Recovering Baseline and Orientation from Essential Matrix

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Abstract: *Certain approaches to the problem of relative orientation in binocular stereo (as well as long-range motion vision) lead to an encoding of the baseline (translation) and orientation (rotation) in a single* 3×3 *matrix called the "essential" matrix. The essential matrix is defined by*

$$\mathbf{E} = \mathbf{BR},$$

where $\mathbf{B}$ *is the skew-symmetric matrix that satisfies* $\mathbf{Bv} = \mathbf{b} \times \mathbf{v}$ *for any vector* $\mathbf{v}$*, with* $\mathbf{b}$ *being the baseline and* $\mathbf{R}$ *the orientation. Shown here is a simple method for recovering the two solutions for the baseline and the orientation from a given essential matrix using elementary matrix operations. The two solutions for the baseline* $\mathbf{b}$ *can be obtained from the equality*

$$\mathbf{bb}^T = \frac{1}{2}\,\mathrm{Trace}(\mathbf{EE}^T)\mathbf{I} - \mathbf{EE}^T,$$

where $\mathbf{I}$ *is the* 3×3 *identity matrix. The two solutions for the orientation can be found using*

$$(\mathbf{b} \cdot \mathbf{b})\,\mathbf{R} = \mathrm{Cofactors}(\mathbf{E})^T - \mathbf{BE},$$

where $\mathrm{Cofactors}(\mathbf{E})$ *is the matrix of cofactors of* $\mathbf{E}$*. There is no need to perform a singular value decomposition, to transform coordinates, or to use circular functions, and it is easy to see that there are exactly two solutions given a particular essential matrix. If the sign of* $\mathbf{E}$ *is reversed, an additional pair of solutions is obtained that are related the two already found in a simple fashion. This helps shed some light on the question of how many solutions a given relative orientation problem can have.*

1. Coplanarity Condition in Relative Orientation

Relative orientation is the well-known photogrammetric problem of recovering the position and orientation of one camera relative to another from five or more pairs of corresponding ray directions [Zeller 52] [Ghosh 72] [Slama *et al.* 80] [Wolf 83] [Horn 86, 87b]. Relative orientation has to be determined before binocular stereo information can be used to recover surface shape. The same is true of the use of image feature correspondences in long-range motion vision (but not in the case of short-range motion vision, where motion can be treated as infinitesimal and rotations can conveniently be represented by vectors [Horn & Weldon 88].)

Let $\mathbf{b}$ be the baseline (translation of the right center of projection with respect to the left center of projection), while $\boldsymbol{\ell}$ and $\mathbf{r}'$ are the rays from the left and right centers of projection to a given point in the scene. These vectors are all measured in the coordinate system of the left camera. The coplanarity condition expresses the fact that the rays from the left and the right centers of projection meet, that is, that for some α and β,

$$\alpha \boldsymbol{\ell} = \mathbf{b} + \beta \mathbf{r}'. \tag{1}$$

The well-known triple product form of the coplanarity condition,

$$[\boldsymbol{\ell}\ \mathbf{b}\ \mathbf{r}'] = 0, \tag{2}$$

is obtained by simply taking the dot-product of both sides of this equation with $\mathbf{b} \times \mathbf{r}'$. Since the triple product is the volume of the parallelepiped formed by the three vectors, we see that this is equivalent to requiring that the vectors be coplanar [Zeller 52] [Thompson 59].

Let $\mathbf{r}$ be the ray direction from the right center of projection measured in the right coordinate system. Then $\mathbf{r}' = \mathbf{R}\mathbf{r}$, where $\mathbf{R}$ is the orientation (the rotation that aligns the right coordinate system with the left coordinate system). The coplanarity condition can then be written in the form

$$\boldsymbol{\ell} \cdot (\mathbf{b} \times (\mathbf{R}\mathbf{r})) = 0, \tag{3}$$

or

$$\boldsymbol{\ell}^T \mathbf{B}\mathbf{R}\mathbf{r} = 0, \tag{4}$$

where $\mathbf{B}$ is a skew-symmetric matrix defined by $\mathbf{B}\mathbf{v} = \mathbf{b} \times \mathbf{v}$ for all vectors $\mathbf{v}$ [Thompson 59]. The coplanarity condition can thus be transformed into

$$\boldsymbol{\ell}^T \mathbf{E}\mathbf{r} = 0, \tag{5}$$

where $\mathbf{E} = \mathbf{B}\mathbf{R}$ is the so-called *essential matrix* [Longuet-Higgins 81] [Tsai & Wang 84].

Note that there must be three constraints on the nine elements of the essential matrix, since there are only six degrees of freedom (three for the baseline and three for the orientation).

The essential matrix can be found given five correspondences between pairs of rays in the left and right coordinate system—we do not, however, discuss here how this may be done [Horn 87b] [Faugeras & Maybank 89] [Holt & Netravali 90]. The essential matrix can also be found using linear methods, when eight pairs of corresponding rays are given [Longuett-Higgins 81]. But such methods do not enforce the required nonlinear constraints on the elements of an essential matrix and thus will produce a matrix that is not decomposable unless the data is absolutely perfect [Longuett-Higgins 84]. They also do not make effective use of the redundant information provided by eight ray pair correspondences.

If $\boldsymbol{\ell}^T \mathbf{E}\mathbf{r} = 0$, then $\boldsymbol{\ell}^T (k\mathbf{E})\mathbf{r} = 0$, for an arbitrary constant k. Thus if $\mathbf{E}$ is an essential matrix for a particular set of corresponding ray pairs, so is $k\mathbf{E}$ for non-zero k. A particular essential matrix has two unique decompositions into a baseline and an orientation. A particular set of ray correspondences, however, does not fix the scale of $\mathbf{E}$. Variations in the scale of the essential matrix are reflected in changes in the magnitude of the implied baseline $\mathbf{b}$. Thus while a particular essential matrix corresponds to a fixed length of baseline, a set of ray correspondences does not. This is referred to as the *scale-factor ambiguity*.

The essential matrix is primarily of theoretical interest, since it is useful only when exactly five ray correspondences have been found. In practice one typically applies least-squares methods to many more than five ray pairs in order to attain reasonable accuracy [Zeller 52] [Ghosh 72] [Slama *et al.* 80] [Wolf 83] [Horn 86, 87b]. If one has more than five ray correspondences, one could use least-squares methods to find a "best-fit" essential matrix, provided one enforces three non-linear conditions that ensure that the matrix is decomposable. This has not proven feasible. If, on the other hand, the three conditions are not enforced, then the resulting matrix will not be decomposable, and the methods described here (or elsewhere) should not be applied, since the elements of the matrix are then inconsistent. Applying an algorithm that assumes that the data is consistent can in this case lead to very poor results. At the very least, one should try to find a baseline and a rotation that yields an essential matrix as close as possible in the least-squares sense to the given inconsistent matrix. This too is a difficult problem.

Since the "real" problem is a least-squares problem, it is important to use a good representation for rotation—unit quaternions are to be preferred to orthonormal matrices in this regard [Horn 87a]. It is also better not to use a two-step approach where an essential matrix is computed as an intermediate term.

This short note addresses the problem of recovering the baseline and the orientation from an essential matrix, despite what has been said above, simply because there still appears to be interest in this problem, and because there has so far been no simple algorithm for doing this.

2. Decomposing the Essential Matrix

The essential matrix is defined by the equation

$$\mathbf{E} = \mathbf{B}\mathbf{R}, \tag{6}$$

where $\mathbf{B}$ is a skew-symmetric matrix that satisfies $\mathbf{B}\mathbf{v} = \mathbf{b}\times\mathbf{v}$, for all vectors $\mathbf{v}$, while $\mathbf{R}$ is an orthonormal matrix (with positive determinant), that is, $\mathbf{R}\mathbf{R}^T = \mathbf{I}$ (and $\text{Det}(\mathbf{R}) = +1$). The matrix $\mathbf{B}$ is singular, since $\mathbf{B}\mathbf{b} = \mathbf{b}\times\mathbf{b}$, so the essential matrix $\mathbf{E}$ is also singular. (Hence one of the constraints on the elements of $\mathbf{E}$ is that $\det(\mathbf{E}) = 0$).

The problem addressed here is how to recover $\mathbf{B}$ (or equivalently $\mathbf{b}$) and $\mathbf{R}$ given $\mathbf{E}$, and to determine how many solutions there are.

2.1 Recovering the Baseline

First of all note that we can separate the problem of recovering $\mathbf{B}$ from that of recovering $\mathbf{R}$ since

$$\mathbf{E}\mathbf{E}^T = \mathbf{B}\mathbf{R}\mathbf{R}^T\mathbf{B}^T = \mathbf{B}\mathbf{B}^T = -\mathbf{B}^2, \tag{7}$$

where we have used $\mathbf{B}^T = -\mathbf{B}$. Now

$$\mathbf{B}^2\mathbf{v} = \mathbf{b}\times(\mathbf{b}\times\mathbf{v}) = \mathbf{b}(\mathbf{b}\cdot\mathbf{v}) - (\mathbf{b}\cdot\mathbf{b})\mathbf{v}, \tag{8}$$

for all vectors $\mathbf{v}$, so

$$\mathbf{B}^2 = \mathbf{b}\mathbf{b}^T - (\mathbf{b}\cdot\mathbf{b})\,\mathbf{I}, \tag{9}$$

where $\mathbf{I}$ is the 3×3 identity matrix. We have further

$$\text{Trace}(\mathbf{B}^2) = (\mathbf{b}\cdot\mathbf{b}) - 3(\mathbf{b}\cdot\mathbf{b}) = -2(\mathbf{b}\cdot\mathbf{b}), \tag{10}$$

so that

$$\mathbf{b}\mathbf{b}^T = \mathbf{B}^2 - \frac{1}{2}\,\text{Trace}(\mathbf{B}^2)\,\mathbf{I}, \tag{11}$$

or finally, using equation (7),

$$\mathbf{b}\mathbf{b}^T = \frac{1}{2}\,\text{Trace}(\mathbf{E}\mathbf{E}^T)\,\mathbf{I} - \mathbf{E}\mathbf{E}^T. \tag{12}$$

Note that $\text{Trace}(\mathbf{E}\mathbf{E}^T)$ is just the sum of the squares of the elements of the essential matrix $\mathbf{E}$.

This simple construction yields the symmetric matrix,

$$\mathbf{b}\mathbf{b}^T = \begin{pmatrix} b_1^2 & b_1b_2 & b_1b_3 \\ b_2b_1 & b_2^2 & b_2b_3 \\ b_3b_1 & b_3b_2 & b_3^2 \end{pmatrix} \tag{13}$$

the rows of which are all parallel to $\mathbf{b} = (b_1, b_2, b_3)^T$ (as are the columns). For numerical accuracy we may pick the largest row (or column) and scale it by dividing by the square root of the element that was on the diagonal (To find the largest row, simply locate the largest diagonal element). So, if the i-th row of the matrix is the largest, we obtain $\mathbf{b}$ by dividing the i-th row by the square root of its i-th element.

Since we can pick either sign for the square root, there are two possible choices for the orientation of $\mathbf{b}$. Indeed, it could not be otherwise, since $(-\mathbf{b})(-\mathbf{b})^T = \mathbf{b}\mathbf{b}^T$. Having found the two possible values for $\mathbf{b}$, we could now proceed to recover the corresponding values of $\mathbf{R}$.

2.2 Alternative Method for Recovering Baseline

But let us first look at another way of obtaining $\mathbf{b}$. Let $\mathbf{R} = \begin{pmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{r}_3 \end{pmatrix}$, where $\mathbf{r}_i$ is the i-th column of $\mathbf{R}$, then

$$\mathbf{E} = \begin{pmatrix} \mathbf{b} \times \mathbf{r}_1 & \mathbf{b} \times \mathbf{r}_2 & \mathbf{b} \times \mathbf{r}_3 \end{pmatrix}. \tag{14}$$

We note that each column of the essential matrix is orthogonal to $\mathbf{b}$, and so $\mathbf{b}$ is parallel to the cross-product of any two columns. To be specific, let $\mathbf{E} = \begin{pmatrix} \mathbf{e}_1 & \mathbf{e}_2 & \mathbf{e}_3 \end{pmatrix}$, where $\mathbf{e}_i$ is the i-th column of $\mathbf{E}$, then

$$\mathbf{e}_1 \times \mathbf{e}_2 = (\mathbf{b} \times \mathbf{r}_1) \times (\mathbf{b} \times \mathbf{r}_2) = [\mathbf{r}_1\ \mathbf{r}_2\ \mathbf{b}]\mathbf{b} = (\mathbf{r}_3 \cdot \mathbf{b})\mathbf{b}. \tag{15}$$

Similarly,

$$\mathbf{e}_2 \times \mathbf{e}_3 = (\mathbf{r}_1 \cdot \mathbf{b})\mathbf{b}, \qquad \text{and} \qquad \mathbf{e}_3 \times \mathbf{e}_1 = (\mathbf{r}_2 \cdot \mathbf{b})\mathbf{b}. \tag{16}$$

For numerical accuracy we may want to choose the largest of these three cross-products and scale it to recover $\mathbf{b}$. The appropriate scale factor can be determined by again noting that the sum of squares of the elements of the essential matrix equals $2(\mathbf{b} \cdot \mathbf{b})$, which also follows from

$$\|\mathbf{b} \times \mathbf{r}_1\|^2 + \|\mathbf{b} \times \mathbf{r}_2\|^2 + \|\mathbf{b} \times \mathbf{r}_3\|^2 = 3(\mathbf{b} \cdot \mathbf{b}) - (\mathbf{b} \cdot \mathbf{b}) = 2(\mathbf{b} \cdot \mathbf{b}). \tag{17}$$

So if $(\mathbf{e}_i \times \mathbf{e}_j)$ is the largest of the three possible pairwise cross-products, then

$$\mathbf{b} = \pm \frac{\mathbf{e}_i \times \mathbf{e}_j}{\left\|\mathbf{e}_i \times \mathbf{e}_j\right\|} \sqrt{\frac{1}{2}\,\mathrm{Trace}(\mathbf{E}\mathbf{E}^T)} \tag{18}$$

gives us the two possible solutions for $\mathbf{b}$.

2.3 Recovering the Orientation

We first note that the matrix of cofactors of $\mathbf{E} = \begin{pmatrix} \mathbf{e}_1 & \mathbf{e}_2 & \mathbf{e}_3 \end{pmatrix}$ is just

$$\mathrm{Cofactors}(\mathbf{E}) = \begin{pmatrix} \mathbf{e}_2 \times \mathbf{e}_3 & \mathbf{e}_3 \times \mathbf{e}_1 & \mathbf{e}_1 \times \mathbf{e}_2 \end{pmatrix}^T, \tag{19}$$

which, using equations (15) & (16) can be written

$$\mathrm{Cofactors}(\mathbf{E}) = \begin{pmatrix} (\mathbf{r}_1 \cdot \mathbf{b})\mathbf{b} & (\mathbf{r}_2 \cdot \mathbf{b})\mathbf{b} & (\mathbf{r}_3 \cdot \mathbf{b})\mathbf{b} \end{pmatrix}^T \tag{20}$$

(So each row of the matrix of cofactors is parallel to $\mathbf{b}$). We can rewrite this in the form

$$\text{Cofactors}(\mathbf{E}) = \left(\mathbf{b}(\mathbf{R}^T\mathbf{b})^T\right)^T, \tag{21}$$

or

$$\text{Cofactors}(\mathbf{E}) = \left((\mathbf{b}\mathbf{b}^T)\,\mathbf{R}\right)^T. \tag{22}$$

Next, note that,

$$\mathbf{BE} = \mathbf{B}^2\mathbf{R} = (\mathbf{b}\mathbf{b}^T)\,\mathbf{R} - (\mathbf{b}\cdot\mathbf{b})\,\mathbf{R}. \tag{23}$$

where we have used equation (9). Clearly then

$$(\mathbf{b}\cdot\mathbf{b})\,\mathbf{R} = \text{Cofactors}(\mathbf{E})^T - \mathbf{BE}. \tag{24}$$

This is a simple convenient formula for the two values of $\mathbf{R}$, given the two possible values of $\mathbf{b}$. (Note that equation (19) provides one way of computing the required cofactors).

2.4 Relationship Between the Solutions

Let $\mathbf{R}_+$ be the rotation associated with the translation $+\mathbf{b}$, and $\mathbf{R}_-$ the rotation associated with the translation $-\mathbf{b}$. Then

$$(\mathbf{b}\cdot\mathbf{b})\mathbf{R}_- = \text{Cofactors}(\mathbf{E})^T + \mathbf{BE}, \tag{25}$$

where $\mathbf{B}$ is the skew-symmetric matrix corresponding to $+\mathbf{b}$. So

$$(\mathbf{b}\cdot\mathbf{b})\mathbf{R}_- = (\mathbf{b}\mathbf{b}^T)\mathbf{R}_+ - (\mathbf{b}\cdot\mathbf{b})\mathbf{R}_+ + (\mathbf{b}\mathbf{b}^T)\mathbf{R}_+ \tag{26}$$

or

$$(\mathbf{b}\cdot\mathbf{b})\mathbf{R}_- = \left(2(\mathbf{b}\mathbf{b}^T) - (\mathbf{b}\cdot\mathbf{b})\mathbf{I}\right)\mathbf{R}_+ \tag{27}$$

Now a rotation $\mathbf{F}$, through π about an axis parallel to $\mathbf{b}$, can be written in the form

$$(\mathbf{b}\cdot\mathbf{b})\mathbf{F} = 2(\mathbf{b}\mathbf{b}^T) - (\mathbf{b}\cdot\mathbf{b})\mathbf{I}. \tag{28}$$

using

$$\mathbf{R} = \cos\theta\,\mathbf{I} + \sin\theta\,\mathbf{\Omega} + (1-\cos\theta)\,\hat{\boldsymbol{\omega}}\hat{\boldsymbol{\omega}}^T,$$

which follows form Rodrigues' formula for rotation of a vector through an angle θ about an axis parallel to the unit vector $\hat{\boldsymbol{\omega}}$. Note that the orthonormal matrix $\mathbf{F}$ is symmetric, and hence its own inverse. We then see that

$$\mathbf{R}_- = \mathbf{F}\mathbf{R}_+ \qquad \text{and} \qquad \mathbf{R}_+ = \mathbf{F}\mathbf{R}_-. \tag{29}$$

So the two possible rotations are related by a rotation through π about an axis parallel to $\mathbf{b}$, as pointed out by [Horn 87b] [Maybank 89] and [Snyder 90]

If $\boldsymbol{\ell}^T\mathbf{E}\mathbf{r} = 0$, then $\boldsymbol{\ell}^T(-\mathbf{E})\mathbf{r} = 0$. Thus if $\mathbf{E}$ is an essential matrix for a particular set of corresponding ray pairs, so is $-\mathbf{E}$. It is easy to see that $-\mathbf{E}$ has two decompositions: the baseline $-\mathbf{b}$ with orientation $\mathbf{R}_+$ and the baseline $+\mathbf{b}$ with orientation $\mathbf{R}_-$. Thus while a particular essential matrix $\mathbf{E}$ has only two decompositions, by noting that a particular set of ray correspondences is also compatible with $-\mathbf{E}$, one finds four possible combinations of baseline and orientation.

2.5 The Number of Solutions

There has been some disagreement about the number of solutions of relative orientation problems [Faugeras & Maybank 89] [Maybank 89] [Netravali *et al.* 89] [Horn 87b] [Snyder 90]. We have seen here that a particular essential matrix always corresponds to exactly two combinations of baseline and orientation (and that the magnitude of the baseline is determined by the scale of the essential matrix). If however we note that any non-zero multiple of a given essential matrix satisfies the same set of coplanarity constraint equations, then the magnitude of the baseline becomes arbitrary. But more importantly, there are then *four* solutions ($\mathbf{b}$ and $-\mathbf{b}$ combined with $\mathbf{R}_+$ and $\mathbf{R}_-$ in either order).

Next, it has been found that sets of five pairs of corresponding rays can give rise to as many as ten distinct essential matrices [Maybank 89] [Netravali *et al.* 89], and as few as none [Horn 87b]. This means that there can be as many as fourty different solutions of the relative orientation problem. Even if we ignore reversals of the baseline direction, there can be as many as twenty solutions [Horn 87b]. It is a mistake, however, to further ignore the fact that an essential matrix is associated with two quite distinct orientations, and to then claim that there are at most ten solutions.

One way to limit the number of apparent "solutions" is to go beyond the essential matrix itself, and to return to the original ray pairs and determine the distances along the rays at which they meet, as implied by the baseline and orientation of a particular solution. These distances are proportional to α and β in equation (1) [Horn 87b]. A solution where the distances for all five pairs of rays are positive is called a *positive* solution. If the data comes from a real situation and is exact, than at least one solution must be positive. Unfortunately, there are cases where more than one solution is positive [Horn 87b] [Netravali & et al 89] [Snyder 90]. Furthermore, with random data there often is no positive solution [Horn 87b].

It should be clear that the question of how many solutions a relative orientation problem has should be treated *separately* from the question of how many of these solutions are positive. It should also be clear that one has to be careful to distinguish the question of how many solutions correspond to a particular essential matrix from the question of how many solution the underlying relative orientation problem has. Overall, it seems that the two-step approach to relative orientation, where one first determines an essential matrix, is the source of both limitations and confusion.

3. Summary and Conclusions

Given the essential matrix

$$\mathbf{E} = \mathbf{B}\mathbf{R},$$

the two solutions for the baseline $\mathbf{b}$ can be obtained from the equality

$$\mathbf{b}\mathbf{b}^T = \frac{1}{2}\,\mathrm{Trace}(\mathbf{E}\mathbf{E}^T)\mathbf{I} - \mathbf{E}\mathbf{E}^T,$$

where $\mathbf{I}$ is the 3×3 identity matrix. The two solutions for the orientation can be found using

$$(\mathbf{b} \cdot \mathbf{b})\,\mathbf{R} = \mathrm{Cofactors}(\mathbf{E})^T - \mathbf{B}\mathbf{E}.$$

There are exactly two solutions for $\mathbf{b}$ and $\mathbf{R}$ given a particular essential matrix. If the sign of $\mathbf{E}$ is reversed, an additional pair of solutions is obtained, in which the baselines and orientation are paired up differently. There is no need to perform a singular value decomposition, to transform coordinates, or to use circular functions.

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