Color

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Many slides courtesy of Victor Ostromoukhov, Leonard McMillan, Bill Freeman, Fredo Durand

Does color puzzle you?

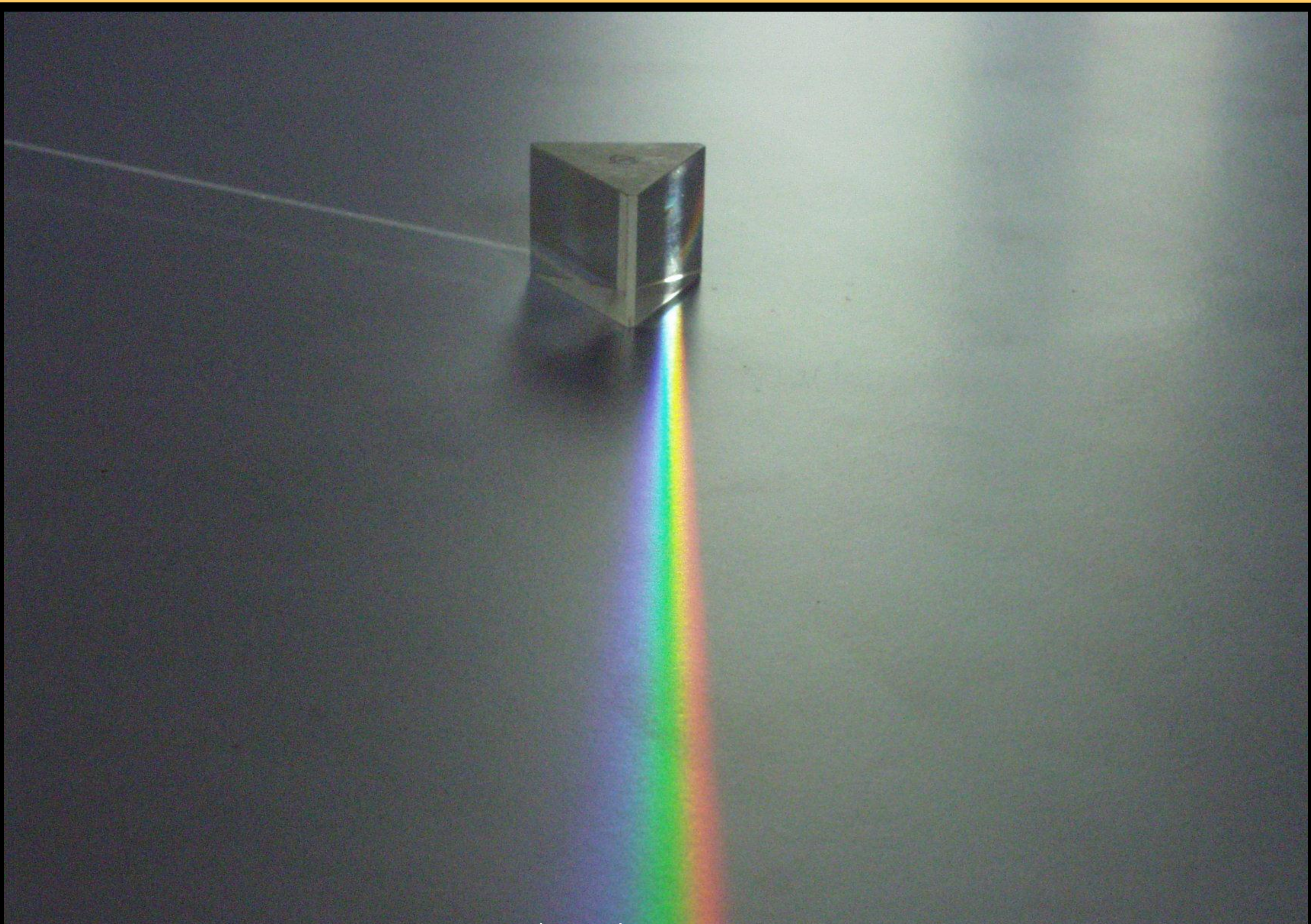
Answer

- It's all linear algebra

Plan

- Spectra
- Cones and spectral response
- Color blindness and metamers
- Color matching
- Color spaces

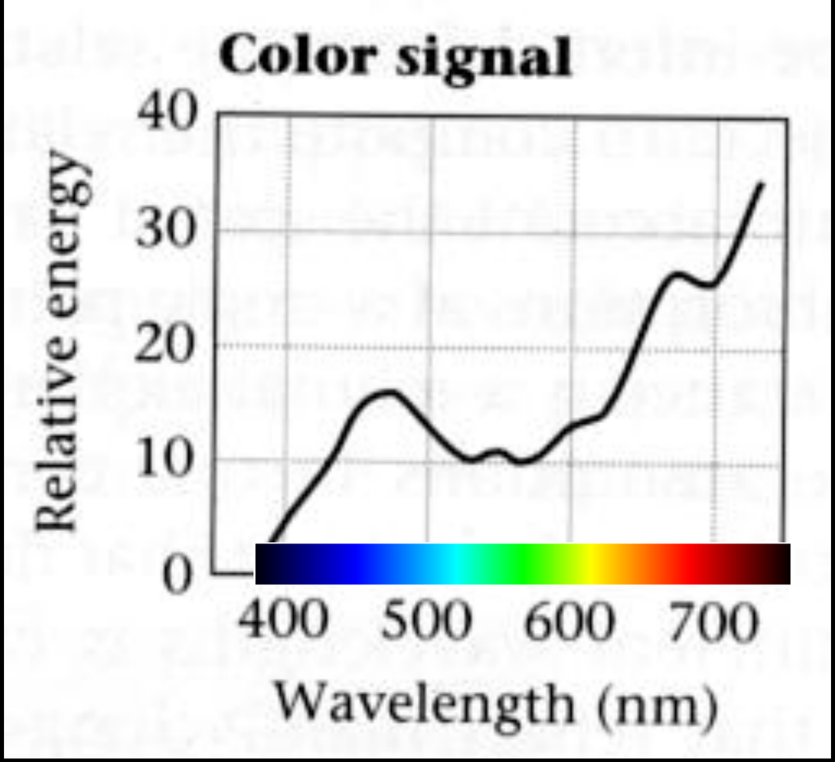
Color



Spectrum

Light is a wave
Visible: between 450 and 700nm

Spectrum



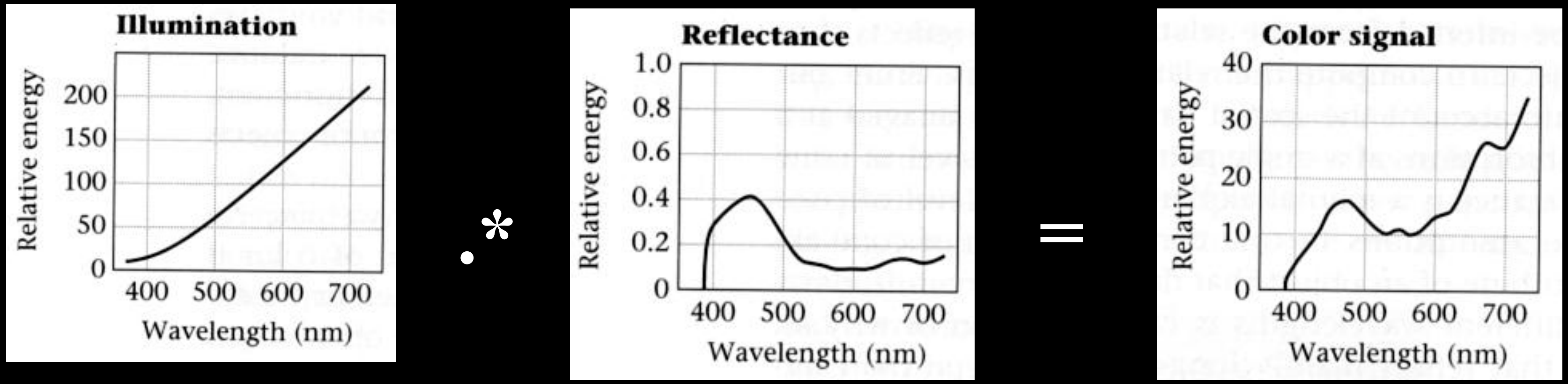
Light is characterized by its spectrum:
the amount of energy at each wavelength
This is a full distribution:
one value per wavelength
(infinite number of values)

Light-Matter Interaction

Where spectra come from:
- light source spectrum
- object reflectance (aka spectral albedo)

get multiplied wavelength by wavelength

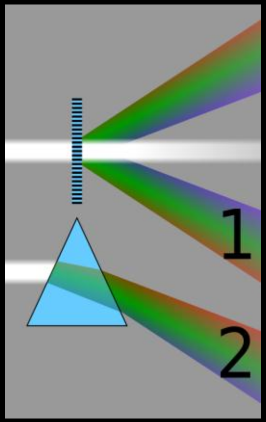
There are different physical processes that explain this multiplication
e.g. absorption, interferences

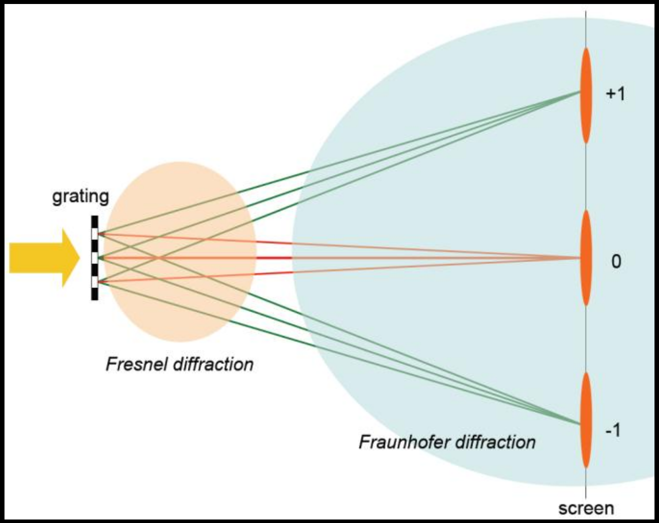


Foundations of Vision, by Brian Wandell, Sinauer Assoc., 1995

Spectrum demo

- Diffraction grating:
  - shifts light as a function of wavelength
  - Allows you to see spectra
  - In particular, using a slit light source, we get a nice band showing the spectrum
- See the effect of filters
- See different light source spectra

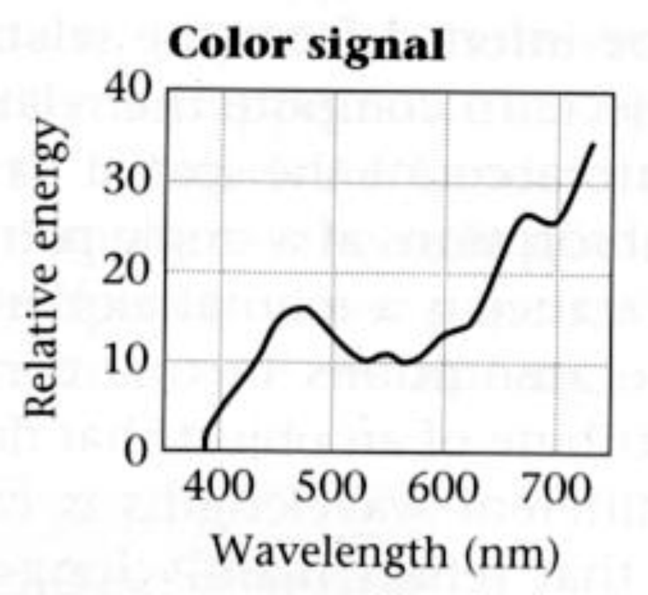




Questions?

So far, physical side of colors: **spectra**

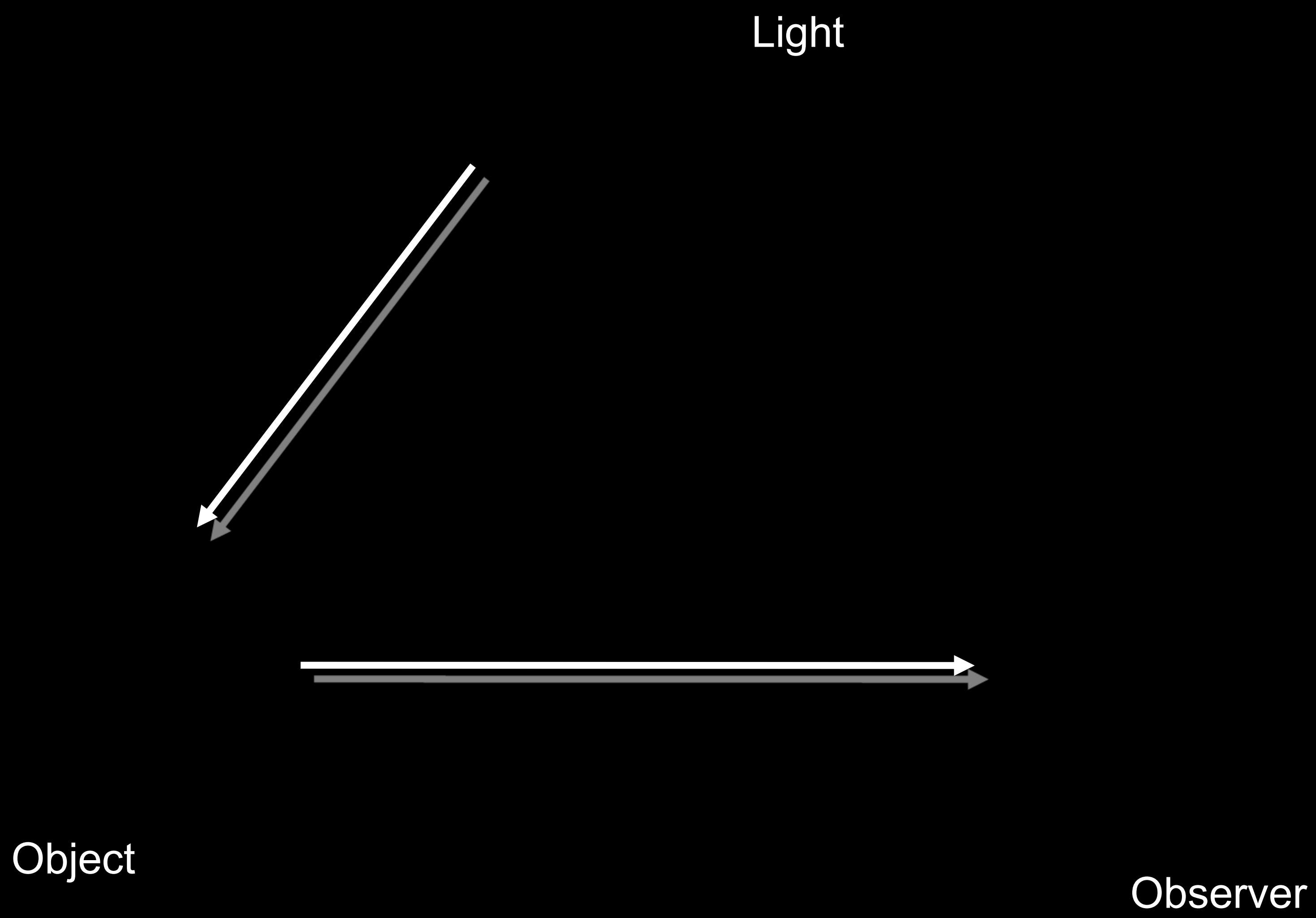
an infinite number of values
(one per wavelength)



Plan

- Spectra
- **Cones and spectral response**
- Color blindness and metamers
- Color matching
- Color spaces

What is Color?



What is Color?

Illumination

Stimulus

Reflectance

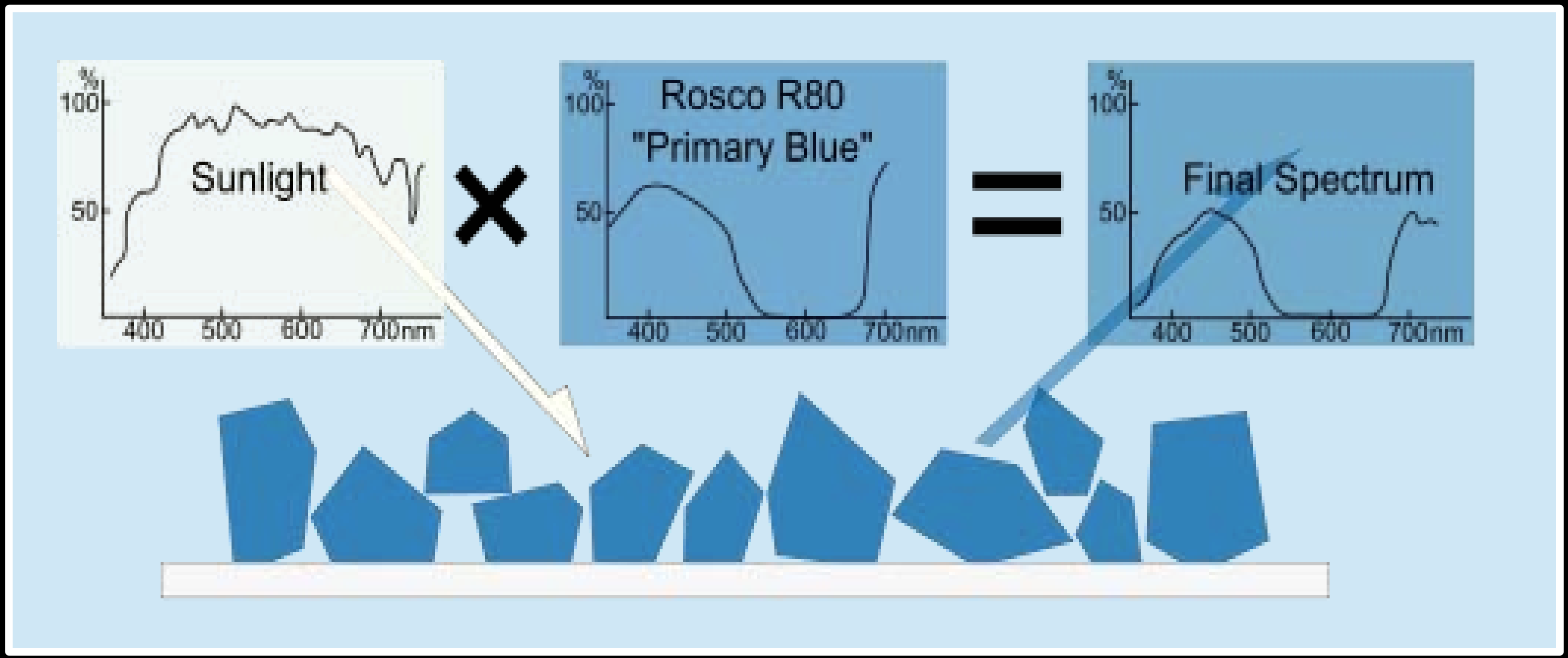
Cone
responses

What is Color?

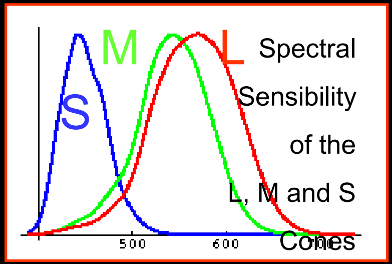
Light Illumination

Object Reflectance

Final stimulus

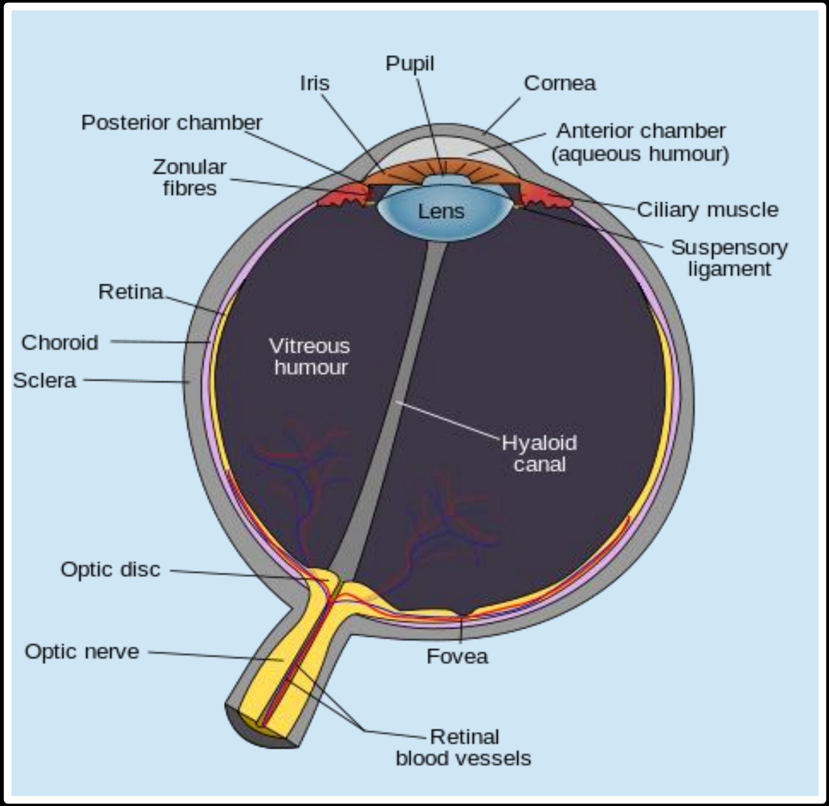


Then the cones in the eye interpret the stimulus



Cones

- We focus on low-level aspects of color
  - Cones and early processing in the retina
- We won't talk about rods (night vision)



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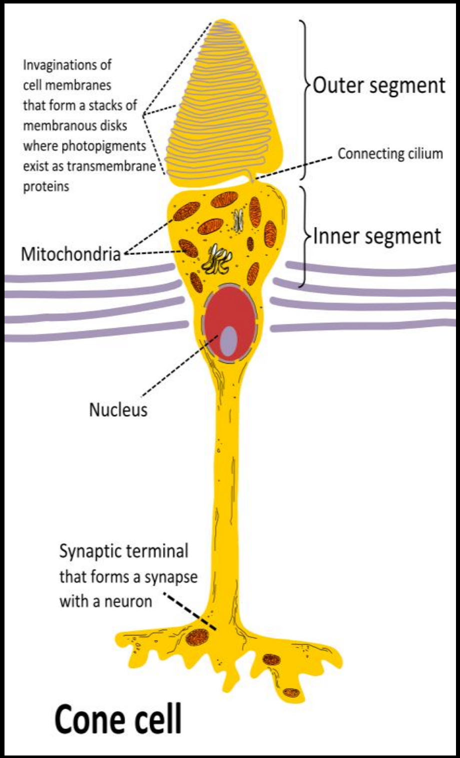
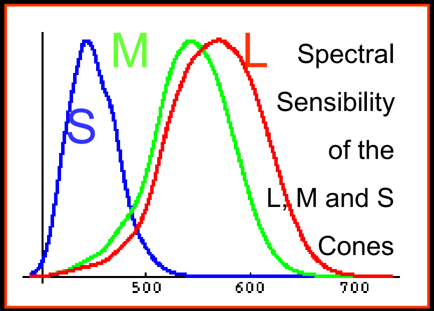


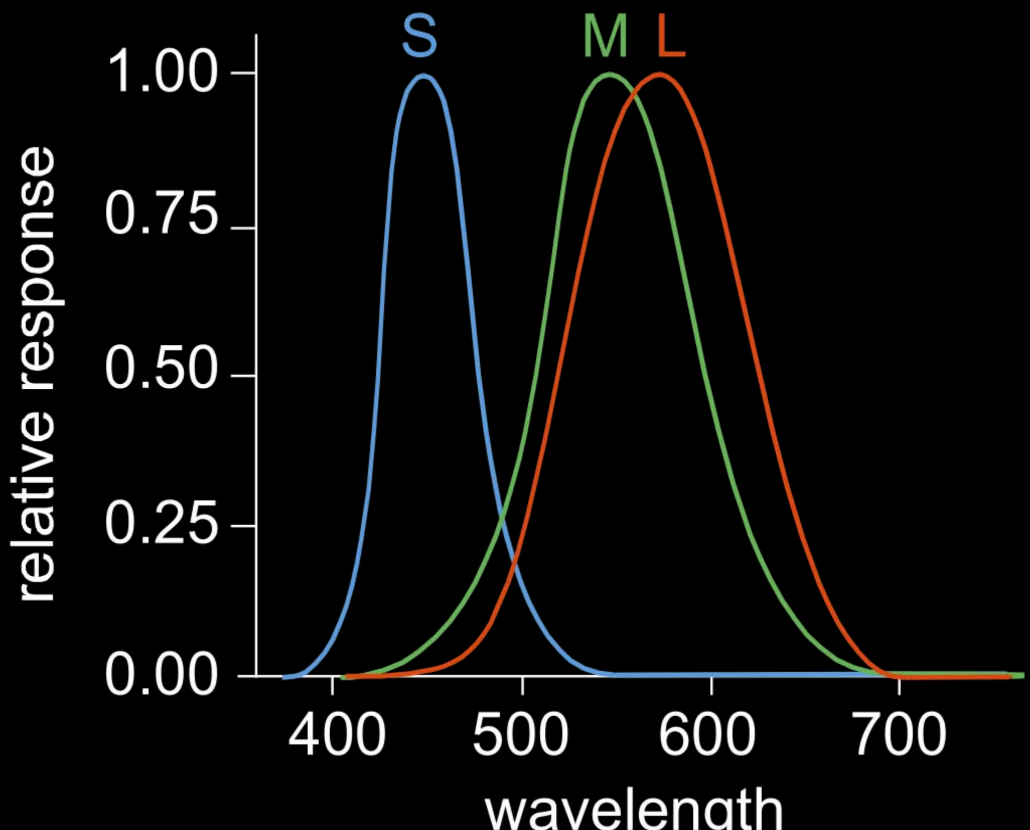


Summary (and time for questions)

- Spectrum: infinite number of values
  - can be multiplied
  - can be added
- Light spectrum multiplied by reflectance spectrum
  - spectrum depends on illuminant
- Human visual system is complicated

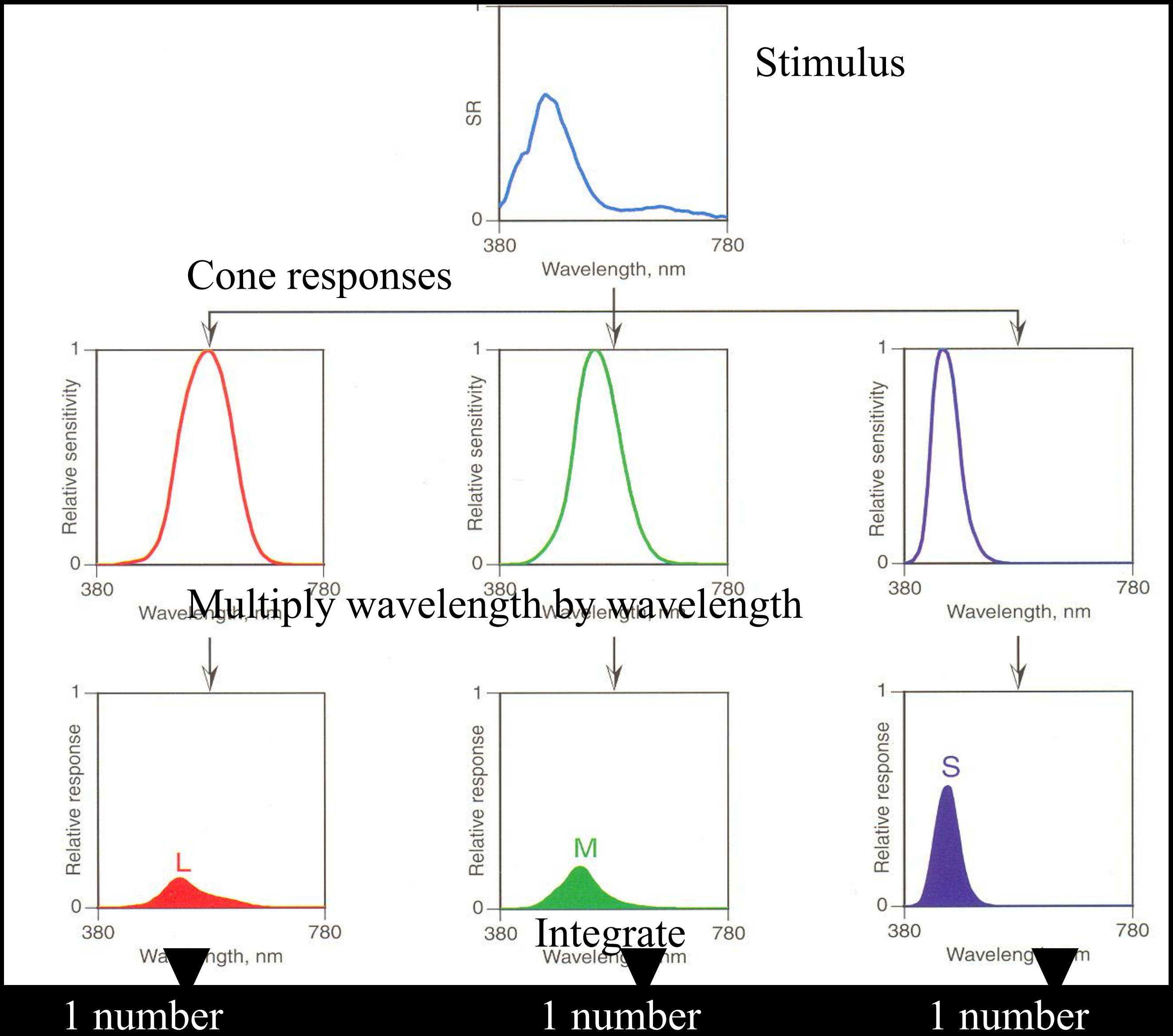
Cone spectral sensitivity

- Short, Medium and Long wavelength
- Response for a cone
  $= \int_\lambda \text{stimulus}(\lambda) * \text{response}(\lambda)\, d\lambda$



Cone response

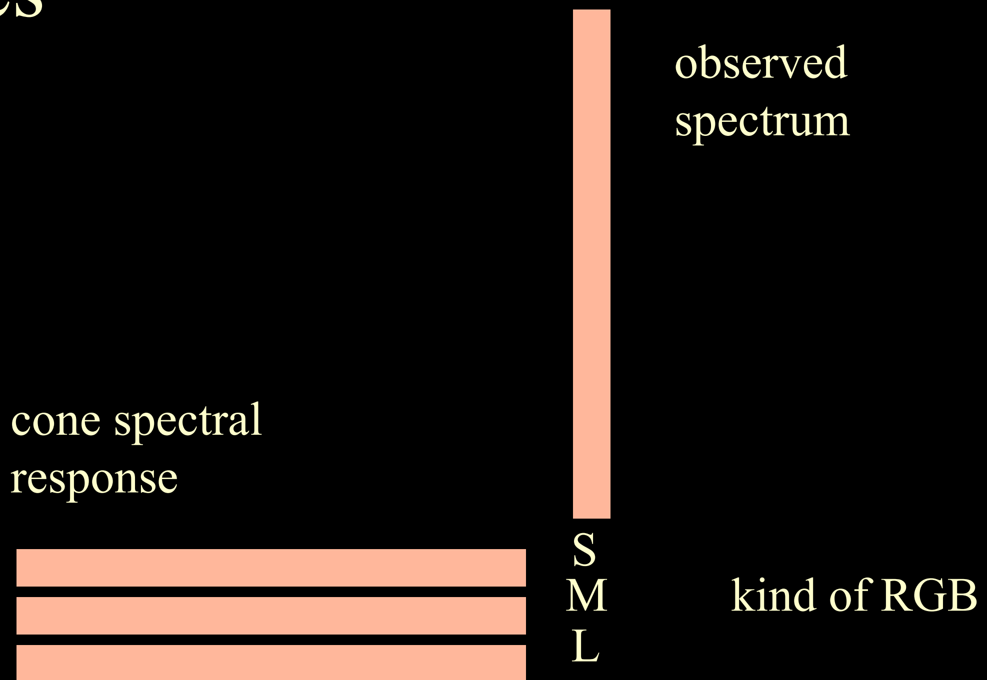
Start from infinite number of values (one per wavelength)



End up with 3 values (one per cone type)

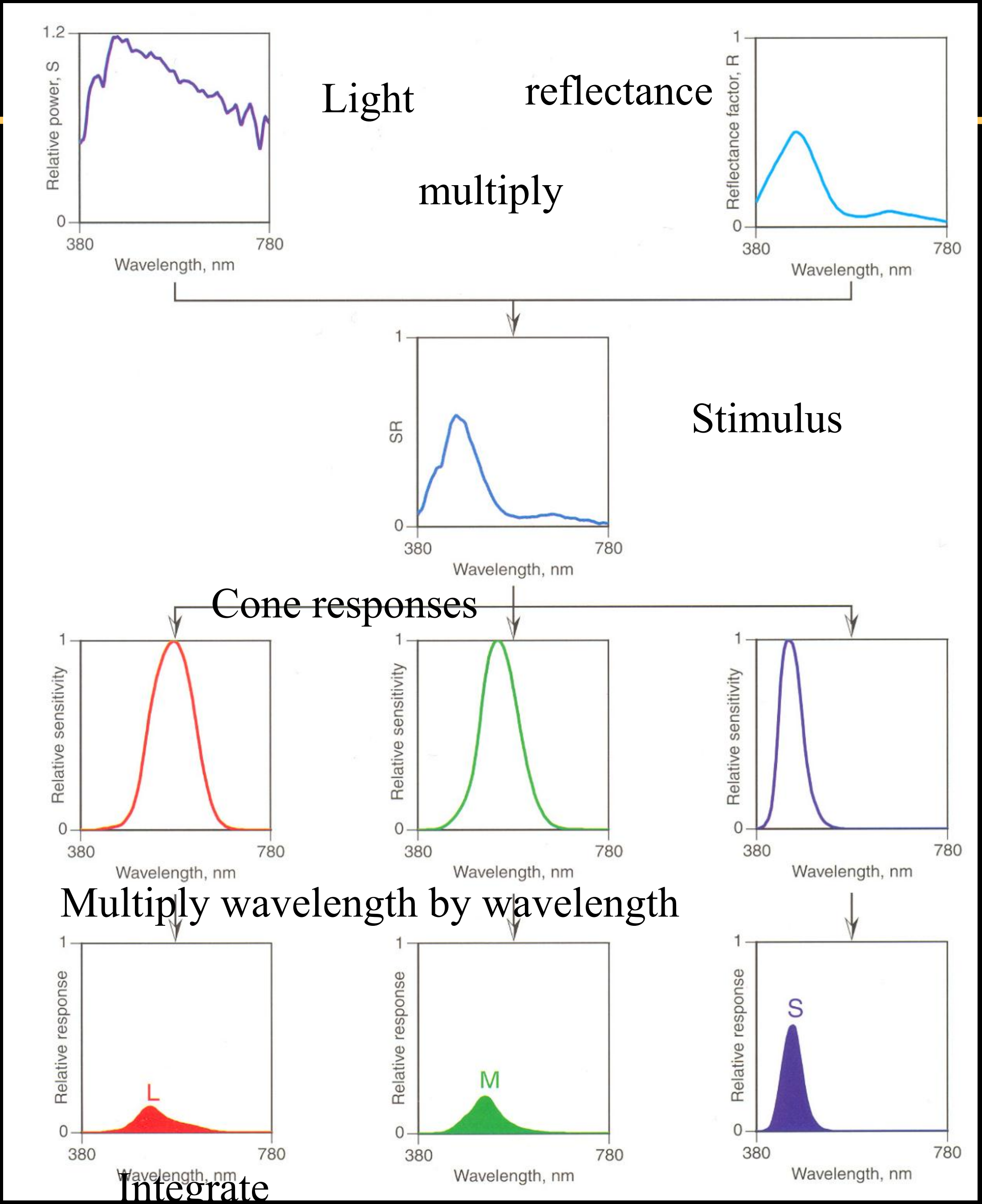
For matrix lovers

- Spectrum: big long vector size N where N=∞
- Cone response: 3xN matrix of individual responses



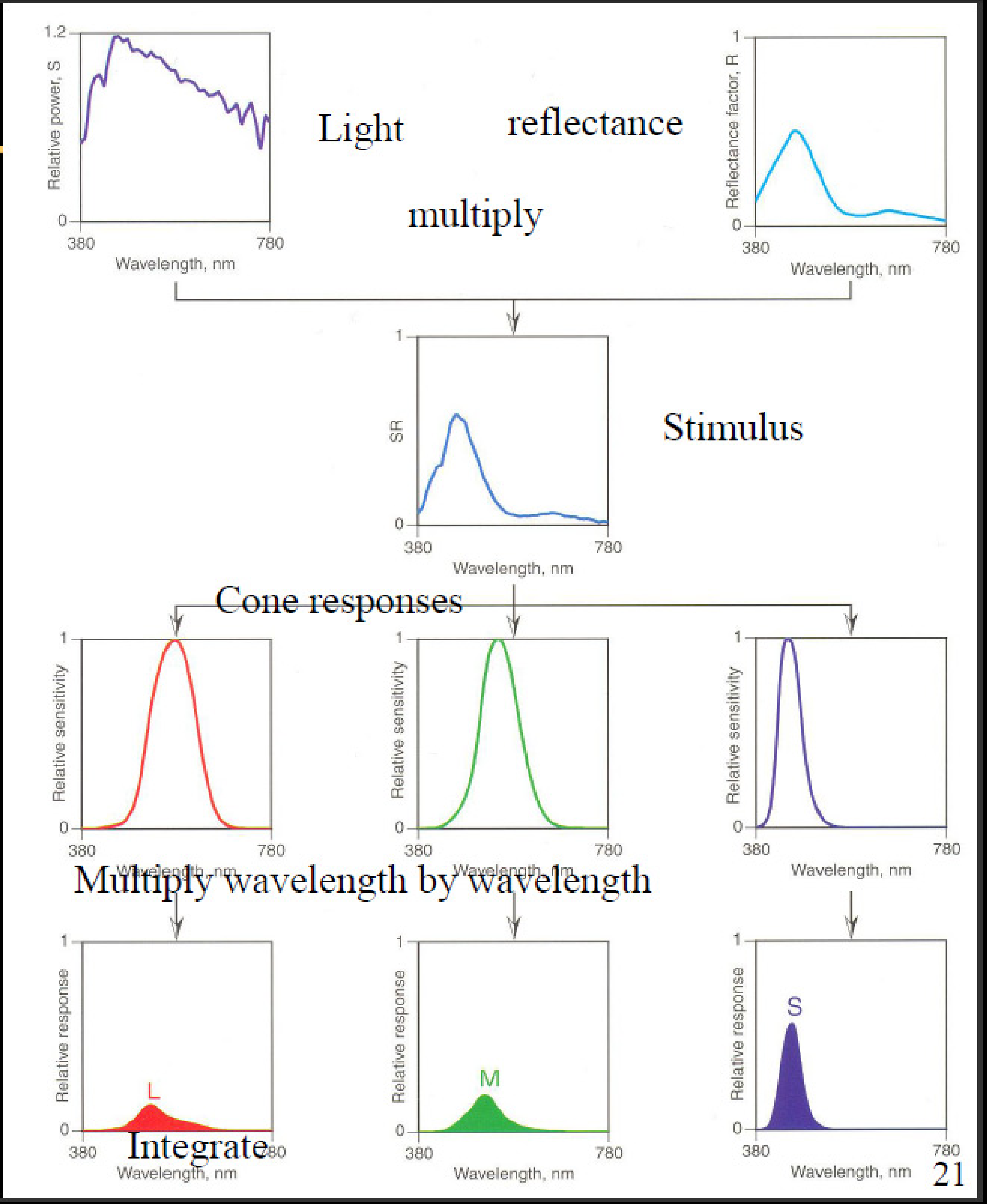
Big picture

- It's all linear!



Big picture

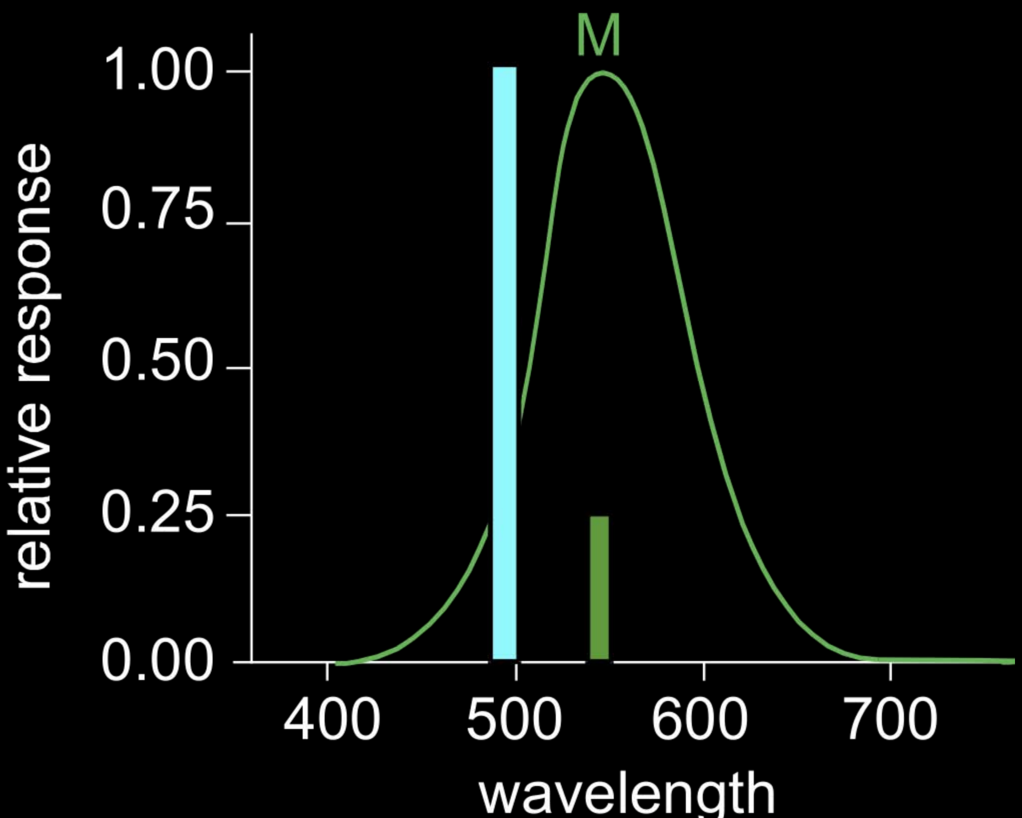
- It's all linear!
  - multiply
  - add
- But
  - non-orthogonal basis
  - infinite dimension
  - light must be positive
- Depends on light source



Questions?

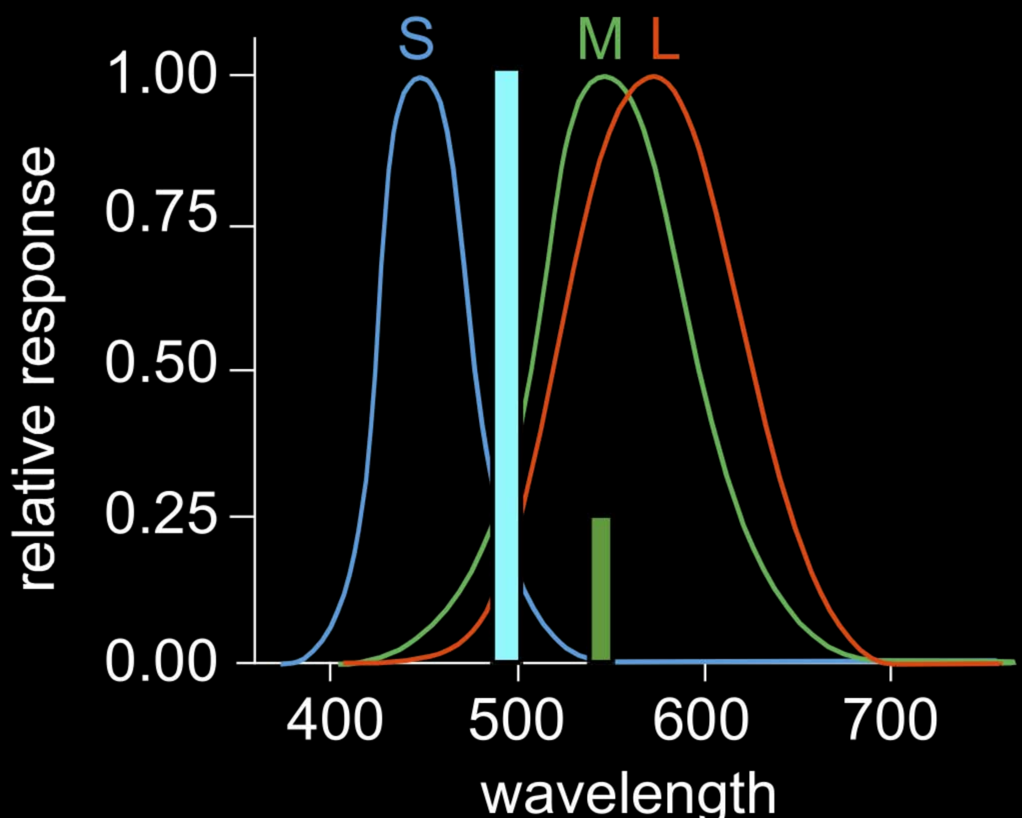
A cone does not "see" colors

- Different wavelength, different intensity
- Same response



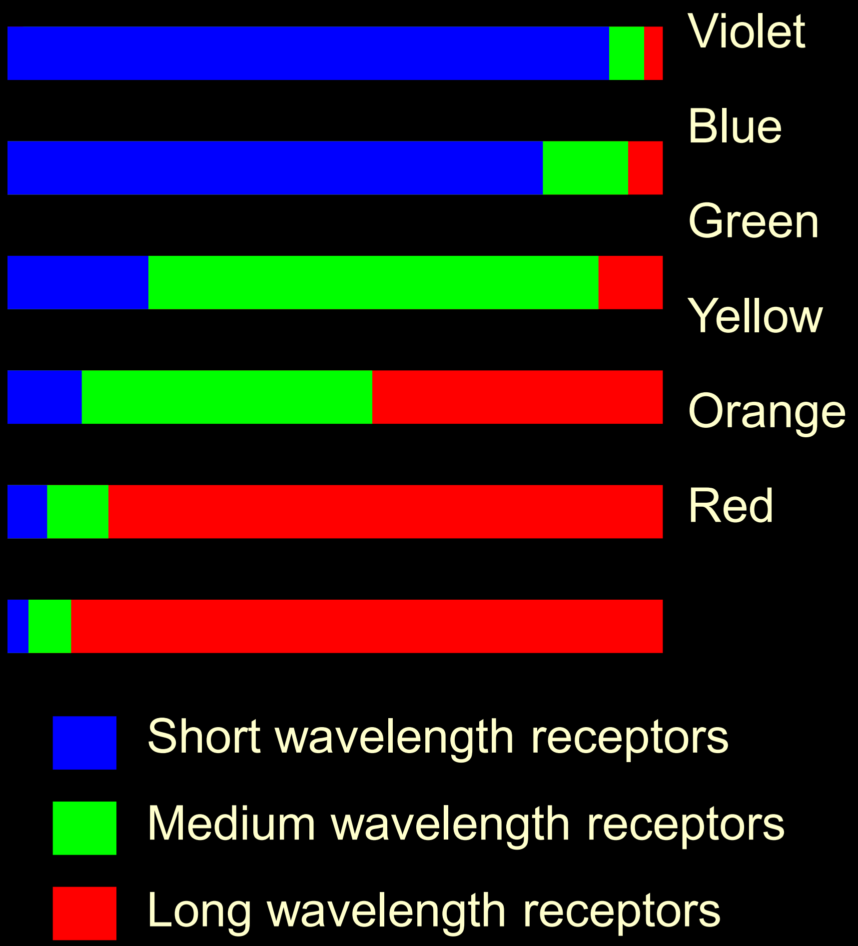
Response comparison

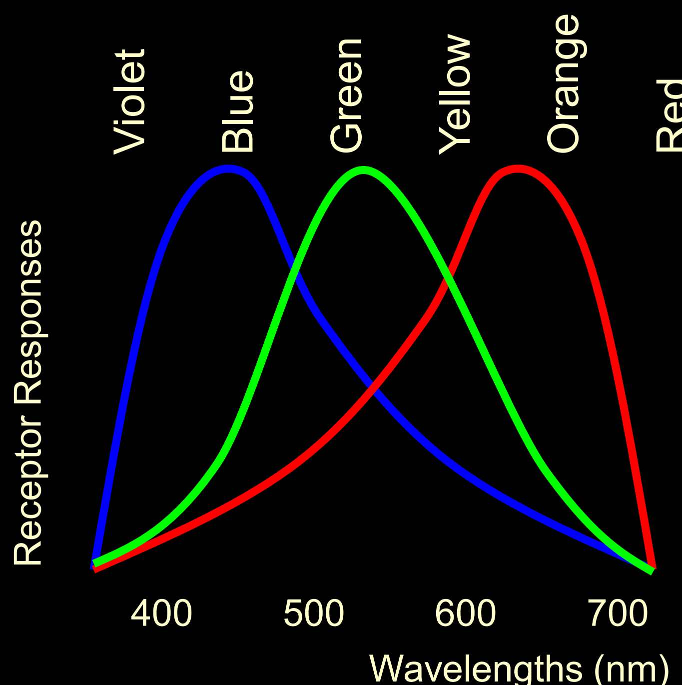
- Different wavelength, different intensity
- But different response for different cones



von Helmholtz 1859: Trichromatic theory

- Colors as relative responses (ratios)





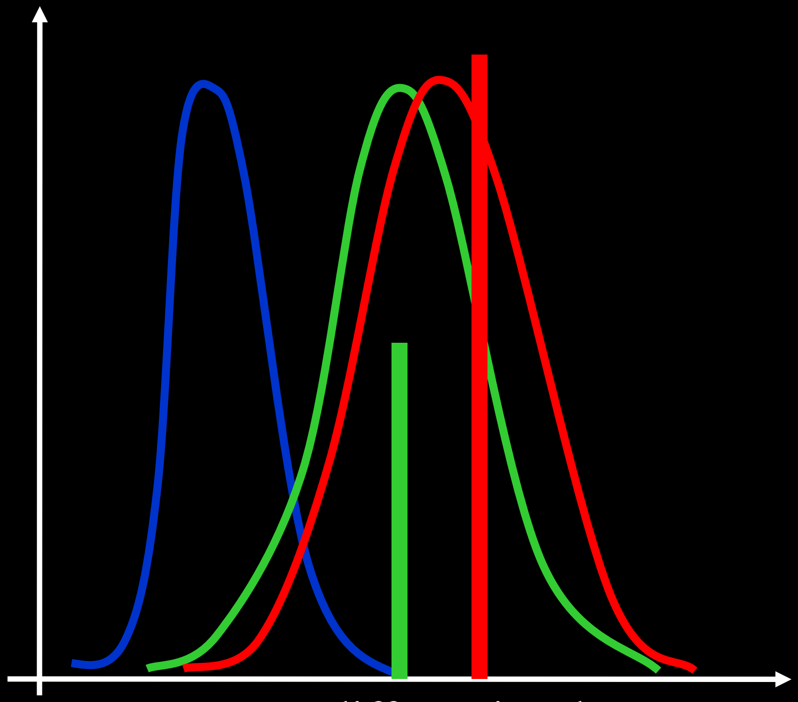
Questions?

Plan

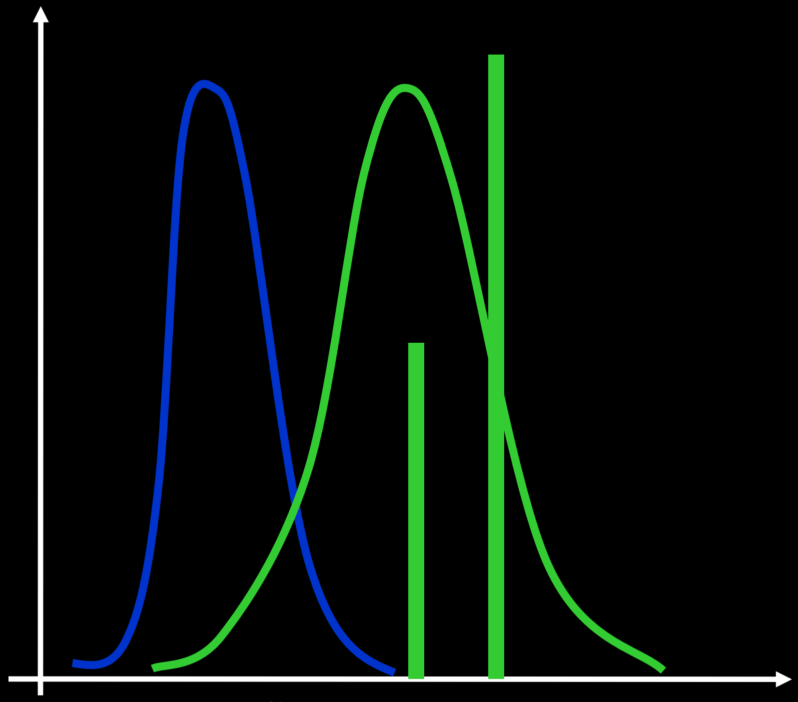
- Spectra
- Cones and spectral response
- **Color blindness and metamers**
- Color matching
- Color spaces

Color blindness

- Classical case: 1 type of cone is missing (e.g. red)
- Makes it impossible to distinguish some spectra



differentiated

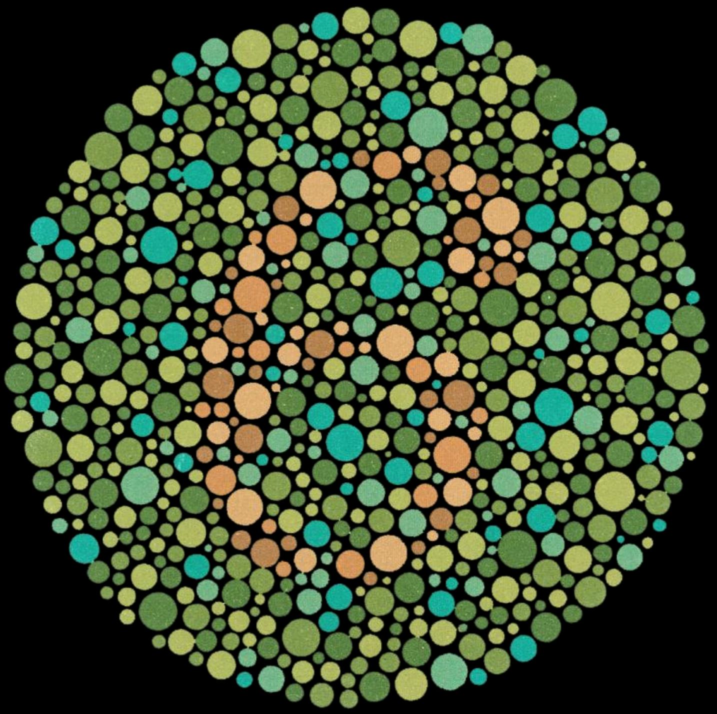


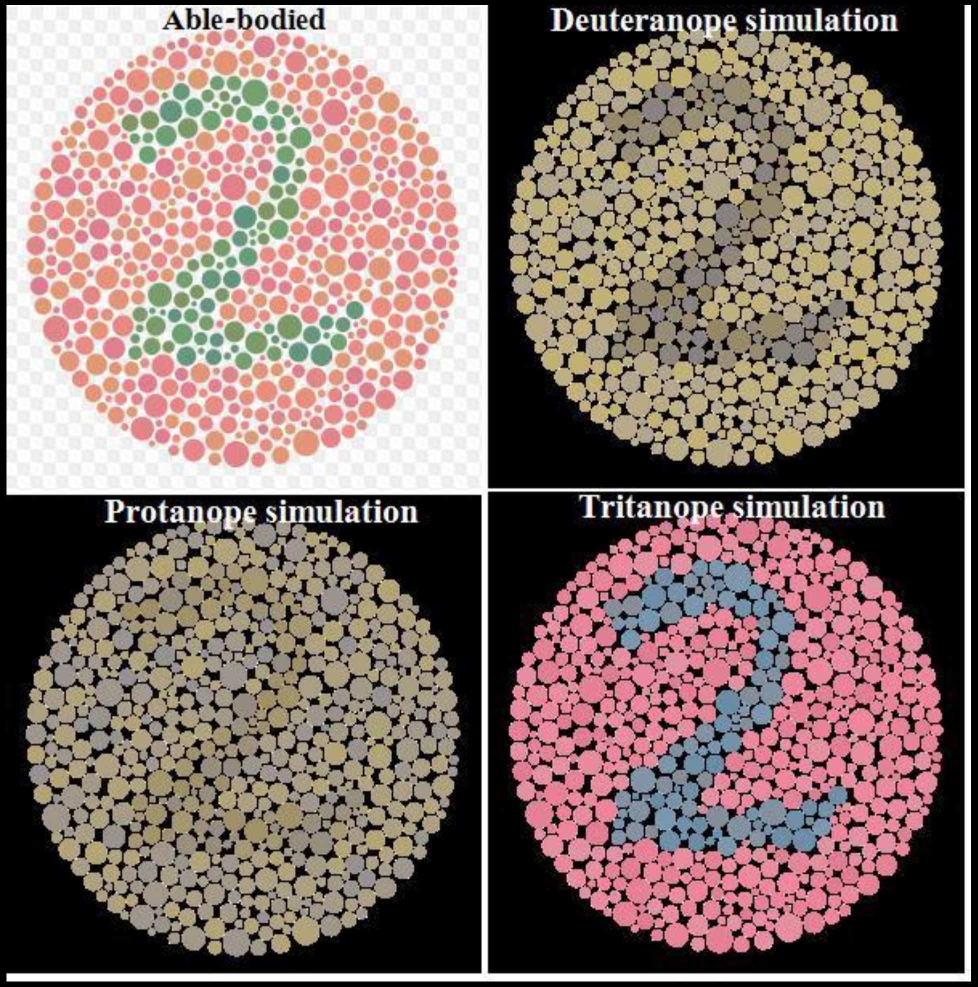
Same responses

Color blindness – more general

- Dalton
- 8% male, 0.6% female
- Genetic
- Dichromate (2% male)
  - One type of cone missing
  - L (protanope), M (deuteranope), S (tritanope)
- Anomalous trichromat
  - Shifted sensitivity

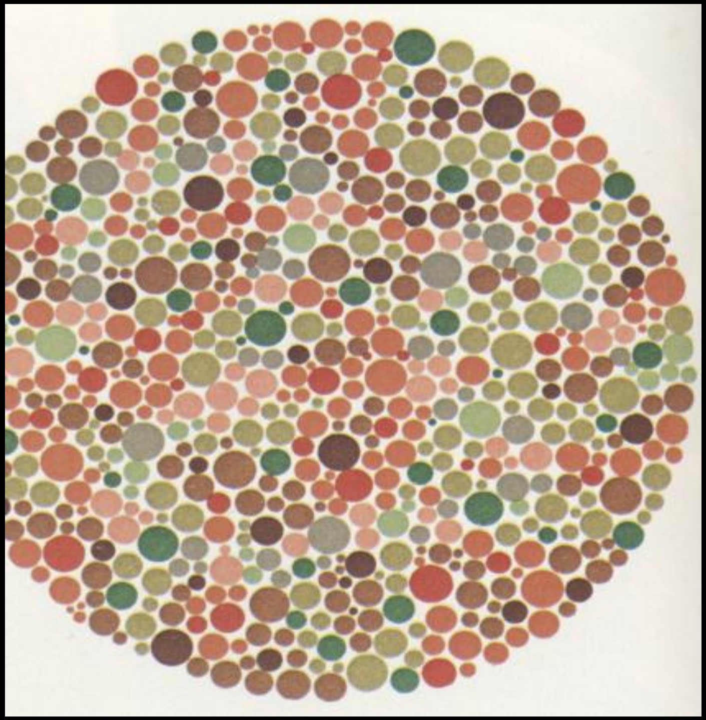
Color blindness test

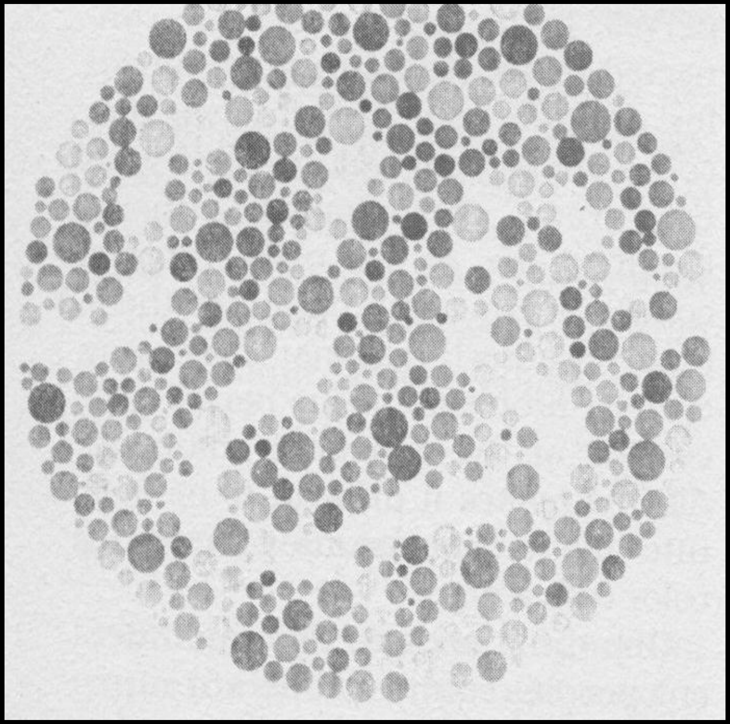




Color blindness test

- Maze in subtle intensity contrast
- Visible only to color blinds
- Color contrast overrides intensity otherwise



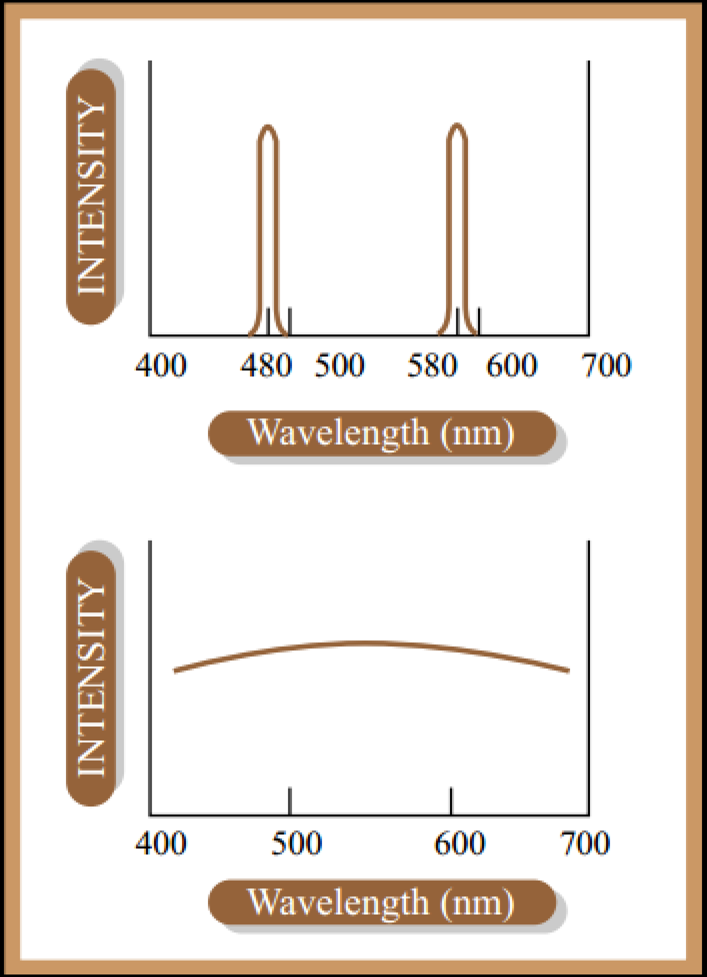


Questions?

- Links:
  - Vischeck shows you what an image looks like to someone who is colorblind.
  - http://www.vischeck.com/vischeck/
  - Daltonize, changes the red/green variation to brightness and blue/yellow variations.
  - http://www.vischeck.com/dalton
  - http://www.vischeck.com/daltonize/runDaltonize.php

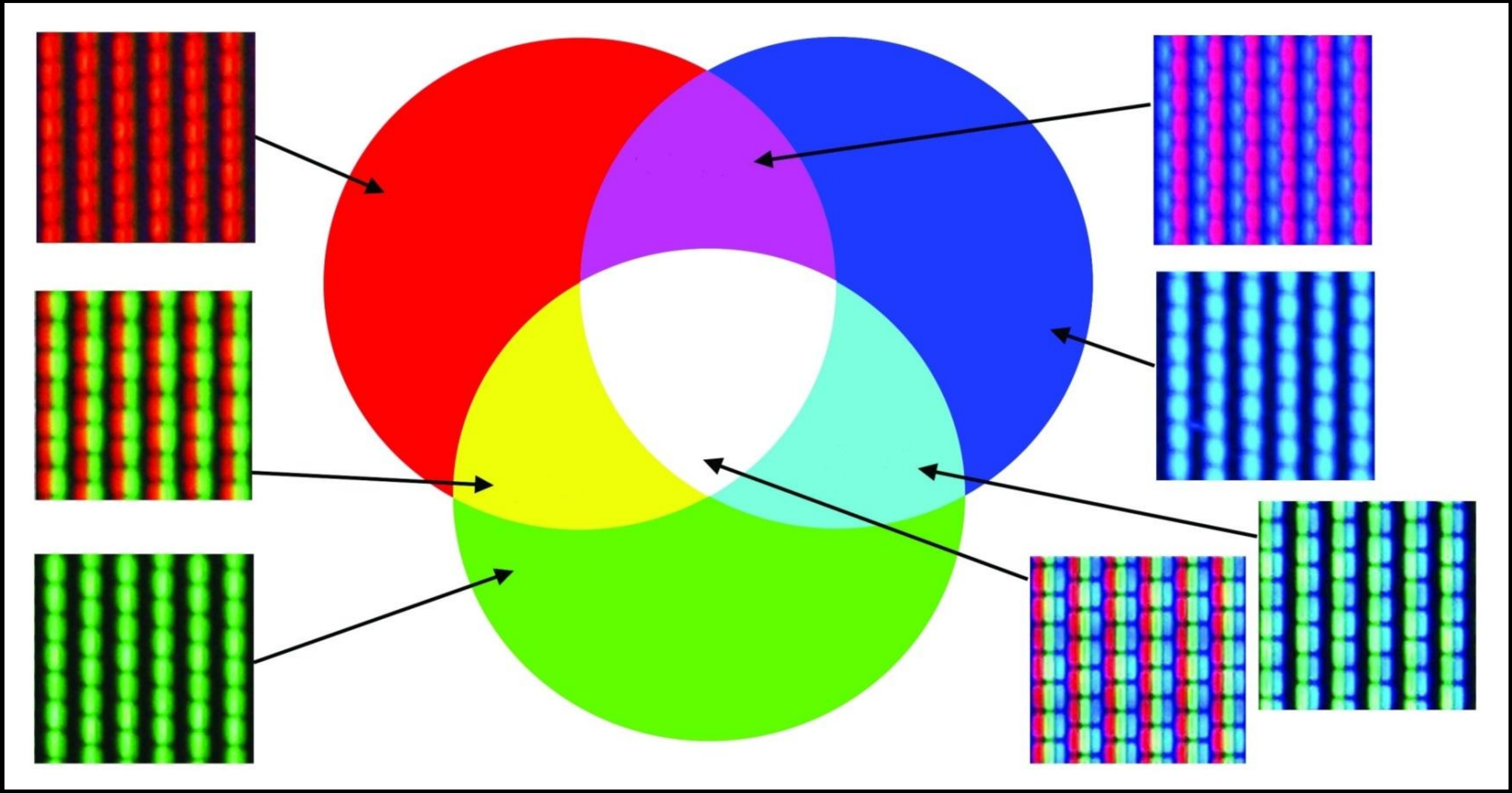
Metamers

- We are all color blind!
- These two different spectra elicit the same cone responses
- Called metamers



Good news: color reproduction

- 3 primaries are (to a first order) enough to reproduce all colors



Recap

- Spectrum: infinite number of values
- projected according to cone spectral response
  => 3 values
- metamers: spectra that induce the same response (physically different but look the same)

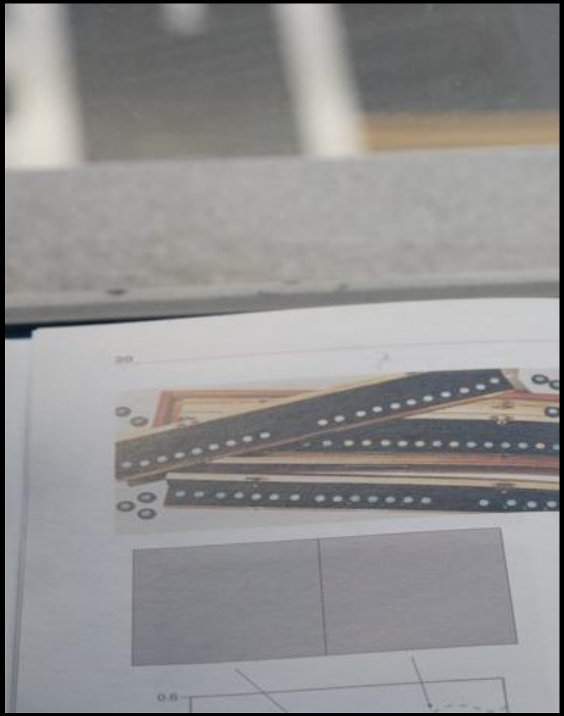
- Questions?

Metamerism & light source

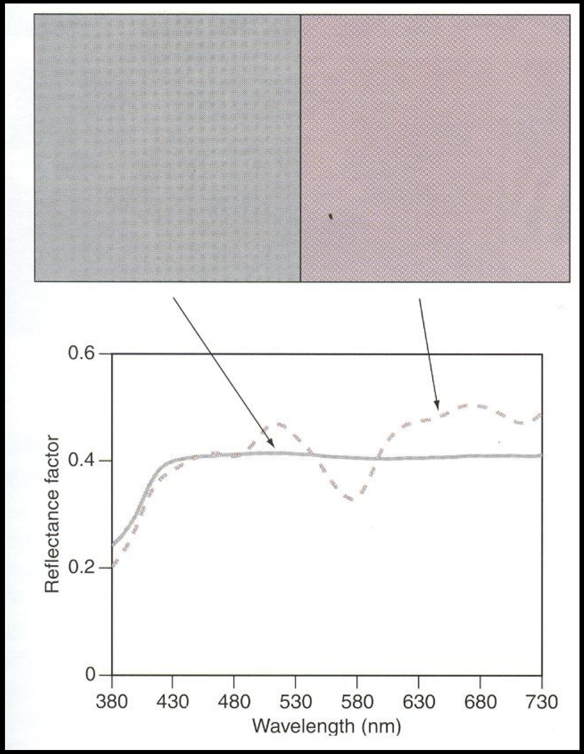
- Metamers under a given light source
- May not be metamers under a different lamp

Illuminant metamerism example

- Two grey patches in Billmeyer & Saltzman's book
  look the same under daylight
  but different under neon or halogen
  (& my camera agrees ;-)



Daylight



Scan (neon)

Hallogen

Bad consequence: cloth matching

- Clothes appear to match in store (e.g. under neon)
- Don’t match outdoor

Recap

- Spectrum is an infinity of numbers
- Projected to 3D cone-response space
  - for each cone, multiply per wavelength and integrate
  - a.k.a. dot product
- Metamerism: infinite-D points projected to the same 3D point
  (different spectrum, same perceived color)
  - affected by illuminant
  - enables color reproduction with only 3 primaries

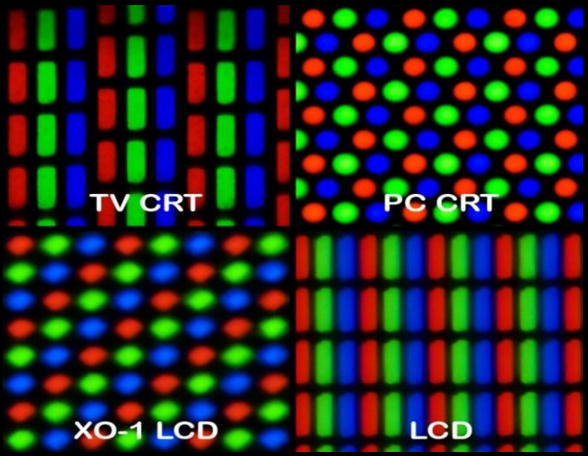
Questions?

Analysis & Synthesis

- Now let's switch to technology
- We want to measure & reproduce color as seen by humans
- No need for full spectrum
- Only need to match up to metamerism

Analysis & Synthesis

- Focus on additive color synthesis
- We'll use 3 primaries (e.g. red green and blue) to match all colors



- What should those primaries be?
- How do we tell the amount of each primary needed to reproduce a given target color?

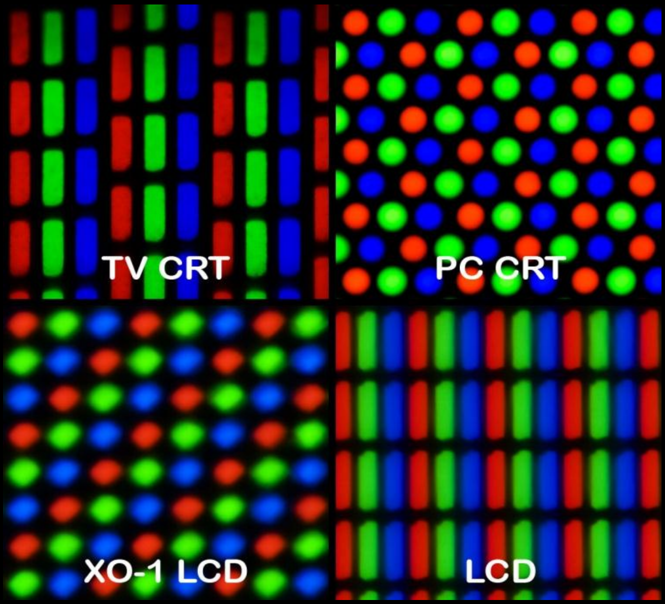
Warning

Tricky thing with spectra & color:

- Spectrum for the stimulus / synthesis
  - Light, monitor, reflectance
- Response curve for receptor /analysis
  - Cones, camera, scanner

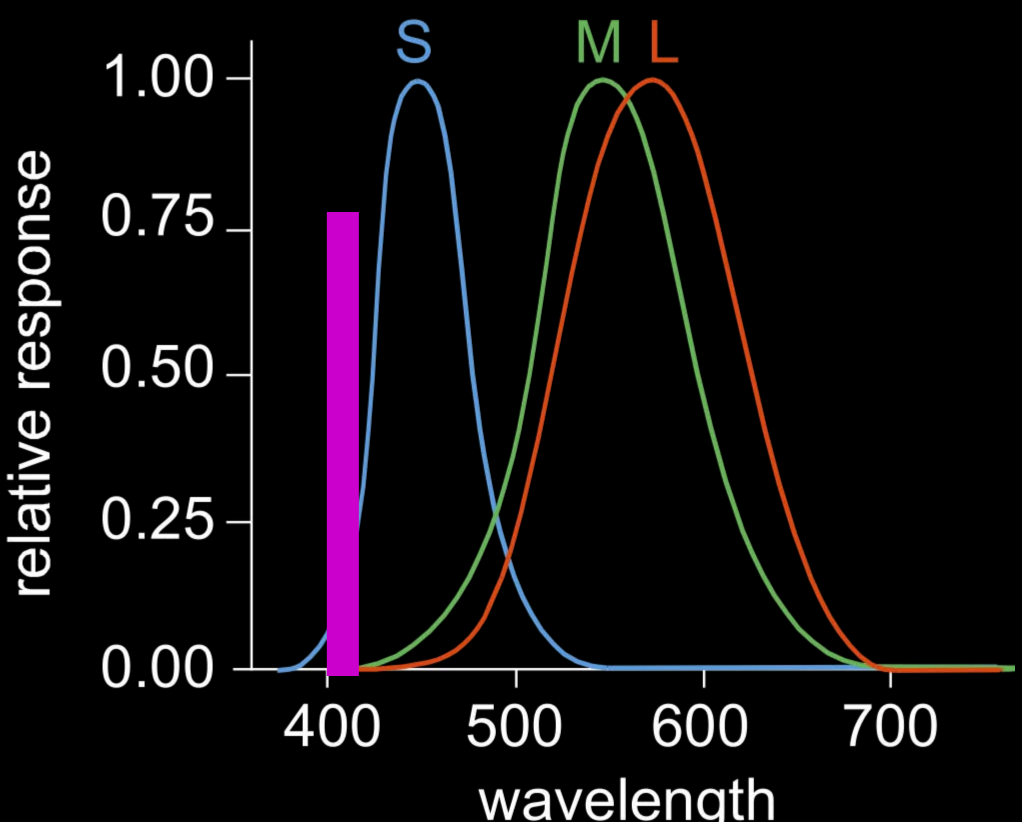
They are usually not the same

There are good reasons for this



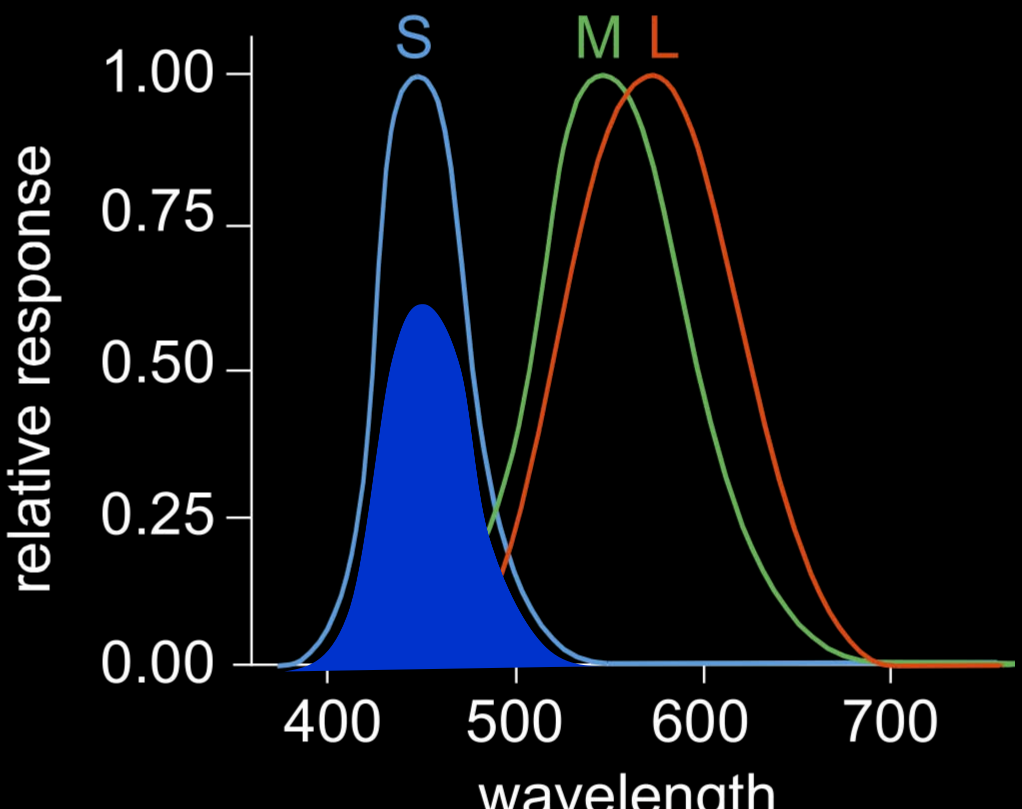
Additive Synthesis - wrong way

- Take a given stimulus and the corresponding responses s, m, l (here 0.5, 0, 0)



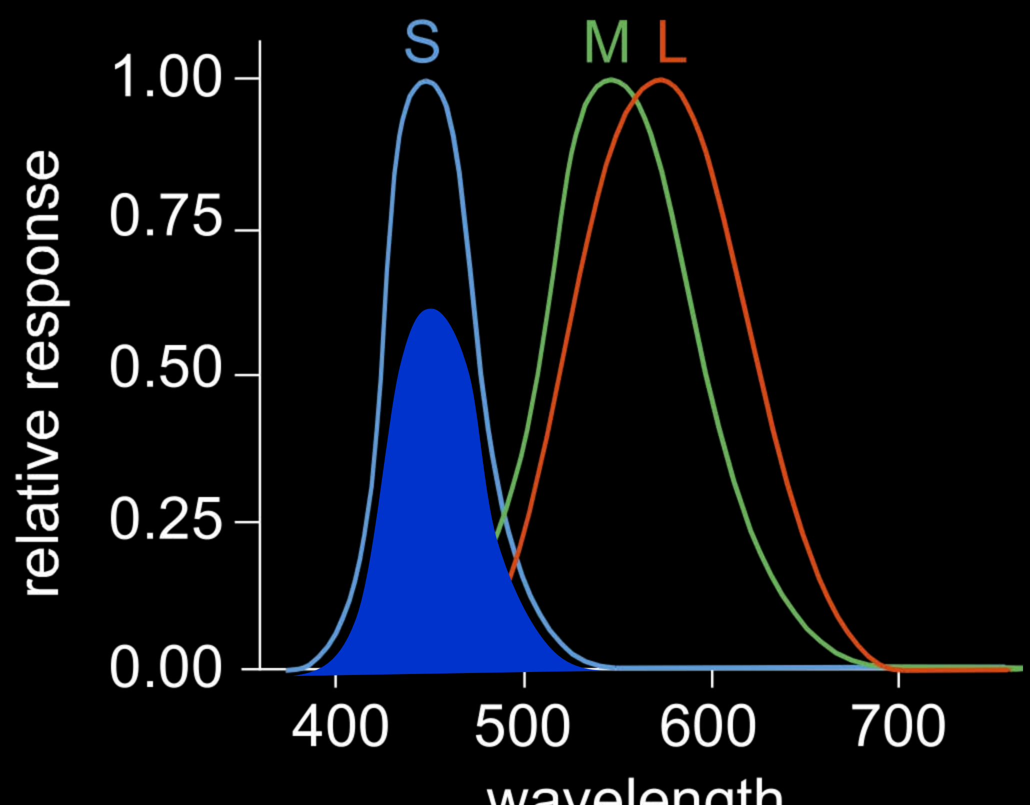
Additive Synthesis - wrong way

- Use it to scale the cone spectra (here 0.5 * S)
- You don't get the same cone response!
  (here 0.5, 0.1, 0.1)



What's going on?

- The three cone responses are not orthogonal
- i.e. they overlap and "pollute" each other



Fundamental problems

- Spectra are infinite-dimensional
- Only positive values are allowed
- Cones are non-orthogonal/overlap

Summary

- Physical color
  - Spectrum
  - multiplication of light & reflectance spectrum
- Perceptual color
  - Cone spectral response: 3 numbers
  - Metamers: different spectrum, same responses
    - Color matching, enables color reproduction with 3 primaries
- Fundamental difficulty
  - Spectra are infinite-dimensional (full function)
  - Projected to only 3 types of cones
  - Cone responses overlap / they are non-orthogonal
    - Means different primaries for analysis and synthesis
  - Negative numbers are not physical

Questions?

Standard color spaces

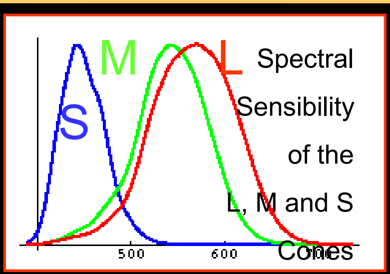
- We need a principled color space
- Many possible definition
  - Including cone response (LMS)
  - Unfortunately not really used,
    (because not known at the time)

- The good news is that color vision is linear and 3-dimensional, so any new color space based on color matching can be obtained using 3x3 matrix
  - But there are also non-linear color spaces
    (e.g. Hue Saturation Value, Lab)

Overview

- Most standard color space: CIE XYZ
- LMS and the various flavor of RGB are just linear transformations of the XYZ basis
  - 3x3 matrices

Why not measure cone sensitivity?



- Less directly measurable
  - electrode in photoreceptor?
  - not available when color spaces were defined
- Most directly available measurement:
  - notion of metamers & color matching
  - directly in terms of color reproduction: **given an input color, how to reproduce it with 3 primary colors?**
  - Commission Internationale de l'Eclairage (International Lighting Commission)
  - Circa 1920

CIE color matching

- Choose 3 synthesis primaries
- Seek to match any monochromatic light (400 to 700nm)
  - Record the 3 values for each wavelength
- By linearity, this tells us how to match any light

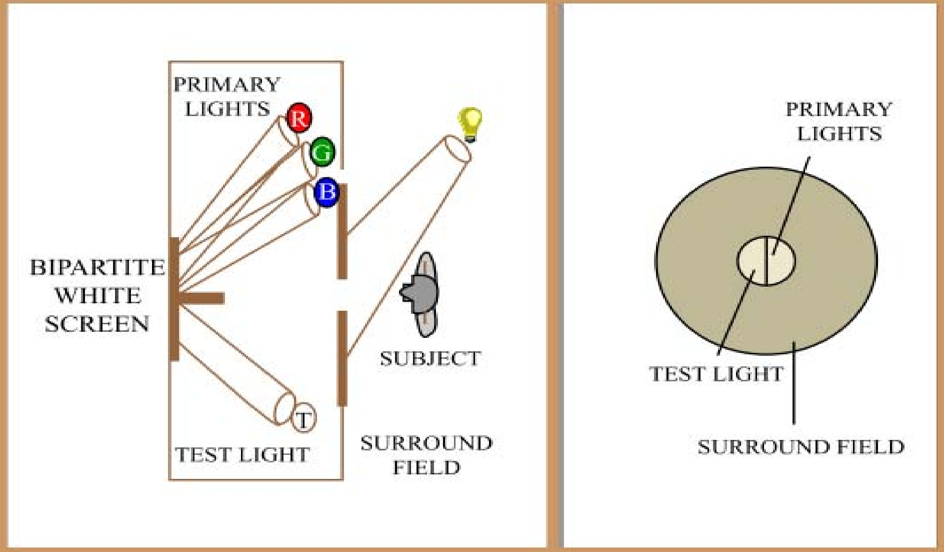
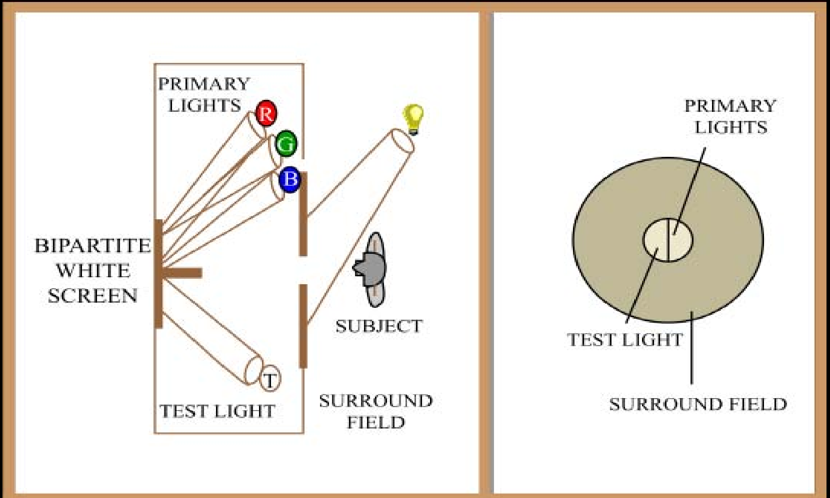


Image adapted from:
Wandell, Brian A. *Foundations of Vision: Behavior, Neuroscience and Computation*. Sinauer Associates, Inc. May 1995. ISBN: 0878938532.

CIE color matching

- Primaries (synthesis) at 435.8, 546.1 and 700nm
  - Chosen for robust reproduction, good separation in red-green
  - Don't worry, we'll be able to convert it to any other set of primaries (Linear algebra to the rescue!)
- Resulting 3 numbers for each input wavelength are called tristimulus values



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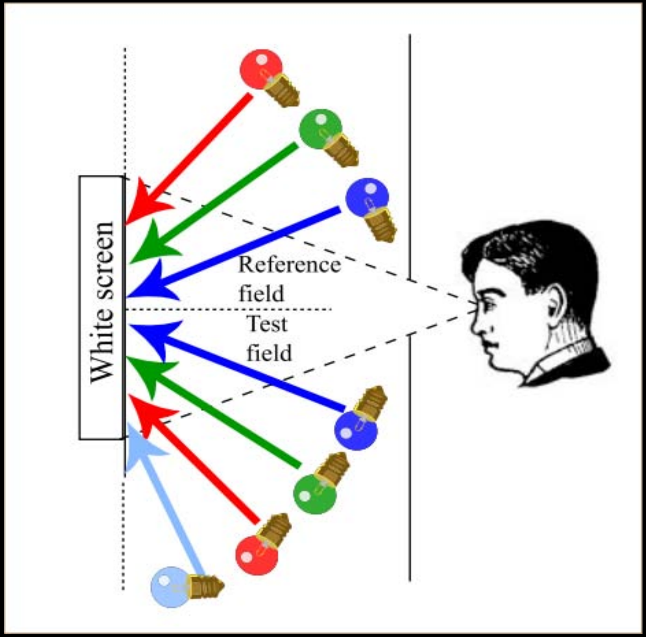
Now, our interactive feature!

You are...

THE LAB RAT

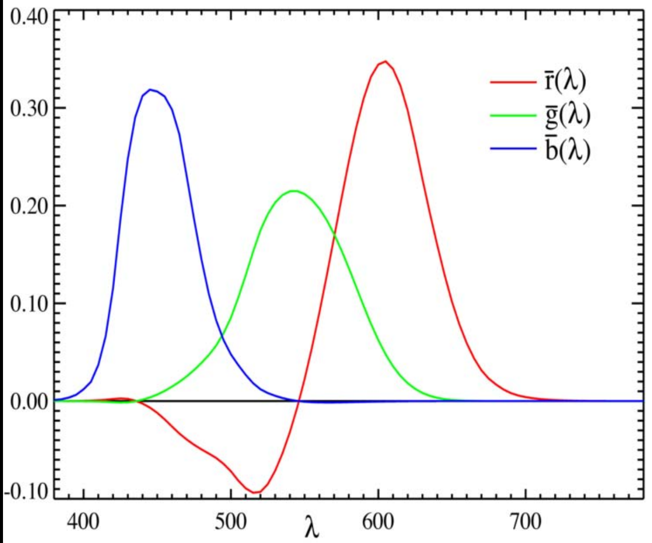
Color Matching Problem

- Some colors cannot be produced using only positively weighted primaries
- Solution: add light on the other side!



CIE color matching

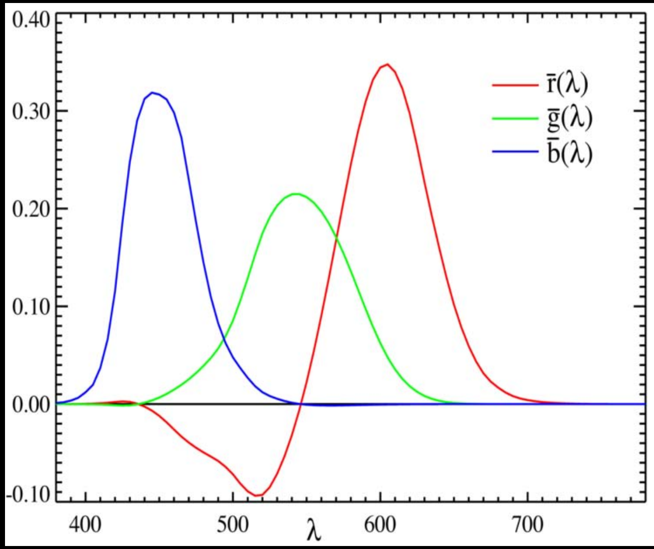
- Meaning of these curves: a monochromatic wavelength λ can be reproduced with $b(\lambda)$ amount of the 435.8nm primary, $+g(\lambda)$ amount of the 546.1 primary, $+r(\lambda)$ amount of the 700 nm primary
- This fully specifies the color perceived by a human
- Careful: this is not your usual rgb



CIE color matching

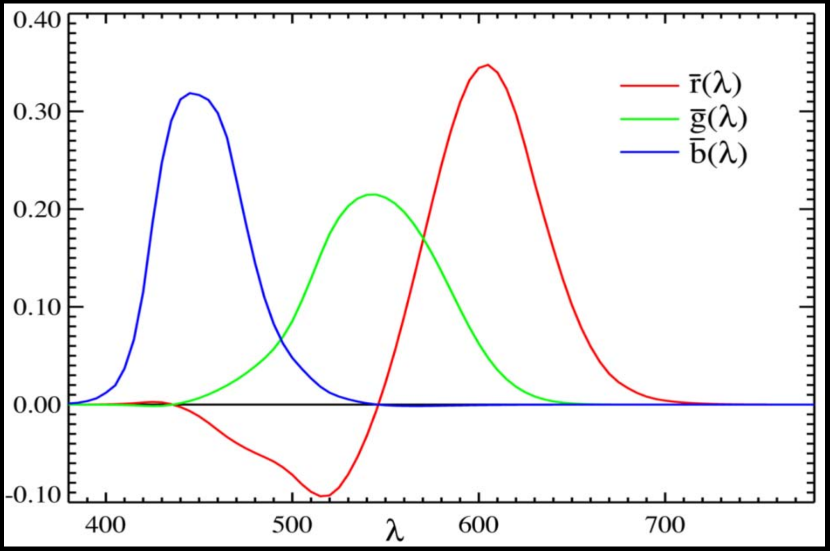
- Meaning of these curves: a monochromatic wavelength λ can be reproduced with b(λ) amount of the 435.8nm primary, +g(λ) amount of the 546.1 primary, +r(λ) amount of the 700 nm primary

- This fully specifies the color perceived by a human
- However, note that one of the responses can be negative
  - Those colors cannot be reproduced by those 3 primaries.



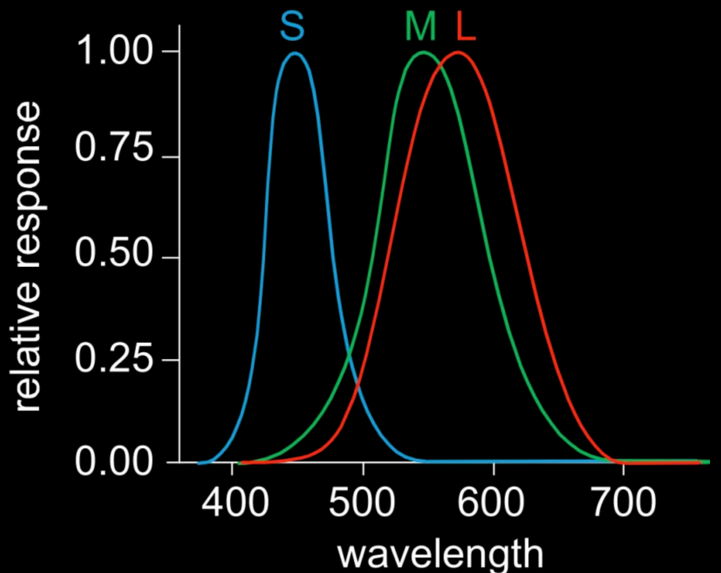
CIE color matching: what does it mean?

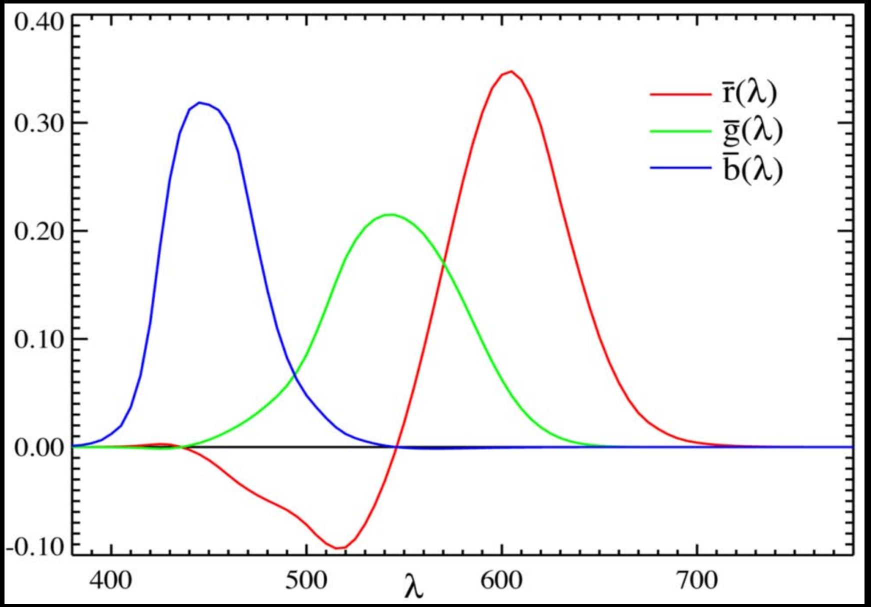
- If I have a given spectrum X
- I compute its response to the 3 matching curves (multiply and integrate)
- I use these 3 responses to scale my 3 primaries (435.8, 546.1 and 700nm)
- I get a metamer of X (perfect color reproduction)



Relation to cone curves

- Project to the same subspace
  - b, g, and r are linear combinations of S, M and L
- Related by 3x3 matrix.
- Unfortunately unknown at that time. This would have made life a lot easier!





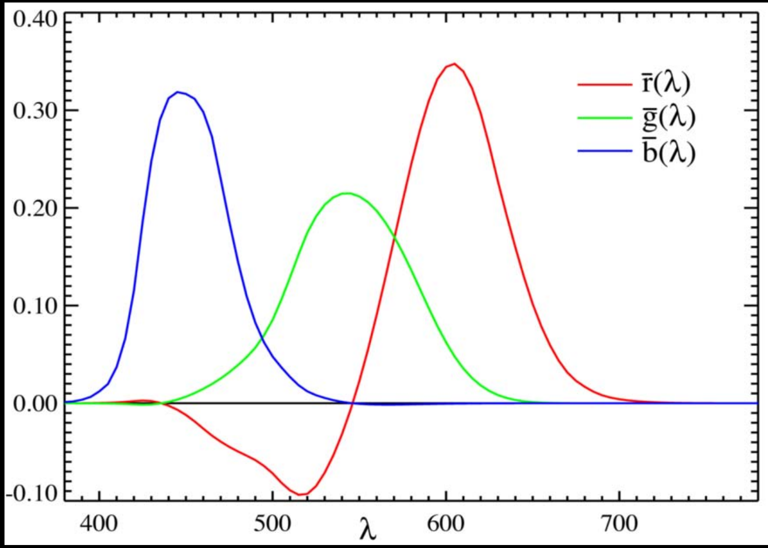
Recap

- Spectra : infinite dimensional
- Cones: 3 spectral responses
- Metamers: spectra that look the same (same projection onto cone responses)
- CIE measured color response:
  - chose 3 primaries
  - tristimulus curves to reproduce any wavelength

- Questions?

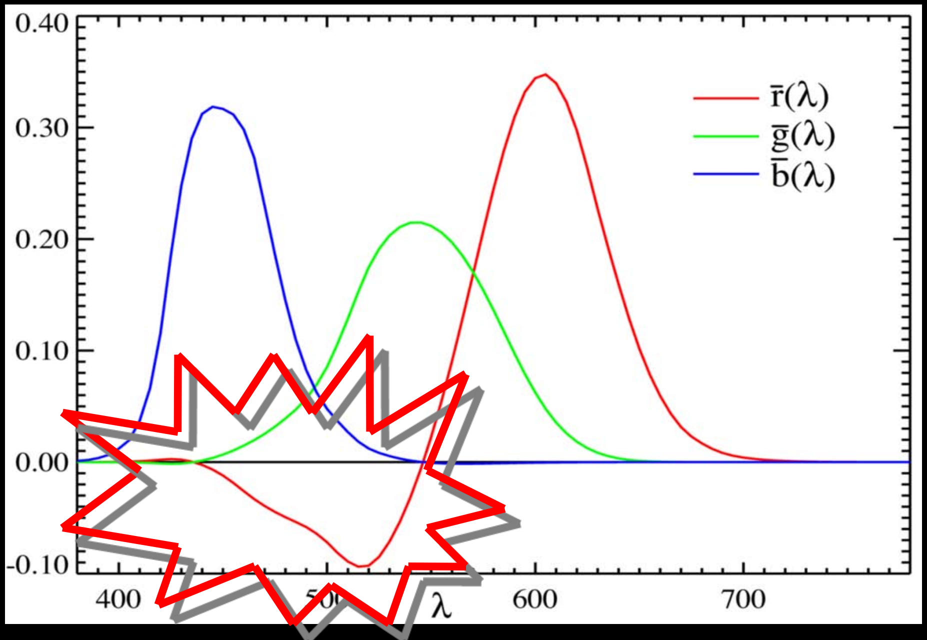
How to build a measurement device?

- Idea:
  - Start with light sensor sensitive to all wavelength
  - Use three filters with spectra b, r, g
  - measure 3 numbers
- This is pretty much what the eyes do!



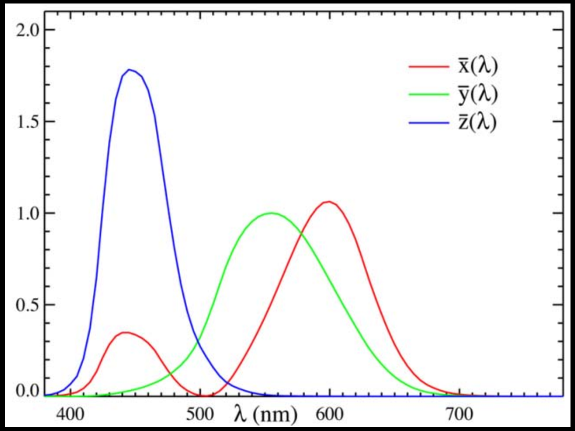
CIE's problem

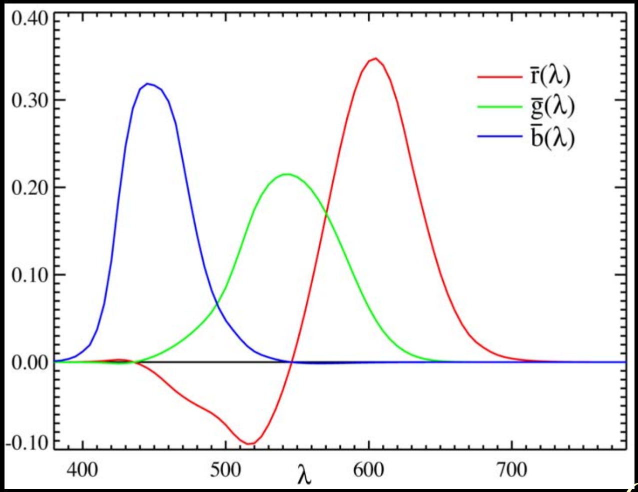
- Idea:
  - Start with light sensor sensitive to all wavelength
  - Use three filters with spectra b, r, g
  - measure 3 numbers
- But for those primaries, we need negative spectra



CIE's problem

- Obvious solution: use cone response!
  - but unknown at the time
- =>new set of tristimulus curves
  - linear combinations of b, g, r
  - pretty much add enough b and g until r is positive





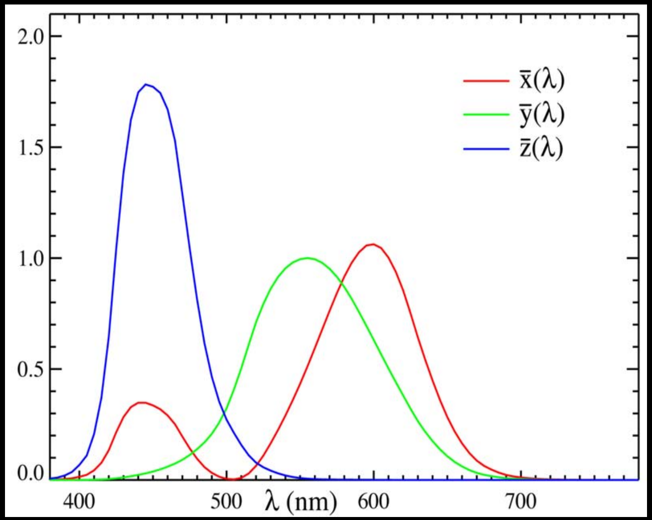
Chromaticity diagrams

- 3D space are tough to visualize
- Usually project to 2D for clarity
- Chromaticity diagram:
  - normalize against X + Y + Z:

$$x = \frac{X}{X+Y+Z}, \quad y = \frac{Y}{X+Y+Z}, \quad z = \frac{Z}{X+Y+Z}$$

CIE XYZ -recap

- THE standard for color specification
- Lots of legacy decision - I wish it were LMS
- Based on color matching
  - 3 monochromatic primaries
  - Subjects matched every wavelength
  - Tricks to avoid negative numbers
  - These 3 values “measure” or describe a perceived color.



Questions?

Other primaries

- We want to use a new set of primaries
  - e.g. the spectra of R, G & B in a projector or monitor
- By linearity of color matching,
  can be obtained from XYZ by a 3x3 matrix

$$\begin{pmatrix} R \\ G \\ B \end{pmatrix} = \begin{pmatrix} 3.24 & -1.54 & -0.50 \\ -0.97 & 1.88 & 0.04 \\ 0.06 & -0.20 & 1.06 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} 0.41 & 0.36 & 0.18 \\ 0.21 & 0.72 & 0.07 \\ 0.02 & 0.12 & 0.95 \end{pmatrix} \begin{pmatrix} R \\ G \\ B \end{pmatrix}$$

one example RGB space

Other primaries

- We want to use a new set of primaries
  - e.g. the spectra of R, G & B in a projector or monitor
- By linearity of color matching, can be obtained from XYZ by a 3x3 matrix
- This matrix tells us how to match the 3 primary spectra from XYZ using the new 3 primaries

$$\begin{pmatrix} R \\ G \\ B \end{pmatrix} = \begin{pmatrix} 3.24 & -1.54 & -0.50 \\ -0.97 & 1.88 & 0.04 \\ 0.06 & -0.20 & 1.06 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

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one example RGB space

XYZ to RGB & back

- e.g.
  http://www.brucelindbloom.com/index.html?Eqn_RGB_XYZ_Matrix.html

- sRGB to XYZ

0.412424	0.212656	0.0193324
0.357579	0.715158	0.119193
0.180464	0.0721856	0.950444

XYZ to sRGB

3.24071	-0.969258	0.0556352
-1.53726	1.87599	-0.203996
0.498571	0.0415557	1.05707

- Adobe RGB to XYZ

0.576700	0.297361	0.0270328
0.185556	0.627355	0.0706879
0.188212	0.0752847	0.991248

XYZ to Adobe RGB

2.04148	-0.969258	0.0134455
-0.564977	1.87599	-0.118373
-0.344713	0.0415557	1.01527

- NTSC RGB to XYZ

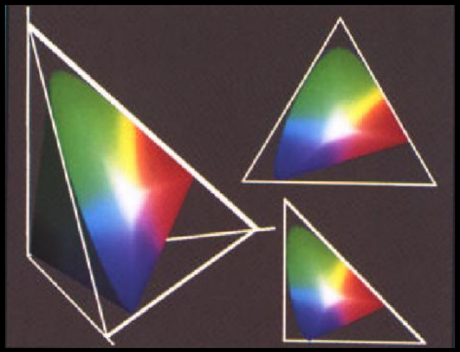
0.606734	0.298839	0.000000
0.173564	0.586811	0.0661196
0.200112	0.114350	1.11491

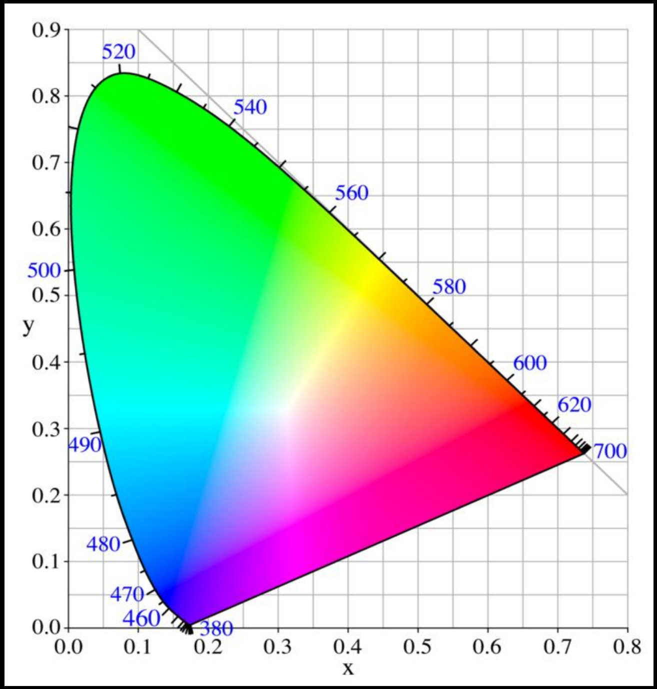
XYZ to NTSC RGB

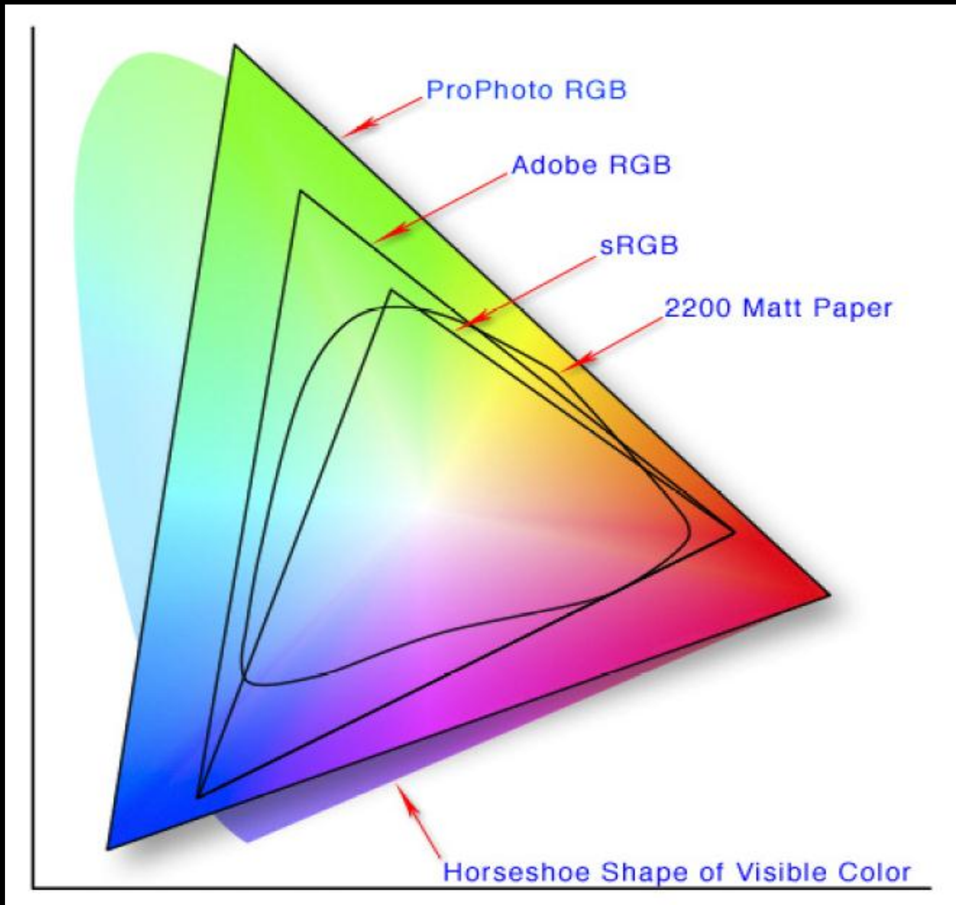
1.91049	-0.984310	0.0583744
-0.532592	1.99845	-0.118518
-0.288284	-0.0282980	0.898611

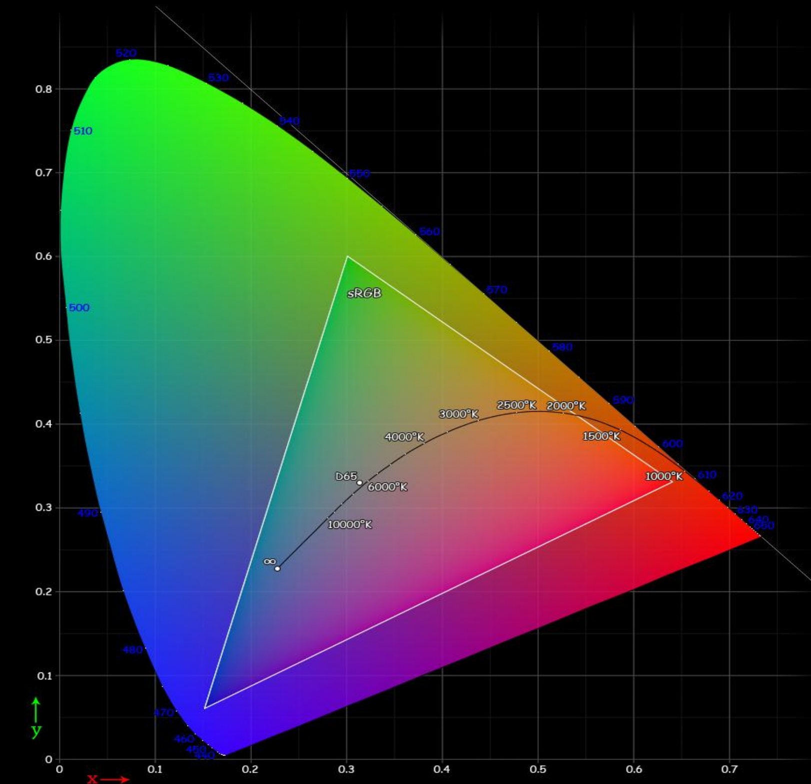
Color gamut

- Given 3 primaries
- The realizable chromaticities lay in the triangle in xy chromaticity diagram
- Because we can only add light, no negative light






ProPhoto RGB
Adobe RGB
sRGB
2200 Matt Paper
Horseshoe Shape of Visible Color


sRGB
D65
10000°K
6000°K
4000°K
3000°K
2500°K
2000°K
1500°K
1000°K
y
x

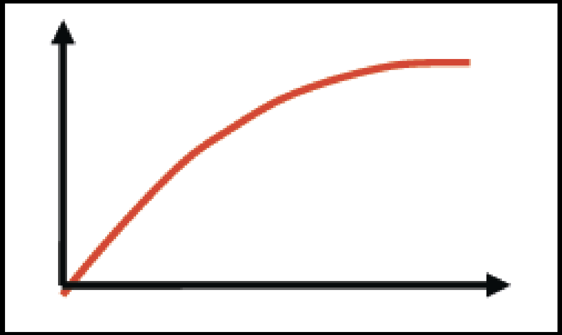
In summary

- It's all about linear algebra
  - Projection from infinite-dimensional spectrum to a 3D response
  - Then any space based on color matching and metamerism can be converted by 3x3 matrix
- Complicated because
  - Projection from infinite-dimensional space
  - Non-orthogonal basis (cone responses overlap)
  - No negative light
- XYZ is the most standard color space
- RGB has many flavors

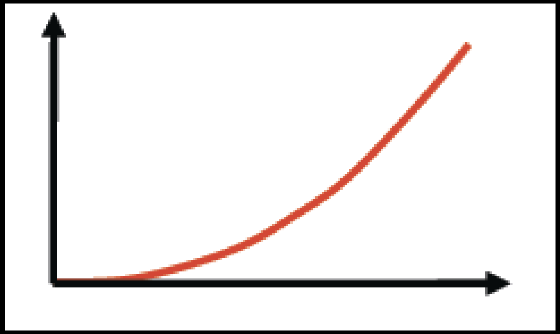
Questions?

Gamma encoding overview

- Digital images are usually not encoded linearly
- Instead, the value **X**1/γ is stored



- Need to be decoded if we want linear values



Color quantization gamma

- The human visual system is more sensitive to ratios
  - Is a grey twice as bright as another one?
- If we use linear encoding, we have tons of information between 128 and 255, but very little between 1 and 2!
- Ideal encoding?

  Log
- Problems with log?

  Gets crazy around zero

Solution: gamma

Color quantization gamma

- The human visual system is more sensitive to ratios
  - Is a grey twice as bright as another one?
- If we use linear encoding, we have tons of information between 128 and 255, but very little between 1 and 2!
- This is why a non-linear gamma remapping of about 2.0 is applied before encoding
- True also of analog imaging to optimize signal-noise ratio

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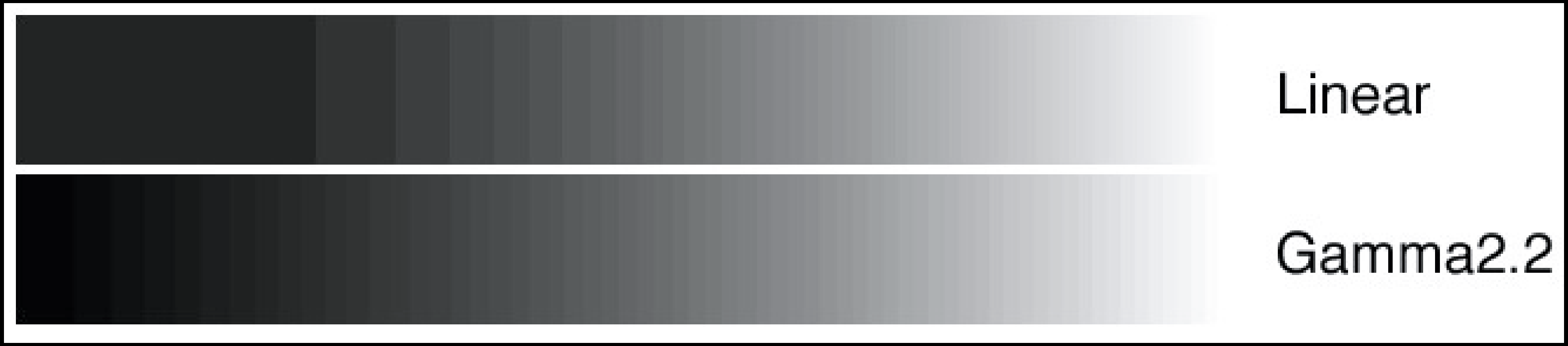
Gamma encoding

- From Greg Ward
- Only 6 bits for emphasis



Important Message

- Digital images are usually gamma encoded
  - Often $\gamma = 2.2$ (but 1.8 for Profoto RGB)
- To get linear values, you must decode
  - apply x => xγ

Questions?

Selected Bibliography

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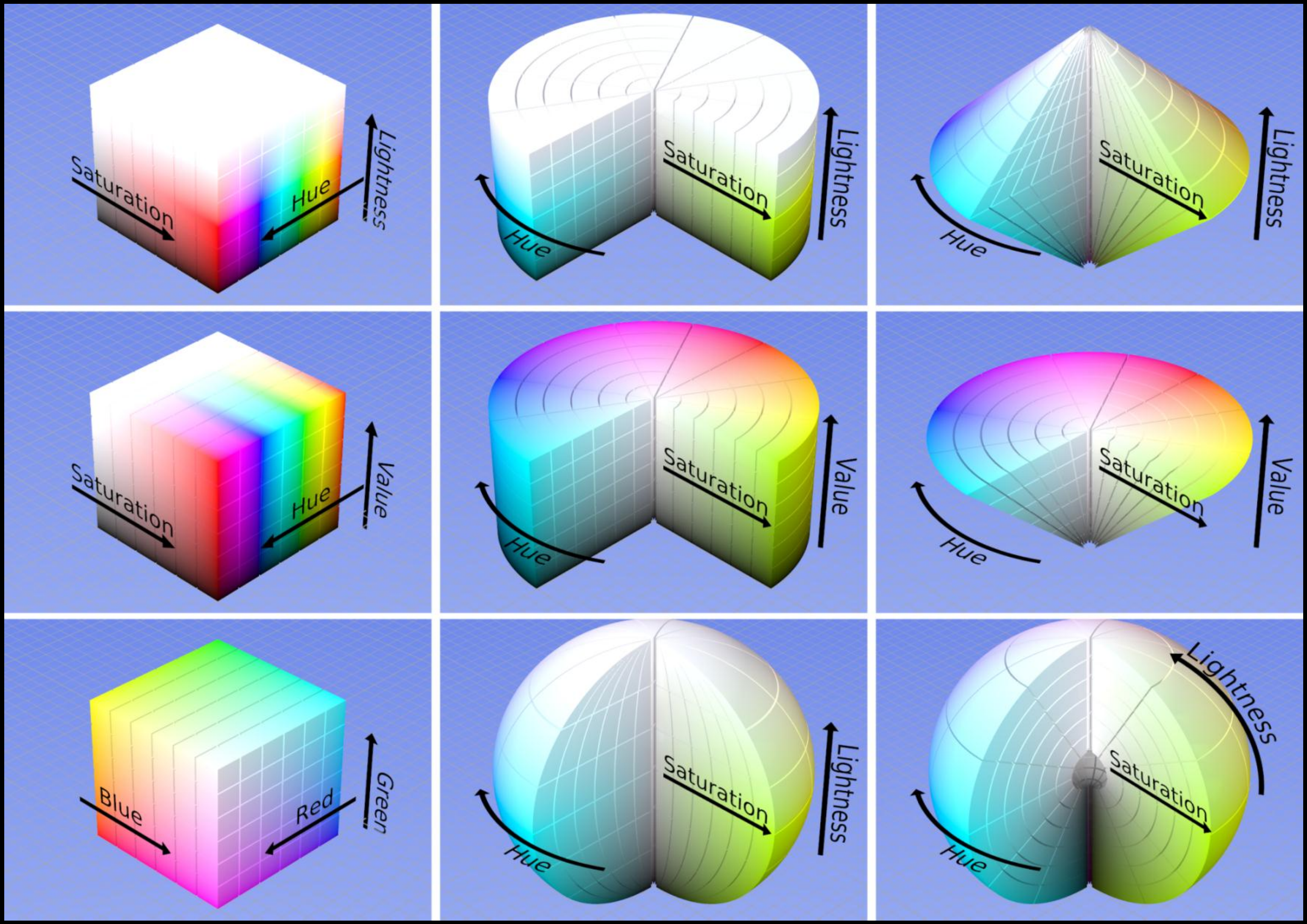
Questions?


Saturation
Hue
Lightness
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Lightness
Lightness
Saturation
Hue