6.728 Applied Quantum and Statistical Physics:

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PROBLEM SET 8

Problem Set Out: 11/01/06 Problem Set Due: 11/8/06 at the beginning of class

Problem 8.1 *A pair of two-level systems* Do problem 19.2 in the text.

Problem 8.2 *Spin* 1/2 *Matrices*

Consider the three Pauli spin matrices σ_i and the identity matrix.

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

(a) Argue that any two by two Hermitian matrix can be written as a linear combination of these four matrices. Consequently, write the Hamitonian

$$\hat{H} = \begin{pmatrix} \bar{E} - \Delta & V \\ V^* & \bar{E} + \Delta \end{pmatrix}$$

in terms of these 4 matrices.

(b) Show that $\sigma_i \sigma_i = I$, and $\sigma_x \sigma_y = i\sigma_z$, $\sigma_y \sigma_z = i\sigma_x$, and $\sigma_z \sigma_x = i\sigma_y$.

(c)The spin of an electron can be written as

$$\mathbf{S} = \frac{\hbar}{2} \left(\sigma_x \mathbf{i_x} + \sigma_y \mathbf{i_y} + \sigma_z \mathbf{i_z} \right)$$

If the Hamiltonian is given by $\mu \mathbf{B} \cdot \mathbf{S}$, where B is the magnetic field, write out the Hamiltonian.

(d) The eigenvectors of the σ_z are $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$. Find the eigenvectors for σ_x and σ_y.