Massachusetts Institute of Technology

Department of Electrical Engineering and Computer Science

6.691 Seminar in Advanced Electric Power Systems

Problem Set 7 Solutions May 7, 2006

Unbalanced line The transmission line has impedances in the symmetrical component network domain:

$$Z_s = j\omega \underline{\underline{A}}\underline{\underline{L}}_{ph}\underline{\underline{A}}^{-1}$$

With some effort we can show that this matrix is

$$Z_s = j\omega \begin{bmatrix} L_{00} & L_{01} & L_{02} \\ L_{10} & L_{11} & L_{12} \\ L_{20} & L_{21} & L_{22} \end{bmatrix}$$

where

$$\begin{aligned}
L_{00} &= L_a + \frac{4}{3}L_{ab} + \frac{2}{3}L_{ac} \\
L_{01} &= \frac{1}{3}a^2 L_{ab} + \frac{1}{3}(1+a)L_{ac} \\
L_{02} &= \frac{1}{3}L_{ab} + \frac{1}{3}(1+a^2)L_{ac} \\
L_{10} &= L_{02} \\
L_{11} &= L_a - \frac{2}{3}L_{ab} - \frac{1}{3}L_{ac} \\
L_{12} &= \frac{2}{3}(1+a)L_{ab} + \frac{2}{3}a^2 L_{ac} \\
L_{20} &= L_{01} \\
L_{21} &= \frac{2}{3}(1+a^2)L_{ab} + /frac23aL_{ac} \\
L_{22} &= L_{11}
\end{aligned}$$

Note this one survives the simple 'idiot check' when $L_{ab} = L_{sc}$ and then:

$$L_s = \begin{bmatrix} L_a + 2L_{ab} & 0 & 0 \\ 0 & L_a - L_{ab} & 0 \\ 0 & 0 & L_a - L_{ab} \end{bmatrix}$$

This is carried out in the script.

The voltage driving the system is:

$$v_1 = \underline{e_{af}} - v_\infty = j(x_d + x_1 + x_t)i_1$$

And note that the generator and transformer combination adds some impedance to the system:

$$z_g = \begin{bmatrix} x_0 + x_t & 0 & 0 \\ 0 & x_d + x_t & 0 \\ 0 & 0 & x_2 + x_t \end{bmatrix}$$

And once we have normalized the line impedance:

$$z_s = \frac{1}{Z_b} Z_s$$

we can solve the problem:

$$\begin{bmatrix} i_0 \\ i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 0 \\ v_1 \\ 0 \end{bmatrix} (z_s + z_g)^{-1}$$

The results of this calculation are shown in the MATLAB outpus: (I have removed a number of spaces).

This first matrix is the symmetrical component impedance matrix:

```
zs  =
   0.0000  +  0.0848i 0.0043  - 0.0025i   -0.0043  - 0.0025i
  -0.0043  - 0.0025i  0.0000  +  0.0099i   -0.0086  +  0.0050i
   0.0043  - 0.0025i  0.0086  +  0.0050i   -0.0000  +  0.0099i
```

Next, we add the impedance of the generator and transformer:

```
z_s  =
   0.0000  +  0.1848i 0.0043  - 0.0025i   -0.0043  - 0.0025i
  -0.0043  - 0.0025i  0.0000  +  2.1099i   -0.0086  +  0.0050i
   0.0043  - 0.0025i  0.0086  +  0.0050i   -0.0000  +  0.3099i
```

Now, applying the (positive sequence) voltage to the inverse of this we get the following currents:

```
i_0  =  0.0279
i_1  =  1.0002
i_2  =  0.0327
```

Note that the zero-sequence current does not flow in the generator (which is why the zero sequence impedance does not reflect the generator impedance) but is carried through the transformer. The negative sequence current of about 3.2% should not be a problem for this generator.

```
% 6.691 Homework Set 7
a = exp(j*2*pi/3);                          % rotation by 120 degrees
A = (1/3) .* [1 1 1;1 a a^2;1 a^2 a]; % symmetrical component transform
om = 377;                                   % 60 Hz
ll = 10000;                                 % length of line (100 km)
La = ll* 2.4e-6;                            % full line inductance in Hy
Lab = ll* 2.06e-6;                          % note it scales by length
Lac = ll * 1.03e-6;
Zph = j*om .* [La Lab Lac;Lab La Lab;Lac Lab La];% phase impedances
Zs1 = A*Zph*inv(A);                         % sym component impedances
Zb = 161^2/100;                             % this is base impedance
zs = Zs1 ./ Zb                              % and here it is in per-unit
x1 = imag(zs(2,2));                         % this is pos sequence line x
xd = 2.0;                                   % internal impedance of generator
x2 = .2;
x0 = .1;
xt = .1;                                    % transformer impedance
v_line = j*(xd+xt+x1);                      % this is positive sequence voltage
zg = j .* [xt 0 0;0 xd+xt 0;0 0 x2+xt]; % generator impedance matrix
z_s = zs + zg                               % full system impedance
vs = [0; v_line; 0];                        % voltage vector driving currents
is = inv(z_s) * vs;                         % vector of current components
i_0 = abs(is(1))
i_1 = abs(is(2))
i_2 = abs(is(3))
```