6.728 Applied Quantum and Statistical Physics:

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PROBLEM SET 4

Problem Set Out: 9/27/06   Problem Set Due: 10/4/06 at the beginning of class

PROBLEMS

Problem 4.1

Resonant tunneling through a double barrier. A multi-layer structure of GaAs | AlGaAs | GaAs | AlGaAs | GaAs is grown with thicknesses 1000Å| 40Å| 100Å| 40Å| 1000Å. We model this layered structure as shown in Figure 1, with $V_0 = 0.3\,\text{eV}$, $b = 50\,\text{Å}$, and $a = 90\,\text{Å}$. The wave function is
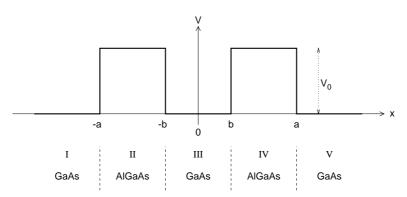

Figure 1: Double barrier potential for resonant tunelling.

piecewise continuous, with

$$
\begin{aligned}
\psi_1 &= e^{ik_1 x} + Be^{-ik_1 x} \\
\psi_2 &= Ce^{ik_2 x} + De^{-ik_2 x} \\
\psi_3 &= Fe^{ik_1 x} + Ge^{-ik_1 x} \\
\psi_4 &= He^{ik_2 x} + Ie^{-ik_2 x} \\
\psi_5 &= Je^{ik_1 x}
\end{aligned}
$$

where

$$
\begin{aligned}
k_1 &= \sqrt{\frac{2mE}{\hbar^2}} \\
k_2 &= \sqrt{\frac{2m\left(E - V_0\right)}{\hbar^2}}
\end{aligned}
$$

1

1. Match the boundary conditions and show that

$$\overline{\overline{M}}\,\overline{C} = \overline{A}$$

where

$$\overline{C} = \begin{pmatrix} B \\ C \\ D \\ F \\ G \\ H \\ I \\ J \end{pmatrix} \qquad \overline{A} = \begin{pmatrix} e^{-ik_1 a} \\ k_1 a e^{-k_1 a} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\overline{\overline{M}} = \begin{pmatrix} -e^{ik_1 a} & e^{-ik_2 a} & e^{ik_2 a} & 0 & 0 & 0 & 0 & 0 \\ k_1 a e^{ik_1 a} & k_2 a e^{-ik_2 a} & -k_2 a e^{ik_2 a} & 0 & 0 & 0 & 0 & 0 \\ 0 & -e^{-ik_2 b} & -e^{ik_2 b} & e^{-ik_1 b} & e^{ik_1 b} & 0 & 0 & 0 \\ 0 & -k_2 b e^{-ik_2 b} & k_2 b e^{ik_2 b} & k_1 b e^{-ik_1 b} & -k_1 b e^{ik_1 b} & 0 & 0 & 0 \\ 0 & 0 & 0 & -e^{ik_1 b} & -e^{-ik_1 b} & e^{ik_2 b} & e^{-ik_2 b} & 0 \\ 0 & 0 & 0 & -k_1 b e^{ik_1 b} & k_1 b e^{-ik_1 b} & k_2 b e^{ik_2 b} & -k_2 b e^{-ik_2 b} & 0 \\ 0 & 0 & 0 & 0 & 0 & -e^{ik_2 a} & -e^{-ik_2 a} & e^{ik_1 a} \\ 0 & 0 & 0 & 0 & 0 & -k_2 a e^{ik_2 a} & k_2 a e^{-ik_2 a} & k_1 a e^{ik_1 a} \end{pmatrix}$$

2. This problem can be solved for the coefficient vector C by

$$\overline{C} = \overline{\overline{M}}^{-1}\overline{A}$$

The transmission coefficient is then

$$T = \left| \overline{C}(8) \right|^2 = |J|^2$$

Use the files rtplot.m and rtwell.m from the course website in the Assignments section to plot T. This program takes a few minutes to run because of the small incremental spacing in energy that is needed. Run the program rtplot.m using the fact that the effective mass of the electron in GaAs and AlGaAs is $0.07 m_e$, where m_e is the mass of the electron. Find the energies below 0.3 eV where $T = 1$. (Hint: use the "ginput" command in Matlab .) What would it be if the electron tunneled through just one barrier?

3. Show that these energies are roughly equal to the particle-in-a-box energies with width $= 2b$. [As you can see, this approximation only applies to a very deep box, but is often used outside its range of applicability in the literature.]

4. If the input electron energy is kept fixed and the voltage, V, is increased across the device, the current will vary across the device. A voltage V causes the voltage on one side to rise by V, the energy level in the box to rise by $V/2$ while the voltage on the other side stays fixed (that is, the voltage only drops across the AlGaAs barrier regions). Assume that the incoming energy is about 0.2eV. Sketch the transmission coefficient as a function of applied voltage.

Problem 4.2

Problem 7.9 from the text. Note that the equation for J_c should read:

$$J_c = \frac{e\hbar n_s}{m_e \zeta} \frac{1}{\sinh\left(2a/\zeta\right)}$$

Problem 4.3

This problem explores the use of the Heisenberg uncertainty relation to make estimates of the ground state behavior of several quantum mechanical systems. In the parts below, assume that the Heisenberg relation is actually an equality:

$$\Delta x \Delta p \equiv \frac{\hbar}{2}$$

We expect that in most systems, it will not turn out to be an equality; however, it is useful as an estimate of ground state behavior. Later, we can compare the full solution with what we obtain from Heisenberg.

(a) Repeat what was done in class and assume a quadratic potential energy $V(x)$ for a particle of mass m:

$$V(x) = \frac{1}{2}Kx^2$$

Find the total energy of a particle with position uncertainty Δx and momentum uncertainty Δp, and find the value of Δx and the corresponding energy at which it is minimum.

(b) Repeat for a potential of the form:

$$V(x) = \frac{1}{6}\tilde{K}x^4$$