Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science
6.685 Electric Machines

Problem Set 8 Solutions — October 24, 2013

Problem 1: Torque-Speed as affected by seventh space harmonic This, too, is a bit artificial. Seventh harmonic is, of course, the first harmonic that will introduce a noticeable kink in the torque-speed curve, but it is usual to include fifth and the harmonics related to slots too. However, we consider here the equivalent circuit shown in Figure 1. The top part of the circuit is as one would expect, but there is also a section for the seventh space harmonic.

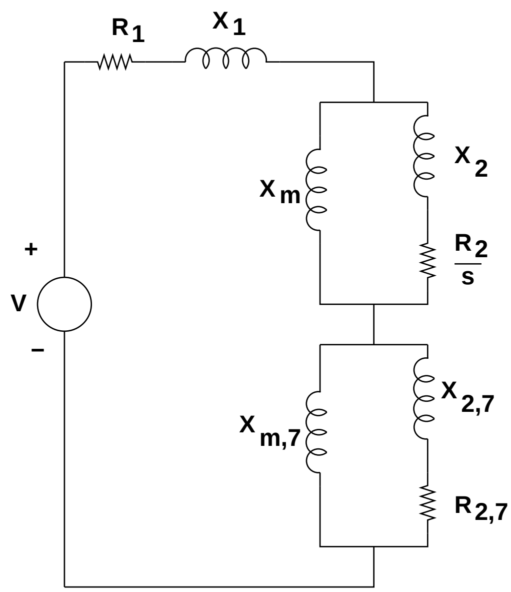

Figure 1: Induction Motor Equivalent Circuit with Seventh Harmonic

Note that the circuit elements in this figure can be estimated to be, assuming that all of the rotor reactance X_2 comes from the slot:

$$
\begin{aligned}
R_{2,7} &= R_2 \left(\frac{k_7}{k_1}\right)^2 \\
X_{2,7} &= X_2 \left(\frac{k_7}{k_1}\right)^2 \\
X_{m,7} &= X_m \left(\frac{k_7}{7k_1}\right)^2
\end{aligned}
$$

The winding factors are purely the breadth factors, since this machine is full pitched. (I didn't recommend this as an admirable machine, did I?)

$$
k_1 = \frac{\sin m\frac{\gamma}{2}}{m \sin\frac{\gamma}{2}}
$$

$$k_7 = \frac{\sin 7m\frac{\gamma}{2}}{m\sin 7\frac{\gamma}{2}}$$

and $\gamma = \frac{2\pi}{12}$

Currents are found in the usual way and torque is:

$$T = 3\frac{p}{\omega}\left(|i_2|^2\frac{R_2}{s} + 7|i_{2,7}|^2\frac{R_{2,7}}{s_7}\right)$$

This torque is shown in Figure 2. It shows a strikingly large blip of torque about the seventh harmonic speed ($\frac{1}{7}$ of synchronous speed). Note this would be a very bad machine, as it would not start, and so be subject to 'asynchronous crawling'.

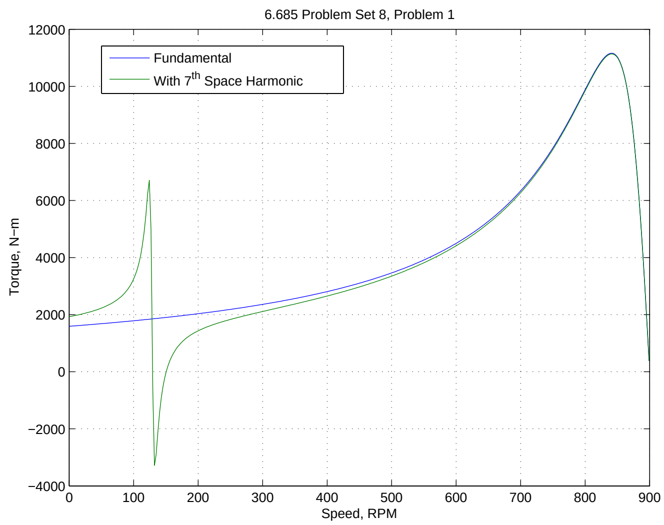

Figure 2: Torque-Speed with Seventh Harmonic

Problem 2: Trolley Car This one is worked entirely with the attached script.

1. First, note that a motor coefficient can be derived that relates back voltage to speed in meters/second. At the rated condition, back voltage must be:

$$E_b = P_m/I = \frac{400,000}{800} = 500V$$

The internal motor resistance is

$$R = \frac{600-500}{800} = 0.125\Omega$$

Then the motor coefficient (in volts/meter/second) must be:

$$G = \frac{E_b}{uI} = \frac{500}{25 \times 800} = .025V-s/A-m$$

Note this is the same as the motor constant if we calculate it based on force: The force that would be produced at 25 m/s and 400,000 watts would be:

$$F = \frac{400,000}{25} = 16,000N$$

and then the motor constant will be:

$$G = \frac{F}{I^2} = \frac{16,000}{800^2} = .025N/A^2$$

Now, if the trolley is drawing 75 kW at 25 m/s, the force at that speed is:

$$F_0 = \frac{75,000}{25} = 3,000N$$

And current required to maintain that speed would be:

$$I = \sqrt{\frac{F}{G}} = \sqrt{\frac{3,000}{.025}} \approx 346A$$

It is also possible to estimate the voltage required as a function of speed by:

$$V = (R + Gu)I$$

and, of course, current is estimated by:

$$I = \sqrt{\frac{F_0}{G}(\frac{u}{u_0})^2}$$

The results are shown in Figure 3

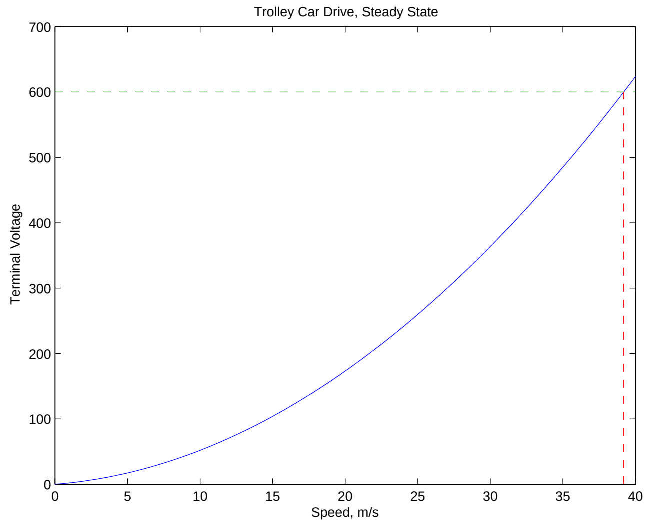

Figure 3: Voltage required vs. speed

2. To find steady speed with a terminal voltage of 600 V:
   The drag coefficient is:
   $$B = \frac{3,000}{25^2} = 4.8N - s^2/m$$
   So then force is:
   $$F = Bu^2 = GI^2$$
   which means that
   $$I = u\sqrt{\frac{B}{G}}$$
   This makes voltage:
   $$V = (R + Gu)I = R\sqrt{\frac{B}{G}}u + \sqrt{BG}u^2$$
   With voltage fixed, we can find a solution for speed:
   $$u = \sqrt{(\frac{R}{2G})^2 + \frac{V}{BG}} - \frac{R}{2G}$$
   and this evaluates to just about 30.2 m/s.
3. So to make a limiting speed of 25 m/s, we need a force of $F = Ku^2$ or, since $K = 4.8N - sec^2/m^2$ and $u = 25m/s$, Force is 3,000 N and current required is 346.41 A. Then back voltage is $E_b = GuI = .025 \times 25 \times 346.41 \approx 216.5V$. Total resistance required is $R_t = \frac{600-216.5}{346.41} \approx 1.107\Omega$. Since internal resistance is $1/8\Omega$, we must add 0.982 Ohms. Dissipation in that is $.982 \times 346.41^2 \approx 118$ kilowatts.

4. If current is limited to 2000 A, the force produced is:

$$F = .025 * 2000^2 = 100,000N$$

And this would be

$$F = Mg \sin \theta$$

So the angle is about $\theta = \sin^{-1} \frac{F}{Mg} \approx 14.8°$. The back voltage must be:

$$E_b = V - RI = 600 - .125 \times 200 \approx 350V$$

and that means speed must be:

$$u = \frac{E_b}{GI} = \frac{350}{.025 \times 2000} = 7m/s$$

5. Simulation of acceleration of the car up a hill is set up in the attached script. The results are shown in Figure 4. Note that the 2,000 A current limit is not reached. As an idiot check, see that to climb a 4° grade, the force required is:

$$F = Mg \sin 4° = 27,378\text{N}$$

and that requires a current of:

$$I = \sqrt{\frac{27,378}{.025}} \approx 1046\text{A}$$

and the simulation of Figure 4 seems to settle out to about that level.

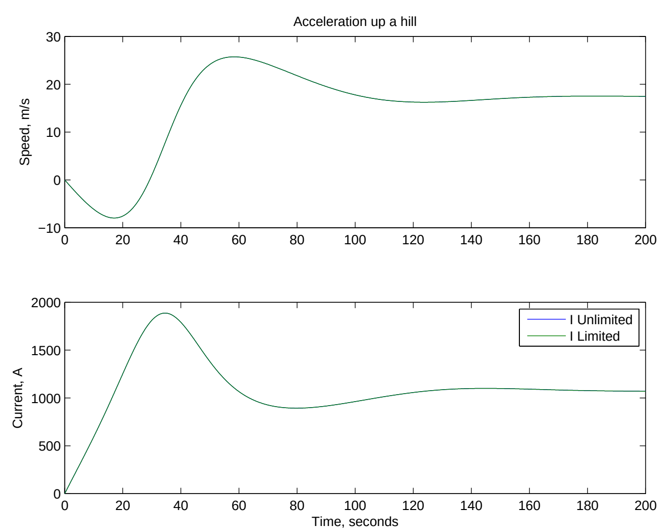

Figure 4: Simulation of Trolley Car Transient

A script for this problem is appended, and the more precise answers printed out by that script are:

```
Trolly Car
Back Voltage = 500
Force Produced = 16000
Force Coefficient G = F/I^2 = 0.025
Back Voltage Coefficient G = E_b/(u I) = 0.025
Drag Coefficient = 4.8 N-sec^2/m^2
Current at 25 m/s = 346.41 A
Maximum Speed at 600 V is 39.1929 M/s
Part 3: to do 25 m/s
Force = 3000 N
Required Current = 346.41 A
Back Voltage = 216.506 V
Total Resistance = 1.10705
Added dropping Resistor = 0.982051
Dissipation in that is 117846 W
Part 4: Maximum Slope at 2000.000 A is 14.7611 degrees
And we can do that at        7 m/s
```

Problem 3: Dynamo The generator will self-excite whenever there is a stable intersection of the excitation curve and the resistance current characteristic of the field winding.

Because of the shape of the curve, the criterion for this intersection is that:

$$\frac{\partial E_{af}}{\partial I_f} > R_f$$

Since the excitation curve is:

$$E_{af} = \frac{N}{N_0}\left(aI_f + b(1 - e^{-\frac{I_f}{I_{f0}}})\right)$$

this means that, for a stable intersection to occur,

$$\frac{N}{N_0}\left(a + \frac{b}{I_{f0}}\right) > 250\Omega$$

And with $a = 5$, $b = 250$ and $I_{f0} = 1$, this makes the minimum speed for self excitation $N \approx 490.2$RPM.

To find the steady state condition at 750 RPM, we have to rely on the nonlinear equation solver. What we do is use the procedure `fzero()` to find a zero of:

$$\frac{N}{N_0}\left(aI_f + b\left(1 - e^{-\frac{I_f}{I_{f0}}}\right)\right) - (R_a + R_f)\,I_f = 0$$

This is written into a function and then the script calls `fzero()`. The script for this is attached. We find, for $\frac{N}{N_0} = 1.5$ that field current is about .946 amperes and resulting internal voltage is about 236 volts.

To get the excitation curve, we repeat this process over a wide range of speeds. The scripts attached show how this is done and the result is shown in Figure 5

To get the output voltage as a function of load current, consider the equivalent circuit shown in Figure 6. An expression for terminal voltage is:

$$V = E_a - R_a\left(I_L + V\frac{R_a}{R_f}\right)$$

And if

$$E_a = \frac{N}{N_0}\left(a\frac{V}{R_f} + b\left(1 - e^{-\frac{V}{R_f}}\right)\right)$$

We have a system that can be easily solved by `fzero()`. The details are shown in the attached scripts.

Now: to compound the machine to achieve zero apparent resistance at zero load, see that if we have a series field with $N_s = \alpha N_f$ turns, we can assign $I_f' = I_f + \alpha I_L$. Voltage is:

$$V\left(1 + \frac{R_a}{R_f}\right) = E(I_f') - R_a I_L$$

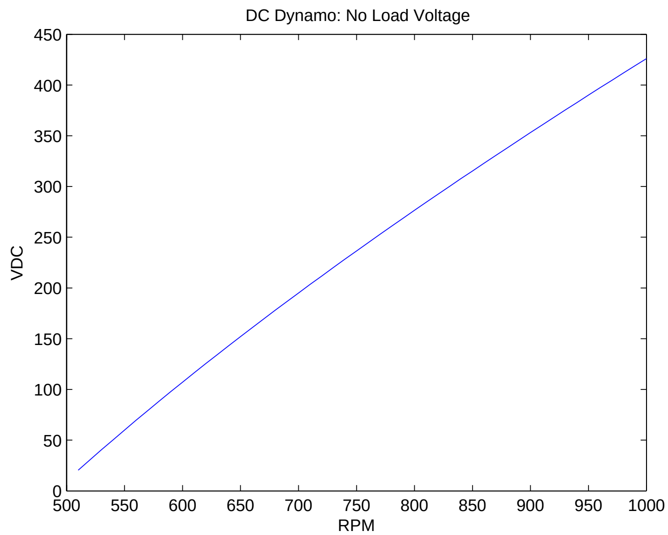

Figure 5: No Load Voltage

Then to achieve zero regulation, note that:

$$\left(1 + \frac{R_a}{R_f}\right) \frac{\partial V}{\partial I_L} = \alpha \frac{\partial E_a}{\partial I_f} - R_a$$

The compounding ratio is then:

$$\alpha = \frac{R_s}{\frac{\partial E_a}{\partial I_f}}$$

To finish this, we compute:

$$\frac{\partial E_a}{\partial I_f} = N_r \left(a + b e^{-\frac{I_f}{I_f 0}}\right)$$

The value of α is found to be just about .0065, so with a 1000 turn field winding, the number of series turns should be about six or seven.

The calculation is carried out by the attached scripts. What is done is to find a solution for voltage: going around the loop:

$$V\left(1 + \frac{R_a}{R_f}\right) - N_r \left(a \quad \frac{V}{R_f} + \alpha I_L \quad + b\left(1 - e^{-\frac{\left(\frac{V}{R_f} + \alpha I_L\right)}{I_f 0}}\right)\right) + R_a I_L = 0$$

This is conveniently done, using `fzero()` with the value of α set to the value calculated here, and then to get the uncompensated value, with α set to zero. The voltage is shown in Figure 7.

Finally, we simulate voltage buildup in the machine, without the compound field winding. This is straightforward, handled by the scripts that are attached and the result is shown in figure 8.

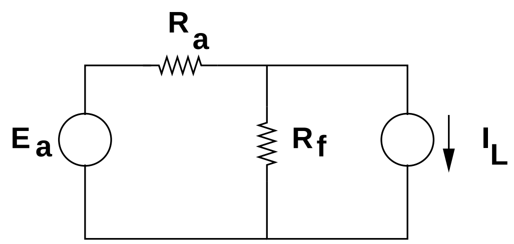

Figure 6: Dynamo Circuit

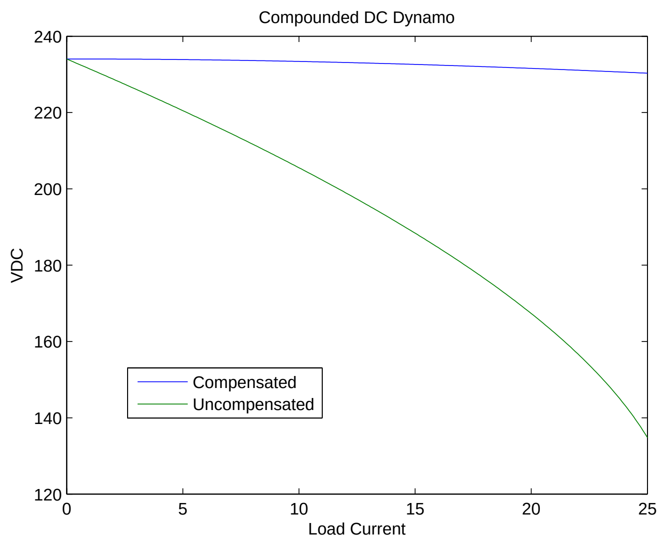

Figure 7: Dynamo Circuit

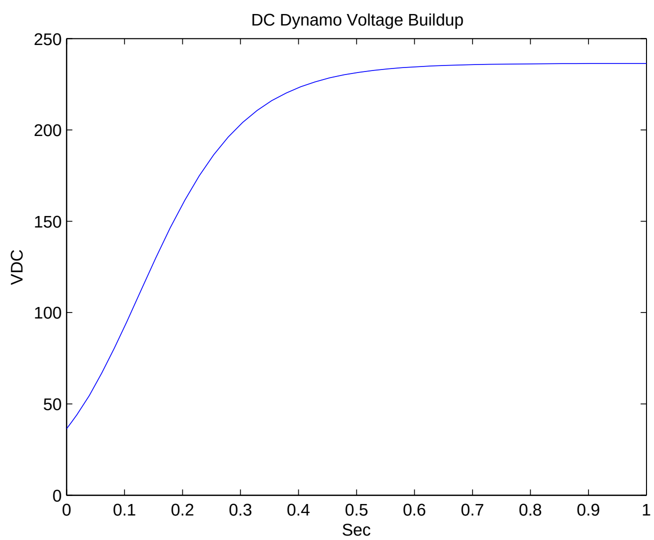

Figure 8: Voltage Buildup

Script for Problem 1:

```
% 6.685 Problem Set 8, Problem 1 (2013)
% this is a 350 kW induction motor
% torque_speed curve as affected by seventh space harmonic

Vz = 600/sqrt(3);       % line-neutral voltage (RMS, line-neutral)
fz = 60;                % Line frequency
p = 4;                  % number of pole pairs
x1z = .038;             % stator leakage reactance
x2z = .114;             % rotor leakage reactance
r1 = .017;              % stator resistance
r2z = .010;             % rotor resistance: low frequency limit
xmz = 10.0;             % magnetizing reactance
Pfw0 = 8000;            % friction and windage base
epsw = 3;               % speed exponent of friction and windage
Pc0 = 10000;            % core base loss (w)
slc = .025;             % stray load coefficient
epsf = 1.8;             % core loss frequency exponent
epsb = 2.2;             % core loss flux exponent
Rcz = 3*Vz^2/Pc0;       % core parallel element
tol = 1e-4;             % tolerance for resistor loops
crit = 2e-3;            % tolerance for getting close to 1/7 speed

% part 1: ordinary torque/speed curve

Rc = Rcz;                          % use the parallel core loss
                                   % element as-is
f = fz;                            % and line frequency
x1 = x1z;                          % so reactances are base
x2 = x2z;
xm = xmz;
V = Vz;

om = 2*pi*f;                       % frequency in radians/second
s = logspace(-3,0,2000);            % use this range of slip

% first, calculate with only the space fundamental
r2 = r2z;                          % this is R2
Zr = j*x2 + r2 ./ s;               % rotor impedance
Zm = j*xm*Rc/(j*xm + Rc);          % magnetizing element impedance
Zag = Zr .* Zm ./ (Zr + Zm);       % air-gap impedance
Zt = j*x1 + r1 + Zag;              % terminal impedance
it = V ./ Zt;                      % terminal current
i2 = it .* Zm ./ (Zm + Zr);        % rotor current
Pag = 3 .* abs(i2) .^2 .* r2 ./ s;% air-gap power
```

```
Tn = (p/om) .* Pag;                % this is torque
omm = (om/p) .* (1 - s);           % mechanical speed
N = (60/(2*pi)) .* omm;            % in RPM, for convenience

% now get the seventh harmonic stuff
m = 2;                             % slots per pole per phase
gama = 2*pi/(6*m);                   % slot angle
k1 = sin(m*gama/2)/(m*sin(gama/2));
k7 = sin(m*7*gama/2)/(m*sin(7*gama/2));
r27 = r2z*(k7/k1)^2;
x27 = x2z*(k7/k1)^2;
xm7 = xmz*(k7/(7*k1))^2;
s7=7 .* s -6;
% correction for stator leakage:
xl7 = xm7*x27/(xm7+x27);
x1 = x1z - xl7;
Zr = j*x2 + r2  ./ s;       % rotor impedance
Zm = j*xm*Rc/(j*xm + Rc);          % magnetizing element impedance
Zag = Zr .* Zm ./ (Zr + Zm);       % air-gap impedance
Zr7 = j*x27 + r27 ./s7;            % rotor impedance at 7th
Zag7 = j*xm7*Zr7 ./(j*xm7+Zr7);      % air-gap impedance at 7
Zt = j*x1 + r1 + Zag + Zag7;       % terminal impedance
it = V ./ Zt;                      % terminal current
i2 = it .* Zm ./ (Zm + Zr);        % rotor current
i27 = it .* j*xm7 ./(j*xm7 + Zr7);  % seventh rotor current
Pag = 3 .* abs(i2) .^2 .* r2 ./ s;
Pag7 = 3 .* abs(i27) .^2 .* r27 ./ s7;% air-gap power
T = (p/om) .* Pag + (7*p/om) .* Pag7;                % this is torque

figure(1)
clf
plot(N, Tn, N, T)
title('6.685 Problem Set 8, Problem 1')
ylabel('Torque, N-m')
xlabel('Speed, RPM')
grid on
legend('Fundamental', 'With 7^{th} Space Harmonic')

\clearpage
\noindent Scripts for Problem 3:
\begin{verbatim}
% 6.685 Problem set 8: trolley car problem

global V_0 M L G R K Ilim
```

```
M = 40000;             % car mass
g = 9.812;             % acceleration due to gravity
u_0 = 25;              % meters per second
u_lim = 25;            % for the part on limiting resistor
V_0 = 600;             % voltage
I_0 = 800;             % current at that (base) condition
P_0 = 400000;          % producing this much power
L =10;                 % winding inductance

P_d = 75000;           % dissipation at 25 m/s
F_d0 = P_d/u_0;        % basic drag force

eps_d = 2;             % force is square law
E_b = P_0/I_0;         % this must be the back voltage
R = (V_0 - E_b)/I_0;   % and this must be armature+field resistance
F = P_0/u_0;           % the motor is making this much force
G = F/I_0^2;           % force coefficient (on base speed)
Gc = E_b/(u_0*I_0);    % just to check

fprintf('Trolly Car\n')
fprintf('Back Voltage = %g\n', E_b)
fprintf('Force Produced = %g\n', F)
fprintf('Force Coefficient G = F/I^2 = %g\n', G)
fprintf('Back Voltage Coefficient G = E_b/(u I) = %g\n', Gc)
% Part 1: Steady Operation
I_s = sqrt(F_d0/G);

% Part 2: calculation of max speed at voltage
K = F_d0/u_0^2;
u_max = sqrt((.5*R/G)^2 + V_0/sqrt(K*G)) - .5*R/G;
fprintf('Drag Coefficient = %g N-sec^2/m^2\n', K)
fprintf('Current at %g m/s = %g A\n', u_0, I_s)
fprintf('Maximum Speed at %g V is %g M/s\n', V_0, u_max)

% part 3: to do u_lim
Force = K*u_lim^2;
Ireq = sqrt(Force/G);
Eback = G*u_lim*Ireq;
Rtot = (V_0-Eback)/Ireq;
Rlim = Rtot - R;
fprintf('Part 3: to do %g m/s\n', u_lim)
fprintf('Force = %g N\n', Force)
fprintf('Required Current = %g A\n', Ireq)
fprintf('Back Voltage = %g V\n', Eback)
fprintf('Total Resistance = %g\n', Rtot)
```

```
fprintf('Added dropping Resistor = %g\n', Rlim)
fprintf('Dissipation in that is %g W\n', Ireq^2 * Rlim)


I_lim = 2000;        % what force can we do?
F_lim = G*I_lim^2;   % this is maximum force
E_lim = V_0 - R*I_lim; % and this is back voltage at that current
u_lim = E_lim/(I_lim*G); % and this is how fast we can go
phi_lim = asin(F_lim/(M*g));  % this is the angle we can climb
phi_deg = (180/pi) * phi_lim; % in degrees
fprintf('Part 4: Maximum Slope at %8.3f A is %g degrees\n',I_lim, phi_deg)
fprintf('And we can do that at %8.3g m/s \n', u_lim)


%So lettuce print what the trolley can do: voltage vs. speed

u = 0:.1:40;        % over this range of speeds
F = F_d0 .* (u ./ u_0) .^eps_d;  % drag force as a function of speed
I = sqrt(F ./ G);    % current required to drive the thing
Eb = G .* I .* u;   % back voltage produced
V = Eb + R .* I;    % and this is terminal voltage

figure(1)
clf
plot(u, V, [0 40], [600 600], '--', [u_max u_max], [0 600], '--')
title('Trolley Car Drive, Steady State')
ylabel('Terminal Voltage')
xlabel('Speed, m/s')

% now we are about to simulate the motor

tt = 0:.1:200;
X0 = [0 0];

Ilim = 10000;       % big enough it doesn't count

[tn, Xn] = ode23('tcsim', tt, [0 0]);

Ilim = 2000;

[tl, Xl] = ode23('tcsim', tt, [0 0]);

in = Xn(:,1);
un = Xn(:,2);
il = Xl(:,1);
```

```
ul = Xl(:,2);

figure(2)
clf
subplot 211
plot(tn, un, tl, ul)
title('Acceleration up a hill')
ylabel('Speed, m/s')
%axis([0 100 -5 20])
subplot 212
plot(tn, in, tl, il)
ylabel('Current, A')
xlabel('Time, seconds')
legend('I Unlimited', 'I Limited')

----------------------
function xdot = tcsim(t, X)

global V_0 M L G R K Ilim

g = 9.812;
th = pi*2/180;          % this is the grade

i = X(1);
u = X(2);
idotp = (V_0 - (G*u + R) *i)/L;

if (i >= Ilim && idotp > 0)
    idot = 0;
else
    idot = idotp;
end


udot = (G*i^2 - u^2*K - M*g*sin(th))/M;

xdot = [idot udot]';
```

Scripts for Problem 3

```
% voltage vs. speed for the DC generator
% and voltage buildup
global Nr a b Rf Lf
N_0 = 500;
N_op = 750;
Rf = 250;
Lf = 10;
a = 5;
b = 250;
i_f = 0:.01:10;

eaf = a .*i_f + b .* (1-exp(-i_f));

ifs = 0:.01:1;
vfs = Rf .* ifs;

figure(1)
plot(i_f, (N_op/N_0) .* eaf)
tit = sprintf('DC Dynamo Exitation Curve at %4.0f RPM', N_op);
title(tit)
ylabel('Armature Voltage, V')
xlabel('Field Current, A')
grid on

% voltage at some speed:
Nr = N_op/N_0;
isubf = fzero('dcf', [.01 10]);
Ea = Nr * (a*isubf + b*(1-exp(-isubf)));

fprintf('At %g RPM, I_f = %g and E_a = %g\n', N_op, isubf, Ea)

% ok now find excitation curve

N = 500:5:1000;

E = zeros(size(N));

for k = 1:length(N)
    Nr = N(k)/N_0;
    isubf = fzero('dcf', [.01 10]);
    E(k) = Nr * (a*isubf + b * (1-exp(-isubf)));
end

figure(2)
```

```
plot(N, E)
title('DC Dynamo: No Load Voltage')
ylabel('VDC')
xlabel('RPM')

%Now, simulate buildup

i_f0 = .1;
t_0 = [0 1];
Nr = 1.5;

[t, i_ft] = ode45('slopes', t_0, i_f0);

figure(3)
plot(t, i_ft)

E_af = Nr .* (a .* i_ft + b .* (1-exp(-i_ft)));

figure(5)
plot(t, E_af)
title('DC Dynamo Voltage Buildup')
ylabel('VDC')
xlabel('Sec')
--------------------
function z = dcf(i_f)
% this one gets zeroed
global Nr a b Ra Rf Lf Rff alf Il

z = Nr * (a*i_f + b*(1-exp(-i_f))) - Rf*i_f;
--------------------
function difdt = slopes(t, x)
global Nr a b Rf Lf

difdt = (Nr * (a*x + b*(1-exp(-x))) - Rf*x)/Lf;
--------------------
% compounding of that odd DC generator
global Nr a b Ra Rf Lf Rff alf Il
N_0 = 500;
Lf = 5;
Rff = 249;
a = 5;
b = 250;
Ra = 1;         % armature part of resistance
Rf = Ra+Rff;    % for original loop
Nr = 1.5;       % 750 RPM
```

```
Il = 0;                             % with zero load current,
alf = 0;                            % need to phony this up
Vz = fzero('dcgf', 300);            % starting point voltage
isubf = Vz/Rff;                     % starting point field current
Eaf = Nr * (a*isubf + b*(1-exp(-isubf))); % voltage at this point

dE = Nr * (a + b * exp(-isubf));   % slope of voltage vs. current

alf = Ra/dE;                        % required turns ratio for series field
fprintf('Dynamo Negative Impedance at Equilibrium Current = %g\n', dE)
fprintf('Zero Load Field Current = %g\n', isubf)
fprintf('Series Field Turns Ratio = %g\n', alf)

I_l = 0:.1:25;                      % load current
V = zeros(size(I_l));               % leave some space

for k = 1:length(I_l)               % here we find compensated voltage
    Il = I_l(k);
    V(k) = fzero('dcgf', Eaf);
end
fprintf('isubf = %g   Eaf = %g\n', isubf, Eaf)
fprintf('Vz = %g   alf = %g\n', Vz, alf)

alf = 0;                            % to generated uncompensated voltage
Vn = zeros(size(I_l));
for k = 1:length(I_l)
    Il = I_l(k);
    Vn(k) = fzero('dcgf', Eaf);
end

figure(4)
plot(I_l, V, I_l, Vn)
title('Compounded DC Dynamo')
ylabel('VDC')
xlabel('Load Current')
legend('Compensated', 'Uncompensated')
-----------------
function z = dcgf(V)
global Nr a b Ra Rf Lf Rff alf Il

% computes the terminal voltage difference
z = V*(1+Ra/Rf) - Nr*(a*(V/Rf + alf*Il) + b*(1-exp(-(V/Rf + alf*Il))))
+ Ra*Il;
```