Problem Set 3

Problem 1 Prove that the two expressions we have derived for torque:

$$T^e = -\frac{3}{2}\frac{p}{\omega}\frac{VE_{af}}{X_d}\sin\delta = -\frac{3}{2}pMII_f\sin\delta_i$$

are consistent.

Problem 2 We are going to make an inductor, using two 'C' cores as shown, in somewhat cartoonish form in Figure 1. Two of these are assembled, together with a non-conducting, non-magnetic ($\mu = \mu_0$) spacer. To start the problem, assume that spacer has thickness $g = 1\text{mm}$ and the total number of turns in the two coils is $N = 100$. For the purposes of this problem you may use the 'nice number theorem' and assume that $1/\mu_0 = 800,000\text{m/A}$.
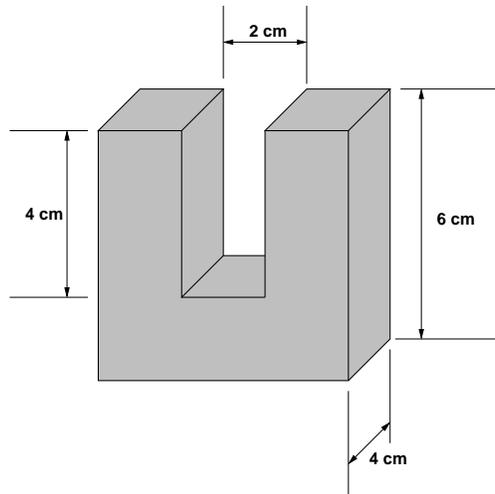
Figure 1: One 'C' core

1. Verify that, assuming the core permeability to be very high, the inductance is very nearly $L = 10mHy$.

2. Now, assume that core permeability is $\mu = 100\mu_0$. Estimate the actual inductance. Assume the effective length of the core magnetic circuit is the length of a line midway between the inside and outside surfaces of the core.

3. There will be two limits to how much current this inductor can carry. Estimate both of those limits.

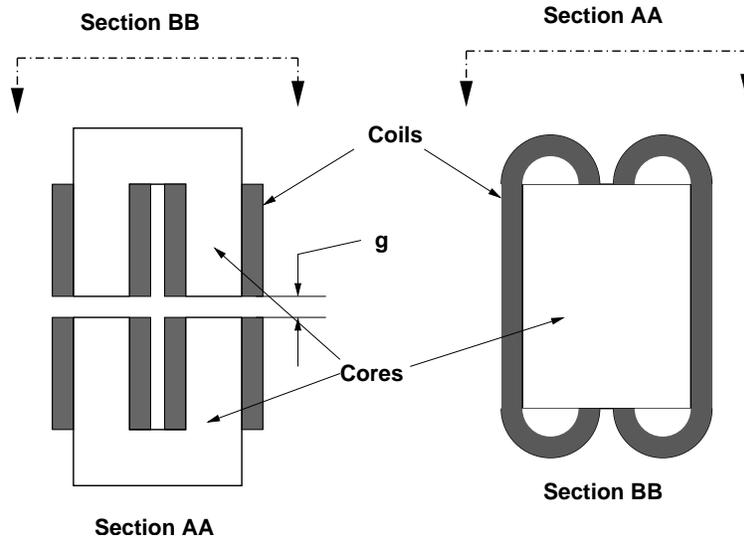   (a) Magnetic Saturation Limit: when the core flux density reaches 2 T.

Figure 2: Inductor made with two 'C' cores and four coils of wire

   (b) Heating Limit: when the current density in the coils reaches $3 \times 10^6 \mathrm{A}/m^2$ Assume this to be the 'superficial' current density over the area of the coils. And assume that the coils occupy the whole space inside the core 'window'. (Figure 2 shows the coils occupying less than the whole window).

  4. Now modify the number of turns and gap so that the magnetic limit and the heating limit are at the same current.

Problem 3 Figure 3 shows, in cartoon-ish form, a transformer core. The dimensions of this transformer are:

Core Central Column Width	W_c	100 mm
Window Width	W	100 mm
Window Height	h	200 mm
Depth	D	60 mm

This core is to be used to make a two-winding transformer similar to one you might see on a distribution pole, supplying power to a number of houses. Primary and secondary voltages are 8,000 and 240 volts, RMS. These numbers correspond with a 13.8 kV, RMS distribution primary voltage and a center tapped ±120 volt feed to the houses. Assume each winding has 5 mm of insulation around it and that the two windings (high and low voltage) take up the same amount of the window. The core is to be operated at a flux density of about 1.25 T (RMS). This is a 60 Hz transformer. Assume a rating of 50 kVA. Along with this problem set there is some data for M-19 electrical steel sheets. While it is highly unlikely anyone would build this type of transformer with M-19, we will assume that core material.

  1. What is the 'superficial' current density in the windings?

  2. What are the required number of primary and secondary turns?

  3. What is the core reactance, referred to the 8000 volt side?

  4. Estimate leakage reactance, assuming the only important parts are associated with the windings actually in the window.

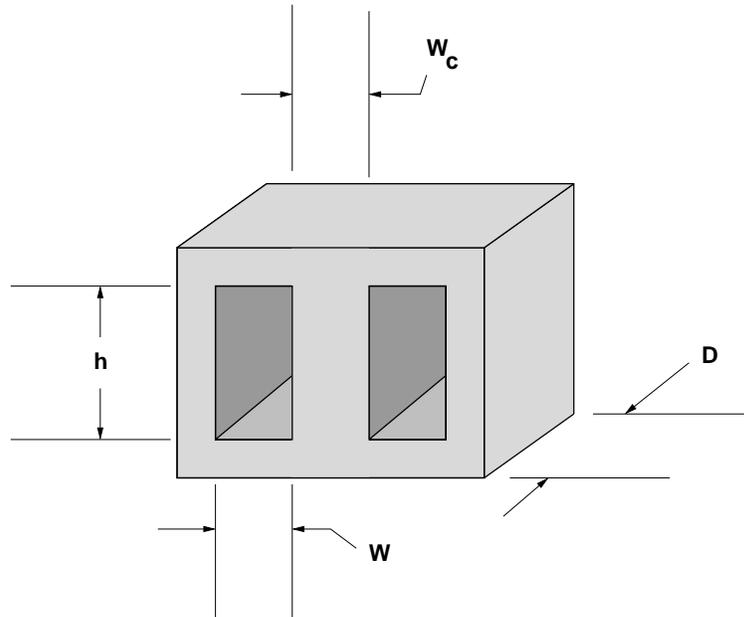 5. Estimate no-load loss for this transformer.

Figure 3: Your Core

6. Estimate full-load loss for this transformer.

Problem 4 The objective of this problem is to look into losses in magnetic iron. As you know, magnetic circuits are typically made up of thin sheets of magnetic steel. Assume you have some steel with the following properties:

Conductivity	3×10^6	S/m
Density	7,200	kg/m^3
Thickness	$\frac{1}{2}$	mm

1. Find the loss density in W/kg for this material operating at a flux density of 1 T, RMS at 60 Hz. Assume the flux density is actually *in* the steel and that it is in a direction that is parallel to the sheet (that is, the favorable direction).

2. Now, suppose you stack a bunch of these sheets of steel with insulating film spacers that are .002" (.05 mm) thick between them. This makes the flux density in the steel higher than the 'superficial' flux density. What is the loss density now, with the 'superficial' flux density at 1 T (RMS).

3. Now consider that situation in which there is also hysteresis loss in the steel. This hysteresis loss is not dependent on sheet thickness, but for the purpose of this problem, assume it is dependent on flux density squared. At 1 T and 60 Hz, assume the hysteresis loss density is 2.5 W/kg. You are now allowed to roll the steel to arbitrary thickness. What is the thickness that minimizes total loss at a superficial flux density of 1 T, 60 Hz, with .05 mm spacers?

Problem 5 This problem concerns a synchronous generator. Here is what we know about the machine. It is 'round rotor', meaning the stator winding inductances are not a function of rotor position and:

- This is a two-pole, 60 Hz, three-phase machine.

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- The rating of this machine is 1,000 MVA with a terminal voltage of 26 kV, RMS, phase-phase. This means that phase voltage is about 15,011 V, RMS.

- The machine is capable of reaching full MVA at a power factor of 0.85, overexcited.

- Under test at rated speed and with the stator winding open, field current required to produce rated terminal voltage is AFNL = 2501 A.

- Also under test with the stator winding terminals shorted together, field current required to produce rated terminal current in the winding is AFSI = 5003 A.

You may neglect armature winding resistance.

1. What is the armature current rating of the machine, per phase?
2. What is the field-to-phase mutual inductance of this machine?
3. What is the synchronous inductance of the machine?

Save your notes on this machine: you will see it again.

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