MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

Receivers, Antennas, and Signals – 6.661

Solutions -- Problem Set No. 5 March 13, 2003

Problem 5.1

a) Photo-excitations can occur anywhere photons are located within a semiconductor, but only when those photons lift electrons to levels where they can cross any imposed voltage barrier do they contribute diode current. See Figure 2.4-7 for a back-biased diode where the n-type semiconductor is sufficiently negative that most electrons thermally excited to the conduction band in the n-type region contribute directly to i_o. In the p-type region electrons excited to the conduction band can exit the positive terminal, while the resulting hole can exit the negative terminal. Thus excitations in **all regions contribute to i_o,** but if hole mobility is less than electron mobility, then those electrons thermally excited in the **n-type region may contribute more to i_o**. In silicon, electron mobility is more than twice that of the holes, and in InSb this ratio exceeds 100.

b) $v_{shot} = R(2Be\ \bar{i})^{0.5}$ and $v_J = (4kTBR)^{0.5}$. Equating them $\Rightarrow$ $\boxed{R = 2kT/(e\ \bar{i})}$

c) The dark current is associated with thermal excitations of electrons across the energy gap. We shall assume the state density (levels per e.v.) is uniform in the conduction band. Although the gap between the acceptor levels and the conduction band is slightly less than E_g, the doping density is generally low so that most excitations jump the full gap. The Boltzmann energy distribution for photons/phonons is exponential $\propto e^{-E/kT}$, and the current is proportional to the integral of the upper tail of this distribution where $E > \sim E_g$. Thus,

$$i_o \propto \int_{Eg}^{\infty} Te^{-E/kT}\, dE = (-kT^2)e^{-E/kGT}\Big|_{Eg}^{\infty} = \boxed{(kT^2)e^{-Eg/kT}.}$$

Because generally $E_g >> kT$, the exponential dependence on T dominates (one e.v. corresponds to $T \cong 11{,}600K$). In silicon $E_g \cong 1.12$ volts, and has a small temperature dependence, ignored here.

Problem 5.2

For both PD's and APD's the CNR is:

$$\frac{\eta P_S/hf2B}{[(1 + P_D/P_S)\langle g^2\rangle/G^2] + 2kThf/(R_L\eta P_S(eG)^2)} \qquad (1)$$

where $\langle g^2\rangle = G^2 = 1$ for PD's (see (2.4.14) in the text), and $\langle g^2\rangle/G^2 \cong G^x$ for APD's $\cong G^{0.3}$ here . APD's perform better than PD's when the corresponding APD denominator is smaller, i.e. when:

$$[(1 + P_D/P_S)G^{0.3}] + 2kThf/(R_L\eta P_S(eG)^2) < 1 + P_D/P_S + 2kThf/(R_L\eta P_S e^2), \text{ or when}$$

$$\boxed{(1 - G^{-2})/(G^{0.3} - 1) > (1 + P_D/P_S)R_L\eta P_S e^2/2kThf \Rightarrow \text{APD's better}}$$

where $(1 - G^{-2})/(G^{0.3} - 1) \cong 1 + G^{-0.3}$ for $G >> 1$.

b) Referring to (1) above, which applies to all photodetectors, PMT's essentially always outperform PT's, all parameters except G being equal. However this advantage diminishes significantly when $\boxed{2kThf \; <\sim R_L\eta P_S e^2}$. PD's and PT's have the same CNR equation, but the quantum efficiency η for PD's is typically several times greater than for PT's, while the dark current power P_D is many times greater. Thus there is a preference for PT's when the dark-current disadvantage of photodiodes outweighs their efficiency advantage: i.e. we prefer PT's over PD's when:

$$(P_{DD}/P_S) + 2kThf/(R_L\eta_D P_S e^2) > (P_{DT}/P_S) + 2kThf/(R_L\eta_T P_S e^2), \text{ or}$$

$$\boxed{(P_{DD} - P_{DT})/(\eta_D - \eta_T) > 2kThf/(R_L e^2\eta_T\eta_D) \Rightarrow \text{PT's better}}$$

Referring to (1) again, we see that PMT's have better CNR's than APD's when the excessive shot noise of APD's is more serious than their advantage in quantum efficiency η_D:

$$G^x(1 + P_{DD}/P_S) + 2kThf/(R_L\eta_D P_S(eG)^2) > 1 + P_{DT}/P_S + 2kThf/(R_L\eta_T P_S(eG)^2)$$

or, $\boxed{G^x[1 + (P_{DD} - P_{DT})/P_S]/[\eta_D - \eta_T] > 1 + 2kThf/(R_L\eta_T\eta_D P_S(eG)^2) \Rightarrow \text{PMT's better.}}$

Although refrigeration can markedly reduce P_{DD} (see problem 5.1c above), it is expensive below ~240K and especially 70K, 20K, or 4K (four common operating zones).

<u>Problem 5.3</u>

Referring to Figures 2.4-13 and 2.4-14 in the text, we see that A = ge, and $I_o = \bar{n}e \propto \langle g \rangle = A_o/e$ here, so in one case we have $g \equiv 1$, and in the other case, g = 2/3 or 4/3 (equally likely). The noise spectral density is proportional to $\langle g^2 \rangle$, or to 1 and 10/9 for the two devices. The rms noise is proportional to the square root of these numbers, so the ratio of the rms noises for the non-uniform device divided by that for the uniform device is $(10/9)^{0.5} = \boxed{1.054.}$

Problem 5.4

a) Referring to (2.4.60) for the desired case where $\tau \cong 1/B$, we have CNR < $\sim\eta S/4B$. Therefore, we need $S > \sim 4B\,(CNR)/\eta = 4\times3000\times100/\eta \cong 1.2\times10^6$ for PD's with $\eta\cong1$. See pp 2-87 to 2-88 of new notes.

b) For $D = 10^5$, we choose a local oscillator power P sufficient to overwhelm D, so the answer to (a) remains unchanged. However, since 2D >> B here, we would not obtain a better CNR if we used a simple detector—see (2.4.62-3). Since 2D>>B, superheterodynes are better

Problem 5.5

a) Heat balance: $I^2R = G_t(T - T_b)$, so $R = G_t(T - T_b)/I^2 = 10^{-7}(22 - 18)/10^{-6} =$ 0.4 ohms

b) Johnson noise $n_J = (4kTBR)^{0.5} = (4 \times 1.38\times10^{-23} \times 22 \times 0.5 \times 0.4)^{0.5} = 1.6\times10^{-11}$ volts

c) $NEP_J = n_J/S$ (see (2.4.23), where (see (2.4.21)) $S = -IT_dR(1 + [I^2T_dR/G_tT^2])/G_tT^2 = -M/(1 + IM)$ where $M = IT_dR/G_tT^2 = 10^{-3} \times 5 \times 0.4/(10^{-7} \times 22^2) = 41.3$, so

$|S| = 41.3/(1 + 10^{-3}\times41.3) = 40$ and

$NEP_J = 1.6\times10^{-11}/40 = 4\times10^{-13}$ [W Hz$^{-0.5}$]

d) See (2.4.27): $NEP_{phonon} \cong (4kG_tT^2)^{0.5} \cong (4\times1.38\times10^{-23}\times10^{-7}\times20^2)^{0.5} =$

$NEP_{phonon} \cong 4.7\times10^{-14}$ [W Hz$^{-0.5}$]

e) See (2.4.51-3): $\Omega = 2\pi$ here. $P_r = A\Omega\,\sigma_{SB}T^4/\pi = 10^{-6} \times 2\pi \times 5.67\times10^{-8} \times 22^4/3.14 = 2.66\times10^{-8}$ watts. $NEP_{photon\ noise} = (P_r16kT)^{0.5} = (2.66\times10^{-8} \times 16 \times 1.38\times10^{-23} \times 22)^{0.5} = NEP_{photon\ noise} = 1.14\times10^{-14}$ [W Hz$^{-0.5}$]

f) Comparing (c-d) we see that phonon noise dominates, followed by Johnson noise. To reduce phonon noise we might simply reduce the diode temperature T and the thermal conductivity G_t, which could be contradictory. The remedy is probably to reduce the bath temperature from 18 to 4K or so, thus dropping T substantially (note that it is squared in (d)) and permitting G_t to drop too if necessary. This also reduces n_J. If such a drop in bath temperature were not feasible, we might halve G_t, thus doubling the temperature difference between the 18K bath and the (now) 26K diode temperature. This reduces $(G_tT^2)^{0.5}$ by a factor of 1.2. We could also drop NEP_J by increasing the bias current I, but that would heat the diode, worsening the phonon noise problem. This problem illustrates why it is difficult to build bolometers with NEP below 10^{-14} without going to extremely low bath temperatures.

Problem 5.6

$P_{rec} = P_{trans}(G_t/4\pi r^2)(G_{rec}\lambda^2/4\pi) = 10(10^5/4\times3.14\times10^{22})(10^7\times0.03^2/4\pi) = 5.7\times10^{-15}$ W

$SNR = P_{rec}/N = 100 = P_{rec}/kT_sB$; $N = kT_sB$

Therefore $B = P_{rec}/[kT_s(SNR)] = 5.7\times10^{-15}/[1.38\times10^{-23}\times20\times100] =$ 206.5 kHz