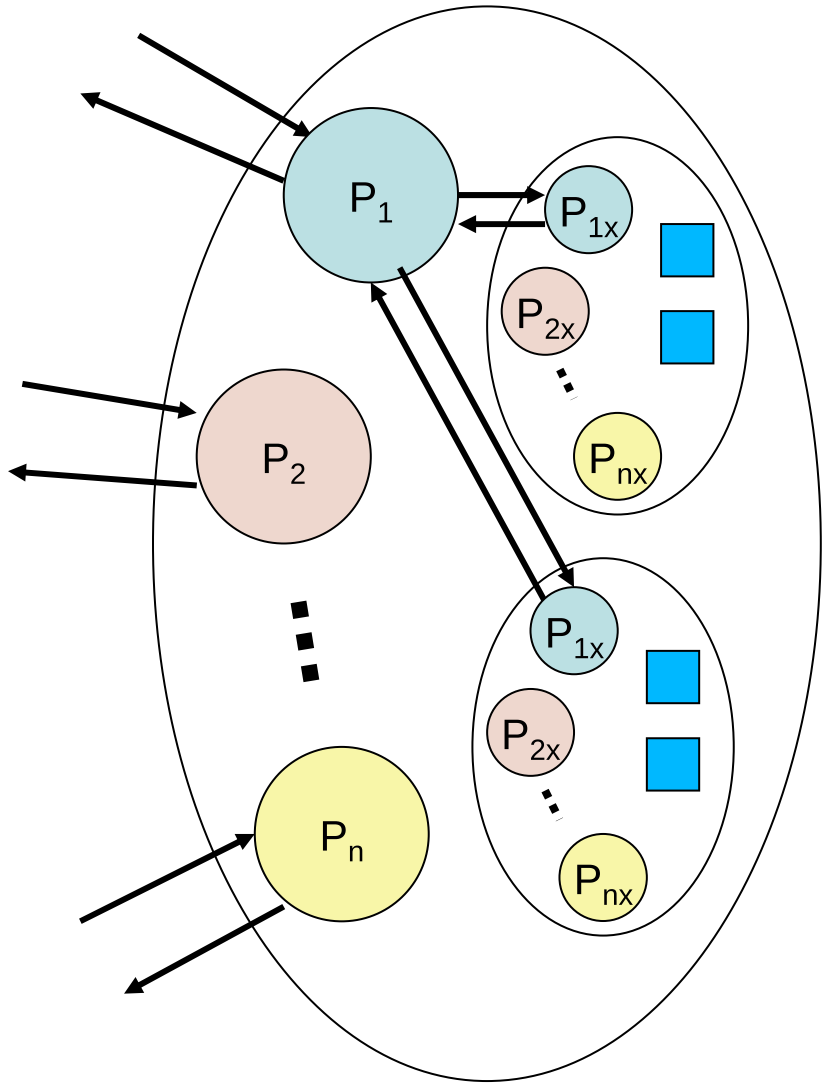
- Can substitute atomic objects for shared variables in a shared-memory system, and the resulting system "behaves the same".
- Theorem: For any execution α of Trans $\times$ U, there is an execution α' of A $\times$ U (the original shared-memory system) such that:
  - $\alpha \mid U = \alpha' \mid U$ (looks the same to the users), and
  - $stop_i$ events occur for the same i in α and α' (same processes fail).



- Needs a technical assumption.
- Construction also preserves liveness:
  - α fair implies α' fair.
  - Provided that the atomic objects don't introduce new blocking.
    - E.g., wait-free.
    - E.g., at most f failures for A and each atomic object guarantees f-failure termination.

Can use Trans to justify:

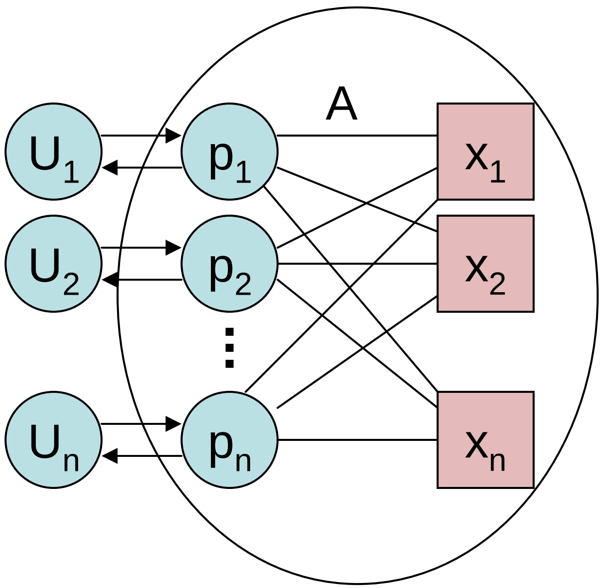
- Implementing fault-tolerant atomic objects using other fault-tolerant atomic objects.
- Building shared-memory systems, including shared-memory implementations of fault-tolerant atomic objects, hierarchically.



Snapshot Atomic Objects

Snapshot Atomic Objects

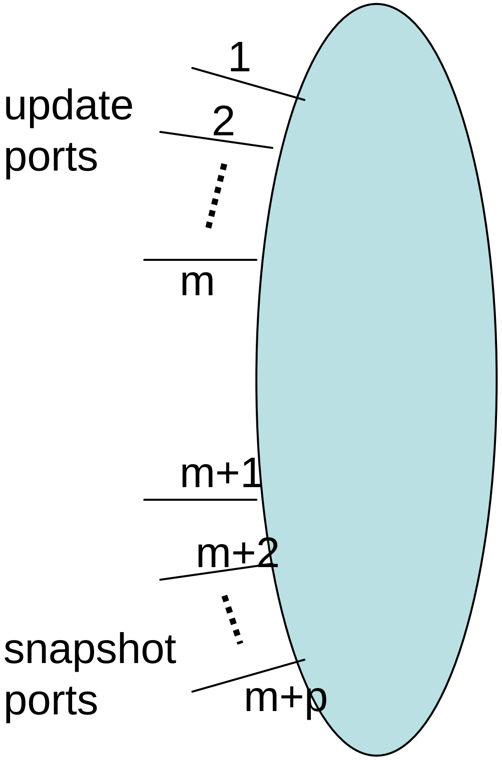
- Most common shared-memory model:
  - Single-writer multi-reader read/write shared variables,
  - Each process writes to one variable, others read it.
- Limitation: Process can read only one variable at a time.
- Atomic snapshot object adds capability for one process to read everyone's variables in one step.



- We will:
  - Define atomic snapshot objects.
  - Show that they do not add any power: they can be implemented using only simple read/write shared variables, with wait-free termination!

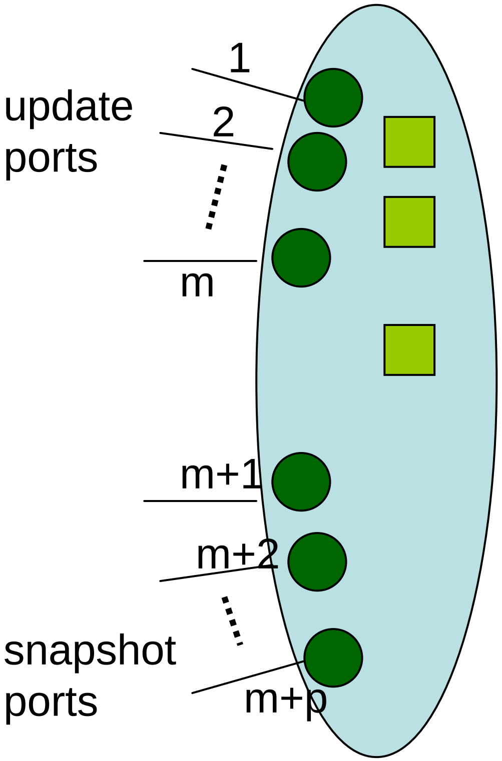
Variable type for snapshot objects

- Assume a lower-level value domain W (for the individual processes to write), with initial value w_0.
- Value domain for the snapshot object: Vectors v of fixed length m, with values in W.
- Initial value: $(w_0, w_0, w_0, \ldots, w_0)$.
- Invocations and responses:
  - update(i,w):
    - Writes value w into component i.
    - Reponds “ack”.
  - snap:
    - Responds with the entire vector.
- External interface: m “update ports”, p “snapshot ports”.
- Each update port i is for updates of vector component i, $\text{update}(i,w)_i$.



Implementing snapshot atomic objects

- Goal: Implement an atomic snapshot object using a shared-memory system, one process per port, with only single-writer multi-reader shared variables.
- Unbounded-variable algorithm [Afek, Attiya, Dolev, Gafni,…]
- Also a bounded-variable version.
- Shared variables:
  - For each update port i, shared variable x(i), written by update process i, read by everyone.
  - Each x(i) holds:
    - val, an element of W.
    - tag, a natural number.
    - Some other stuff, we'll see shortly.
- Processes use these separate read/write variables to implement a single snapshot atomic object.



Idea 1

- update(w,i)$_i$:
  - To write w to vector component i, update process i writes it in x(i).val.
  - Adds a tag that uniquely identifies the update (a sequence number, starting with 1).
- snap:
  - Read all the x(i)s, one at a time.
  - Read them all again.
  - If the two read passes yield the same tags, then return the vector of x(i).val values.
    - The vector actually appears in the memory at some point in real time.
    - That can be the serialization point for the snap.
  - If not, then keep trying, until two consecutive read passes yield the same tags.
  - This is correct, if it completes.
  - But the snap might never complete, because of continuing concurrent updates.

Idea 2 (Clever)

- Suppose the snap sees the same x(i) variable with four different tag values t_1, t_2, t_3, t_4.
- Then it knows that the interval of the update operation that wrote t_3 is entirely contained in the interval of the snap.
- Why:
  - Since the snap sees t_1, i's update with tag t_2 doesn't finish before the snap starts.
  - So i's update with tag t_3 starts after the snap starts.
  - Since the snap sees t_4, i's update with tag t_4 must start before the snap finishes.
  - So i's update with tag t_3 finishes before the snap finishes.
- So, modify update process i:
  - Before it writes to x(i), executes its own embedded-snap subroutine, which is just like a snap.
  - When it writes (val, tag) to x(i), also writes the result of its embedded-snap.
- Now, a snap that sees four different tags t_1, t_2, t_3, t_4, in x(i) returns the recorded value of the embedded-snap associated with t_3.
- Embedded-snap behaves the same.

In more detail:

- x(i) contains:
  - val in W, initially w_0
  - tag, a natural number, initially 0
  - view, a vector indexed by { 1,…,m } of W, initially $(w_0)^m$.
- snap:
  - Repeatedly read all x(i)s (any order) until one of the following:
    - 2 passes yield the same x(i).tag for every i.
      - Then return the common vector of x(i).val values.
    - For some i, four distinct x(i).tags are seen.
      - Then return x(i).view from the third x(i).tag.
- update(i,w):
  - Perform embedded-snap, same as snap.
  - Write to x(i):
    - val := w
    - tag := next sequence number (local count)
    - view := vector returned by embedded snap
  - Return ack.

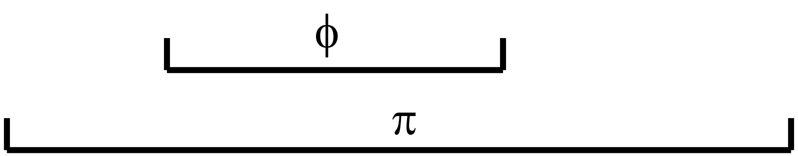
Correctness

- Theorem: This algorithm implements a wait-free snapshot atomic object.

- Proof:
  - Well-formedness: Clear.
  - Wait-free termination: Easy---always returns by one case or the other.
  - Atomicity:
    - Show we can insert serialization points appropriately.
    - By Lemma 13.10, it's enough to consider executions in which all operations complete.
    - So, fix an execution α of the algorithm + users, and assume that all operations complete in α.
    - Insert serialization points:
      - For update: Just after the write step.
      - For snap: We need a more complicated rule:

Serialization points for snaps

- Assign serialization points to all snaps/embedded-snaps.
- For every snap/embedded-snap that terminates by performing two read passes with the same tags (type 1):
  - Choose any point between end of the first pass and beginning of the second pass.
- For all the snap/embedded-snaps that terminate by finding four distinct tags for some x(i) (type 2):
  - Insert serialization points 1 by 1, in order of operation completion.
  - For each snap/embedded-snap π in turn:
    - The vector returned comes from some embedded-snap ϕ (from some update) whose interval is completely contained within the interval of π:



    - By the ordering, ϕ has already been assigned a serialization point.
    - Insert serialization point for π right after that for ϕ.

Correctness of serialization points

- All serialization points are in the required intervals:
  - updates:
    - Obvious.
  - Type 1 snaps/embedded-snaps (terminate with two identical read phases):
    - Obvious.
  - Type 2 snaps/embedded-snaps (terminate with four distinct values):
    - Argue inclusion by induction on the number of response events.
    - Use the containment property.
- Result of shrinking operations to their serialization points is a serial trace:
  - Because each snap returns the "correct" vector at its serialization point (result of all writes up to that point).
  - Easy for Type 1 snaps.
  - For Type 2 snaps, use induction on number of response events.

Complexity

- Shared memory size:
  - m variables, each of unbounded size (because of x(i).tag).
  - m variables for length m vector.
- Time for snapshot:
  - $\leq (3m+1)\, m$ shared memory accesses
  - $O(m^2\, l)$ time
- Time for update:
  - Also $O(m^2\, l)$, because of embedded-snap.

Algorithm using bounded variables

- Also by [Afek, Attiya, Dolev, Gafni,…], based on ideas by Peterson.
- Uses bounded tags.
- Involves a slightly tricky handshake protocol.
- See [Book, Section 13.3.3].

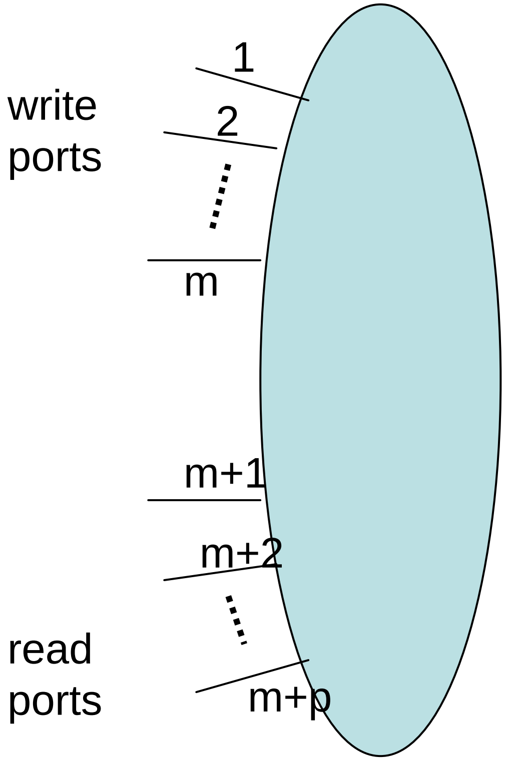
- Other snapshot algorithms have been developed, improving further on complexity, more complicated.

- Moral: Wait-free snapshot atomic objects can be implemented from simple wait-free read/write registers.
- So they don't add extra computing power.

Read/Write Atomic Objects

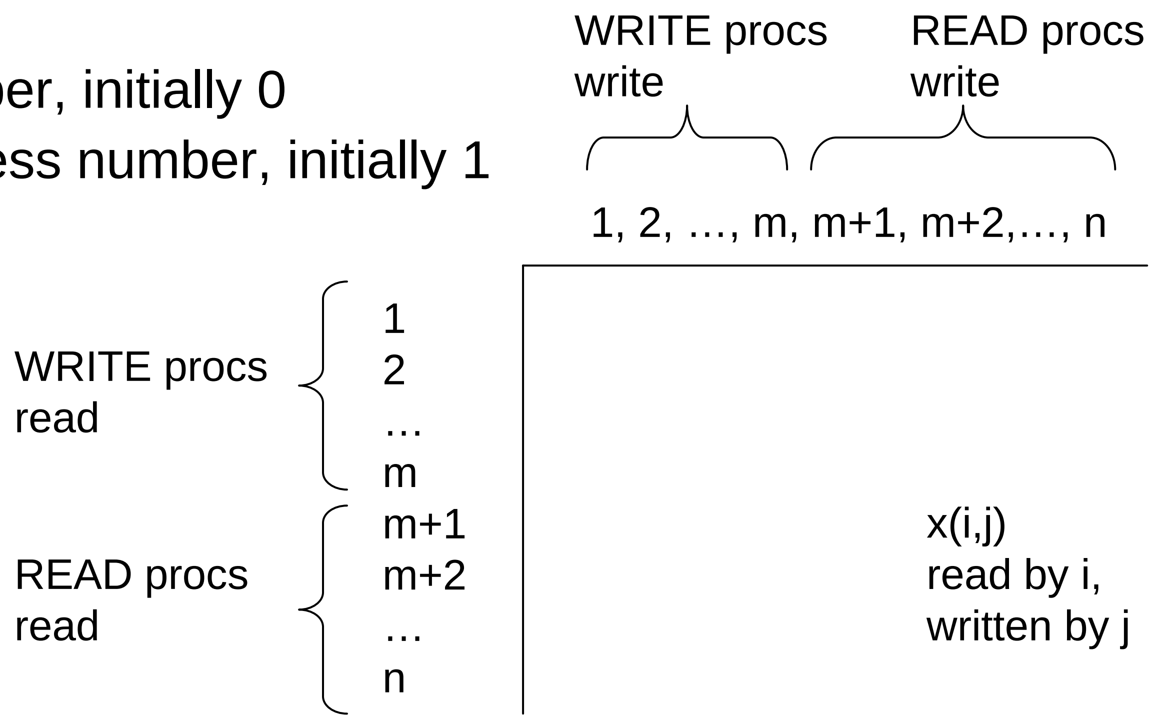
Read/write atomic objects

- Consider implementing an atomic m-writer, p-reader register, using lower-level primitives.
- Q: What lower-level primitives?
- Try 1-writer, 1-reader registers.
- Several published algorithms, some quite complicated.
- Show a simple one, with unbounded tags [Vitanyi, Awerbuch].
- Caution: Bounded-tag algorithm in that paper is incorrect.



Vitanyi-Awerbuch algorithm

- m-writer, p-reader read/write atomic objects from 1-writer, 1-reader read/write registers.
- Use n^2 shared variables, $n = m + p$:
- Caps for high-level operations
- $x(i,j)$ has:
  - val in V, initially v_0
  - tag, a natural number, initially 0
  - index, a write process number, initially 1



Vitanyi-Awerbuch algorithm

- $\mathrm{WRITE}(v)_i$:
  - Process i reads all variables in its row.
  - Let k = largest tag it sees.
  - Writes to each variable in its column:
    - val := v, tag := k+1, index := i
  - Responds “ack”.
- READ_i:
  - Process i reads all variables in its row.
  - Let (v,k,j) be a triple with maximum (tag,index) (lexicographic order).
  - “Propagates” this information by writing to each variable in its column:
    - val := v, tag := k, index := j
  - Finally, responds v.

Correctness

- Theorem: Vitanyi-Awerbuch implements a wait-free m-writer p-reader read/write atomic object.
- Well-formed, wait-free: Easy.
- Atomicity:
  - Proceed as in snapshot proof, describing exactly where to put the serialization points?
  - But not so obvious where to put them:
    - E.g., each WRITE and READ does many write steps.
    - Contrast: Each update in snapshot algorithm does just one write step.
    - Placement of serialization points seems to be sensitive to “races” between processes reading their rows and other processes writing their columns.
  - Use a different proof method:
    - Define a partial ordering of the high-level operations, based on (tag,index), and prove that the partial order satisfies certain conditions:

A useful lemma

- Let β be a (finite or infinite) sequence of invocations and responses for a read/write atomic object, that is well-formed for each i, and that contains no incomplete operations.
- Let Π be the set of operations in β.
- Suppose there is an irreflexive partial order $<$ of Π satisfying:
  1. For any operation π in Π, there are only finitely many operations ϕ such that $\phi < \pi$.
  2. If the response for π precedes the invocation for ϕ in β, then we don't have $\phi < \pi$.
  3. If π is a WRITE in Π and ϕ is any operation in Π then either $\pi < \phi$ or $\phi < \pi$.
  4. Value returned by each READ is the one written by the last preceding WRITE, according to $<$. (Or v_0, if there is no such WRITE.)
- Then β satisfies the atomicity property.

Proof of lemma

- Insert serialization points using the rule:
  - Insert serialization point for π just after the latest of the invocations for π and for all operations ϕ with $\phi < \pi$.
  - Condition 1 implies this is well-defined.
  - Order contiguous serialization points consistently with <.
- Claim 1: The order of the serialization points is consistent with the < ordering on Π; that is, if $\phi < \pi$ then the serialization point for ϕ precedes the serialization point for π.
- Claim 2: The serialization point for each π is in the interval of π.
  - Obviously after the invocation of π.
  - Could it be after the response of π?
  - No: If it were, then the invocation for some $\phi < \pi$ would come after the response of π, violating Condition 2.
- Claim 3: Each READ returns the value of the WRITE whose serialization point comes right before the READ's serialization point.
  - Condition 3 says all WRITES are ordered w.r.t. everything.
  - Condition 4 says that the READ returns the result written by the last preceding WRITE in <.
  - Since order of ser. pts. is consistent with <, that's the right value to return.

Using lemma to show atomicity for [Vitanyi, Awerbuch] algorithm

- Consider any execution α of V-A, assume no incomplete operations.
- Construct a partial order based on (tag,index) pairs:
  - $\pi < \phi$ iff
    - π writes (or propagates) a smaller tag pair than ϕ, or
    - π and ϕ write (or propagate) the same tag pair, π is a WRITE and ϕ is a READ.
  - That is, iff
    - tagpair(π) < tagpair(ϕ), or
    - tagpair(π) = tagpair(ϕ), π is a WRITE and ϕ is a READ.
- Show this satisfies the Properties 1-4.
- Condition 1 follows from Condition 2 and the fact that there are no incomplete operations.
- Show Condition 2:

Condition 2

- Claim: The (tag,index) pairs in any particular variable x(i,j) never decrease during α.
- Proof of Claim 1:
  - x(i,j) is written only by process j.
  - j's high-level operations are sequential.
  - Each operation of j involves reading row j, choosing a tag pair $\geq$ the maximum one it sees, then writing it to column j.
  - Among the variables j reads is the diagonal x(j,j), so j's chosen pair is $\geq$ the one in x(j,j).
  - Since x(i,j) contains the same pair as x(j,j), j's chosen pair is also $\geq$ the one in x(i,j).
  - Writes x(i,j) with this tag pair, nondecreasing.

Condition 2

- Condition 2: If the response for π precedes the invocation for ϕ in β, then we can't have $\phi < \pi$.
- Proof:
  - Suppose we have:

  - Then before the response event, π has written tagpair(π) to its entire column.
  - So (by Claim 1), ϕ reads a tagpair $\geq$ tagpair(π).
  - Then (by the way the algorithm works), ϕ chooses a tag pair, tagpair(ϕ), that is $\geq$ tagpair(π); furthermore, if ϕ is a WRITE, then tagpair(ϕ) > tagpair(π).
  - Then we can't have $\phi < \pi$:
    - Since tagpair(ϕ) $\geq$ tagpair(π), the only way we could have $\phi < \pi$ is if tagpair(ϕ) = tagpair(π), ϕ is a WRITE and π is a READ (by definition of <).
    - But in this case, tagpair(ϕ) > tagpair(π), contradiction.

Condition 3

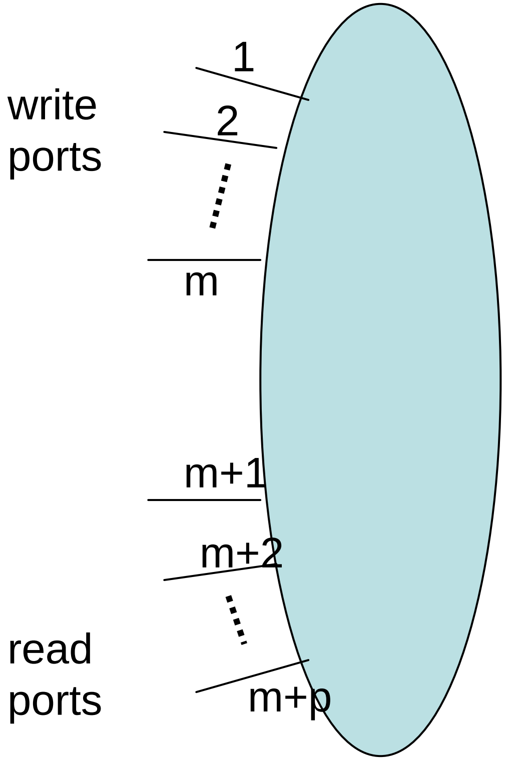
- Condition 3: WRITEs are ordered with respect to each other and with respect to all READs.
- Proof:
  - Follows because all WRITEs get distinct tagpairs.
  - Why distinct?
    - Different ports: Different indices.
    - Same port i:
      - WRITEs on port i are sequential.
      - Each WRITE by i reads its previous tag in its own diagonal variable x(i,i) and chooses a larger tag.
- Condition 4: LTTR
- Apply the Lemma, implies that V-A satisfies atomicity, as needed.

Complexity

- Shared memory size:
  - n^2 variables, each of unbounded size (because of x(i).tag).
- Time for read:
  - $\leq 2\ (m + p)$ shared memory accesses
  - $O((m + p)\ l)$ time
- Time for write:
  - Also $O((m + p)\ l)$

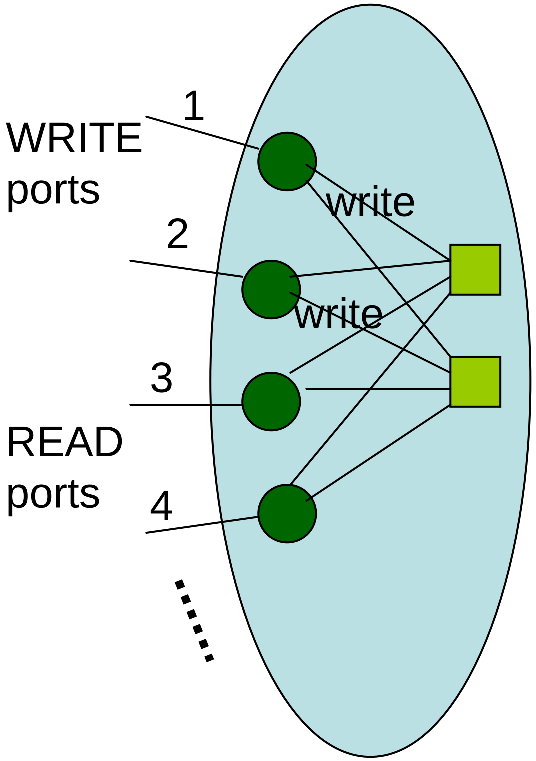
More on read/write atomic objects

- [Vitanyi, Awerbuch] algorithm is not too costly in time, but uses unbounded variables.
- Q: Can we implement multi-writer multi-reader atomic objects in terms of single-writer single-reader registers, using bounded variables?
- A: Yes. Several published algorithms:
  - [Burns, Peterson]
  - [Dolev, Shavit]
  - [Vidyasankar]
  - …
  - Bounded-tag algorithm in [Vitanyi, Awerbuch] incorrect.
- Fairly complicated, costly.
- Usually divide the problem into:
  - 1-writer multi-reader from 1-writer 1-reader.
  - Multi-writer multi-reader from 1-writer multi-reader.



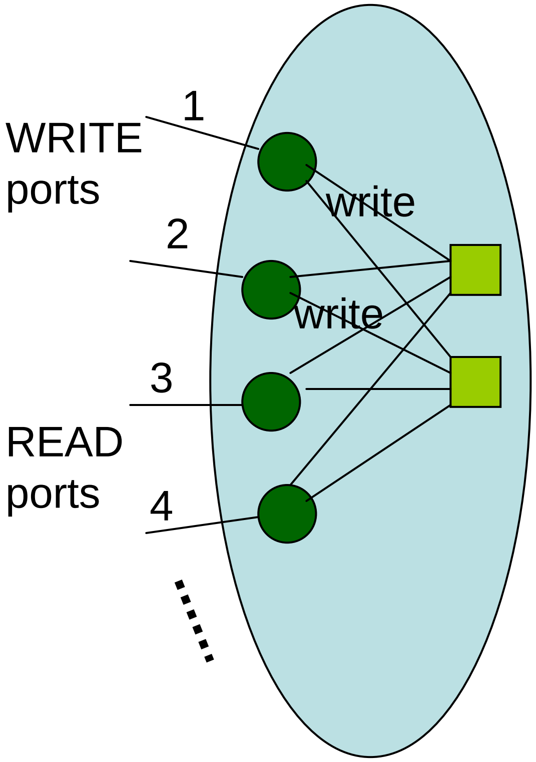
Bloom algorithm

- A simple special case, illustrates:
  - Typical difficulties that arise
  - Interesting proof methods
- 2-writer multi-reader register from 1-writer multi-reader registers
- Shared variables:
  - x(1), x(2), with:
    - val in V, initially v_0
    - tag in {0,1}, initially 0
  - x(1) written by WRITER 1, read by everyone
  - x(2) written by WRITER 2, read by everyone



Bloom algorithm

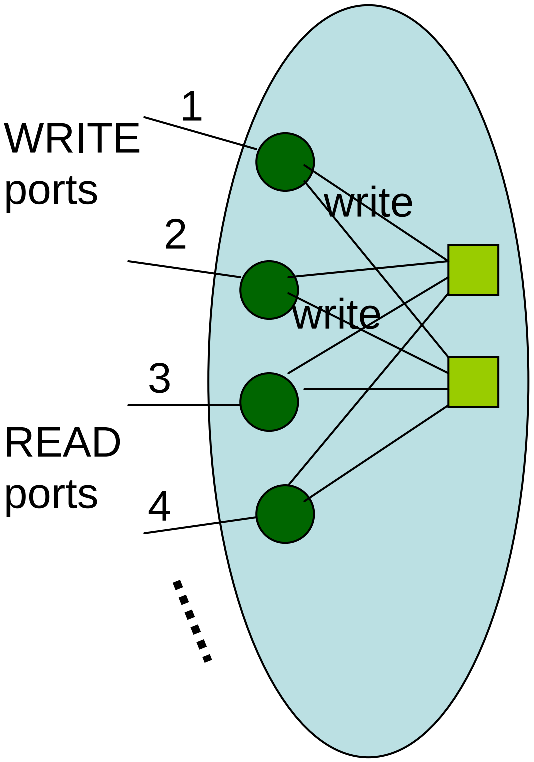
- WRITE(v)$_1$:
  - Read x(2).tag, say b
  - Write:
    - x(1).val := v,
    - x(1).tag := 1 - b
  - Tries to make tags unequal.
- WRITE(v)$_2$:
  - Read x(1).tag, say b
  - Write:
    - x(2).val := v,
    - x(2).tag := b
  - Tries to make tags equal.
- READ:
  - Read both registers.
  - If tags are unequal then reread and return x(1).val.
  - If tags are equal then reread and return x(2).val.



Correctness

- Well-formedness, wait-freedom: Clear
- Atomicity:
  - Could use:
    - Explicit serialization points, or
    - Partial-order lemma
  - Instead, use a simulation relation, mapping the algorithm to a simpler unbounded-tag version

Unbounded-tag algorithm



- Shared variables:
  - x(1), x(2), with:
    - val in V, initially v_0
    - tag, a natural number; initially x(1).tag = 0, x(2).tag = 1
- WRITE$(v)_1$:
  - Read x(2).tag, say t
  - Write x(1).val := v, x(1).tag := t + 1
- WRITE$(v)_2$:
  - Read x(1).tag, say t
  - Write x(2).val := v, x(2).tag := t + 1
- READ:
  - Read both registers, get tags t_1 and t_2.
  - If $| t_1 - t_2 | \leq 1$ then reread the register x(i) with the higher tag and return x(i).val.
  - Else reread and return either (choose nondeterministically)

Why the nondeterministic choice?

- Extra generality needed to make the simulation relation from the Bloom algorithm work correctly.
- The integer algorithm works even with the nondeterministic choice.
- The nondeterminism doesn’t significantly complicate the integer algorithm.
- Doesn’t complicate the proof at all; in fact, makes it a little easier to see what’s needed.

Proof for integer algorithm

- Invariant:
  - x(1).tag is always even
  - x(2).tag is always odd
  - | x(1).tag – x(2).tag | = 1
- Well-formedness, wait-freedom: Clear
- Atomicity:
  - E.g., use the partial-order lemma.
  - Define the partial order < using the tags:
    - Order WRITEs by the tags they write.
    - Break ties (must be sequential operations by the same WRITER) in temporal order.
    - Insert each READ just after the WRITE whose value it gets.
  - Check Conditions 1-4 of the partial order lemma.

E.g., Condition 2

- Condition 2: If the response for π precedes the invocation for ϕ in β, then we can't have $\phi < \pi$.
- Proof:
  - Suppose we have:

  - Consider cases based on the types of π and ϕ.
  - Most interesting case: π is a WRITE, ϕ is a READ.
  - Suppose WRITE π is done by WRITER i, writes tag t.
  - Must show we can't have $\phi < \pi$.
  - That is, we must show that READ ϕ must return either the result written by WRITE π or one by some other WRITE ψ with $\pi < \psi$.

Proof of Condition 2, cont'd

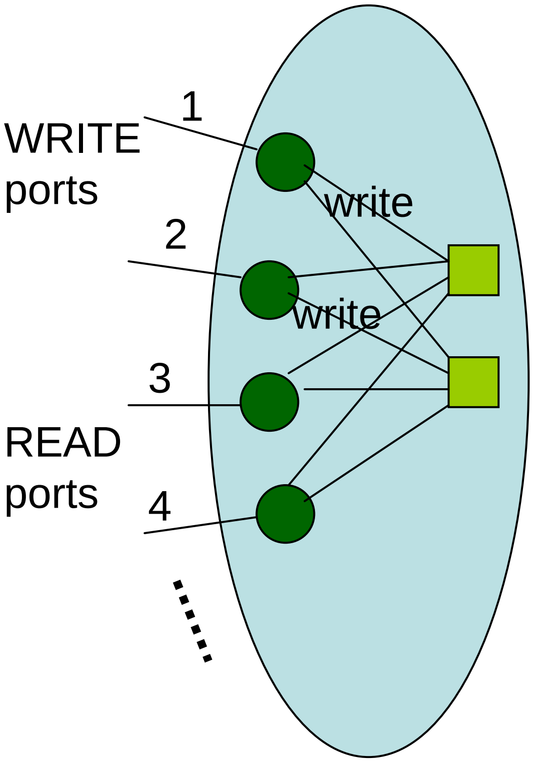
π ϕ

WRITE by i, tag t READ

- Show ϕ must return either the result written by π or one by some other WRITE ψ with $\pi < \psi$.
- When READ ϕ is invoked, x(i).tag $\geq$ t, by monotonicity.
- At that point, x(2-i).tag $\geq$ t – 1, by invariant.
- Only possible problem: ϕ returns the value of a WRITE with tag t – 1.
- Suppose it does; then ϕ must see x(2-i).tag = t – 1 on its initial read of x(2-i), and also on the third read.
- What might ϕ see for x(i).tag on its initial read of x(i)?
- 2 possibilities:
  - ϕ sees x(i).tag = t.
    - Then it would reread x(i), contradiction.
  - ϕ sees x(i).tag > t.
    - Then by the time it sees this, x(2-i).tag is already > t – 1.
    - So ϕ couldn't see x(2-i).tag = t-1 on the third read, contradiction.

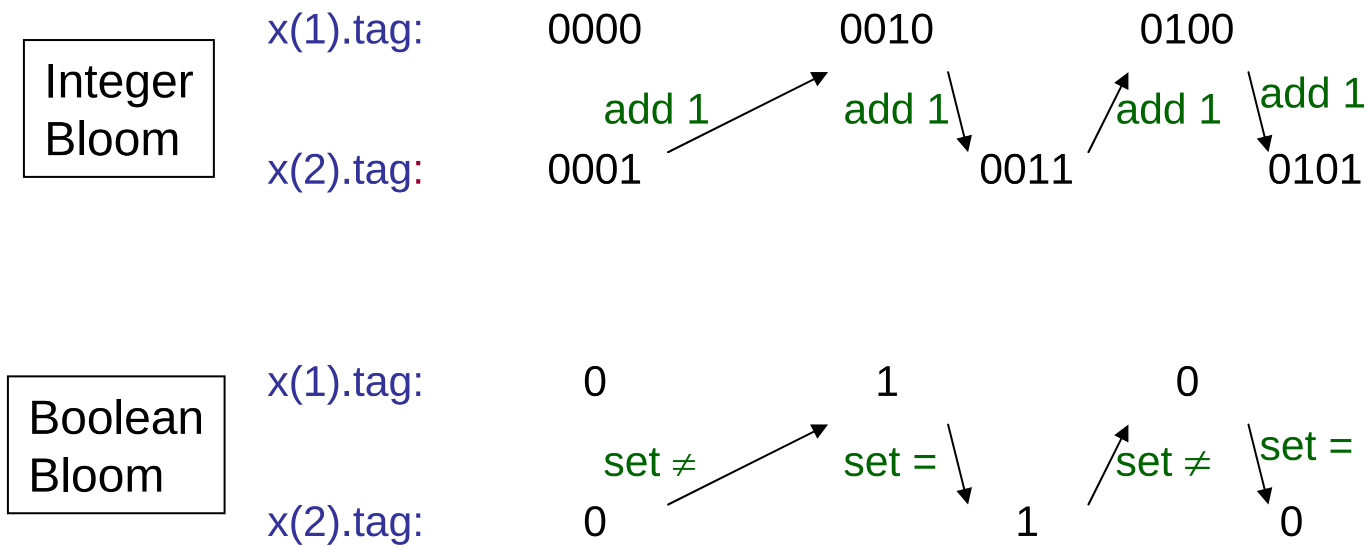
Where are we?

- Integer version of Bloom algorithm (IB) implements a 2-writer multi-reader atomic object from 1-writer multi-reader registers.
- Now show that the original Boolean Bloom algorithm (BB) implements the integer version.
- Use a simulation relation from BB to IB.



Simulation relation from BB to IB

- If s is a state of Boolean Bloom system, u a state of IntegerBloom system, then define (s,u) in R exactly if:
  – Each occurrence of a tag in BB is exactly the second low-order bit of the corresponding tag occurrence in IB.
  – All other state components are identical in the two systems.
- Note this is multivalued: Each state of BB corresponds to many states of IB.
- Example:



R is a simulation relation

- Proof:
  - Start states related:
    - Second low-order bit of 0000 is 0
    - Second low-order bit of 0001 is 0
  - Step condition:
    - For any step (s, π, s′) in BB, and any state u of IB such that (s,u) in R, the corresponding step of IB is almost the same:
      - Same kind of action, same process, same register…
    - Must show:
      - The IB step is enabled, and
      - The state correspondence is preserved.
    - Key facts:
      - The write step of a WRITE preserves the state correspondence.
      - The third read of a READ is always enabled in IB (on same register).

First key fact

- The write step of a WRITE operation preserves the state correspondence.
- Proof:
  - E.g., a WRITE by process 1.
  - Writes to x(1).tag:
    - 1-b, where b is the value read from x(2).tag, in BB.
    - t+1, where t is the value read from x(2).tag, in IB.
  - By relation R on the pre-states, b is the second low-order bit of t.
  - We need to show that 1-b is the second low-order bit of t+1.
  - Follows because:
    - t is odd (by an invariant, process 2's tag is always odd), and
    - Incrementing an odd number always flips the second low-order bit.
  - Example:
    - t = 101, b = 0
    - t + 1 = 110, 1-b = 1

  - Argument for process 2 is similar.

Second key fact

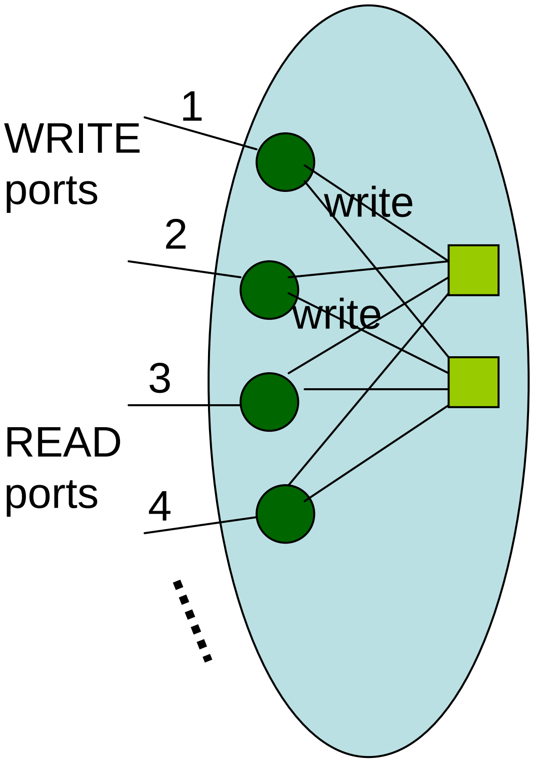
- IB allows reading the same third register as BB.
- Proof:
  - Choice of register is based on the tags read in the first two reads.
    - In BB: Read x(1).tag = b_1, x(2).tag = b_2.
    - In IB: Read x(1).tag = t_1, x(2).tag = t_2.
    - By state correspondence, b_1 and b_2 are the second low-order bits of t_1 and t_2, respectively.
  - Consider cases:
    - $t_1 = t_2 + 1$
      - Then IB reads from x(1) on third read.
      - Since t_1 is even and t_2 is odd, second low-order bits are unequal.
      - Thus, $b_1 \neq b_2$, and so BB also reads from x(1) on third read.
    - $t_2 = t_1 + 1$
      - Symmetric, both read from x(2) on the third read.
    - Neither of these holds.
      - Then IB allows either to be read.

Now where are we?

- Argued simulation relation from Bloom to IB.
- Implies every trace of Bloom is a trace of IB.
- Earlier, showed that IB satisfies atomicity.
- Trace inclusion implies that Bloom also satisfies atomicity.

- Theorem: The Bloom algorithm implements a 2-writer multi-reader atomic object from 1-writer multi-reader registers.

- Unfortunately…
- This algorithm doesn't appear to extend to three or more writers.
- Algorithms exists that do this, but they are much more complicated.



Next time…

- Wait-free computability
- The wait-free consensus hierarchy
- Reading:
  - [Herlihy, Wait-free synchronization],
  - [Attiya, Welch, Chapter 15]

6.852J / 18.437J Distributed Algorithms
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