

MIT OpenCourseWare
<http://ocw.mit.edu>

6.641 Electromagnetic Fields, Forces, and Motion
Spring 2009

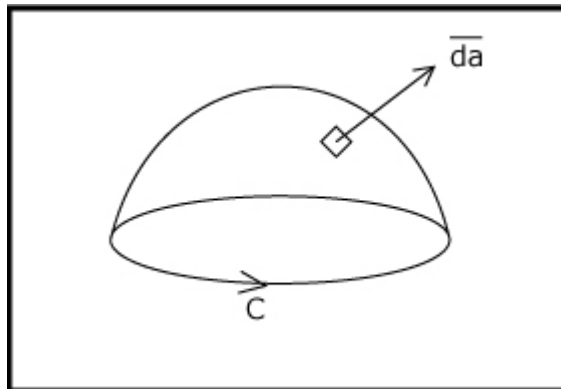
For information about citing these materials or our Terms of Use, visit: <http://ocw.mit.edu/terms>.

Lecture 1: Integral Form of Maxwell's Equations

I. Maxwell's Equations in Integral Form in Free Space

1. Faraday's Law

$$\underbrace{\oint_C \vec{E} \cdot d\vec{s}}_{\text{Circulation of } \vec{E}} = - \frac{d}{dt} \underbrace{\int_S \mu_0 \vec{H} \cdot d\vec{a}}_{\text{Magnetic Flux}}$$



$\mu_0 = 4\pi \times 10^{-7}$ henries/meter
 [magnetic permeability of free space]

EQS form: $\oint_C \vec{E} \cdot d\vec{s} = 0$ (Kirchoff's Voltage Law, conservative electric field)

MQS circuit form: $v = L \frac{di}{dt}$ (Inductor)

2. Ampère's Law (with displacement current)

$$\underbrace{\oint_C \vec{H} \cdot d\vec{s}}_{\text{Circulation of } \vec{H}} = \underbrace{\int_S \vec{j} \cdot d\vec{a}}_{\text{Conduction Current}} + \underbrace{\frac{d}{dt} \int_S \epsilon_0 \vec{E} \cdot d\vec{a}}_{\text{Displacement Current}}$$

MQS form: $\oint_C \vec{H} \cdot d\vec{s} = \int_S \vec{j} \cdot d\vec{a}$

EQS circuit form: $i = C \frac{dv}{dt}$ (capacitor)

3. Gauss' Law for Electric Field

$$\oint_S \epsilon_0 \vec{E} \cdot \vec{da} = \int_V \rho dV$$

$$\epsilon_0 \approx \frac{10^{-9}}{36\pi} \approx 8.854 \times 10^{-12} \text{ farads/meter}$$

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} \approx 3 \times 10^8 \text{ meters/second (Speed of electromagnetic waves in free space)}$$

4. Gauss' Law for Magnetic Field

$$\oint_S \mu_0 \vec{H} \cdot \vec{da} = 0$$

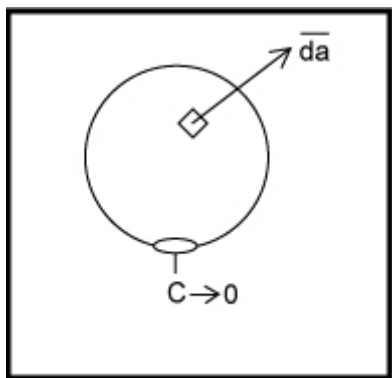
In free space:

$$\vec{B} = \mu_0 \vec{H}$$

\swarrow magnetic flux density (Teslas)	\searrow magnetic field intensity (amperes/meter)
---	---

5. Conservation of Charge

Take Ampère's Law with displacement current and let contour $C \rightarrow 0$



$$\lim_{C \rightarrow 0} \oint_C \vec{H} \cdot \vec{ds} = 0 = \oint_S \vec{J} \cdot \vec{da} + \frac{d}{dt} \underbrace{\oint_S \epsilon_0 \vec{E} \cdot \vec{da}}_{\int_V \rho dV}$$

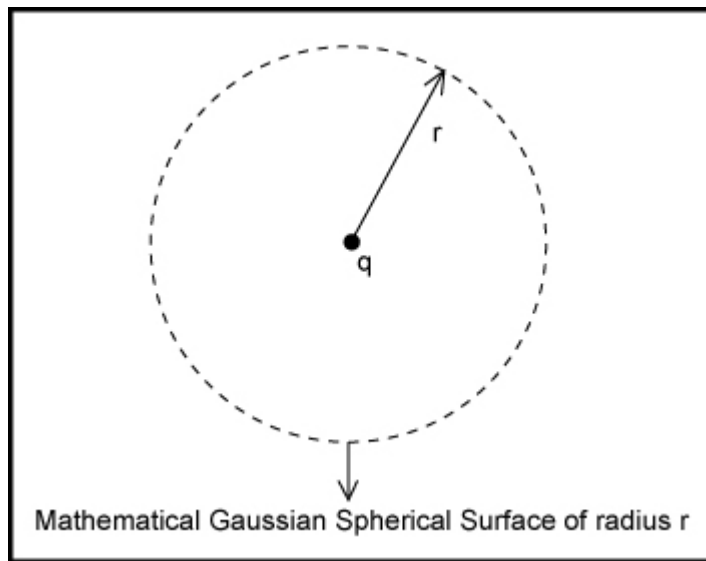
$$\underbrace{\oint_S \vec{J} \cdot d\vec{a}} + \frac{d}{dt} \underbrace{\int_V \rho dV}_{\text{Total charge inside volume}} = 0$$

Total current leaving volume through surface Total charge inside volume

6. Lorentz Force Law

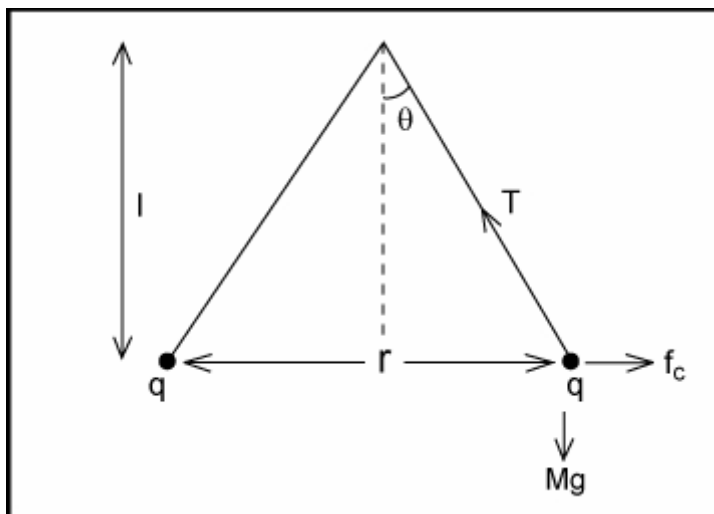
$$\vec{f} = q(\vec{E} + \vec{v} \times \mu_0 \vec{H})$$

II. Electric Field from Point Charge



$$\oint_S \epsilon_0 \vec{E} \cdot d\vec{a} = \epsilon_0 E_r 4\pi r^2 = q$$

$$E_r = \frac{q}{4\pi \epsilon_0 r^2}$$



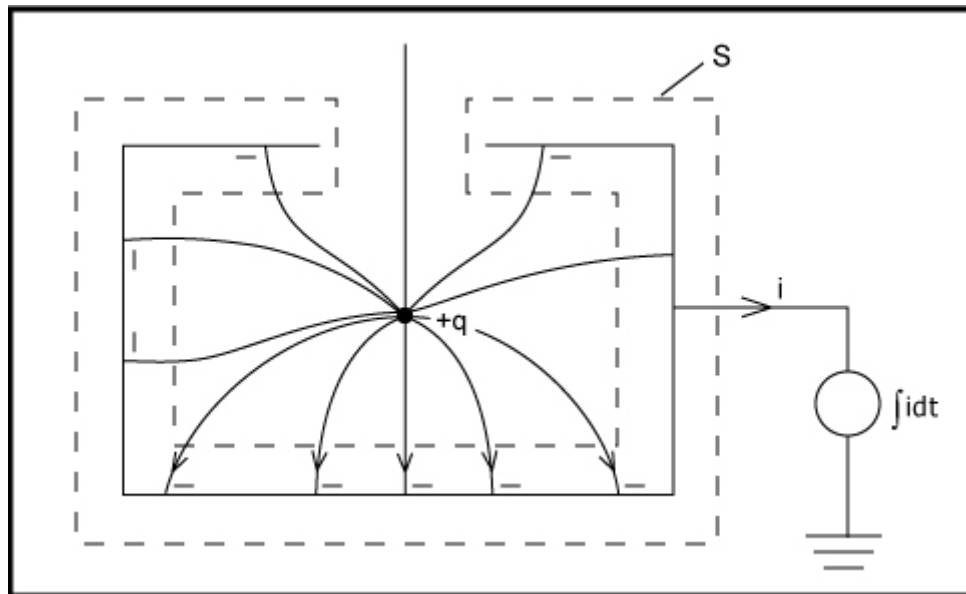
$$T \sin \theta = f_c = \frac{q^2}{4\pi \epsilon_0 r^2}$$

$$T \cos \theta = Mg$$

$$\tan \theta = \frac{q^2}{4\pi \epsilon_0 r^2 Mg} = \frac{r}{2l}$$

$$q = \left[\frac{2\pi \epsilon_0 r^3 Mg}{l} \right]^{1/2}$$

III. Faraday Cage



$$\oint_S \vec{J} \cdot d\vec{a} = i = -\frac{d}{dt} \int \rho dV = -\frac{d}{dt} (-q) = \frac{dq}{dt}$$

$$\int idt = q$$

IV. Boundary Conditions

1. Gauss' Continuity Condition

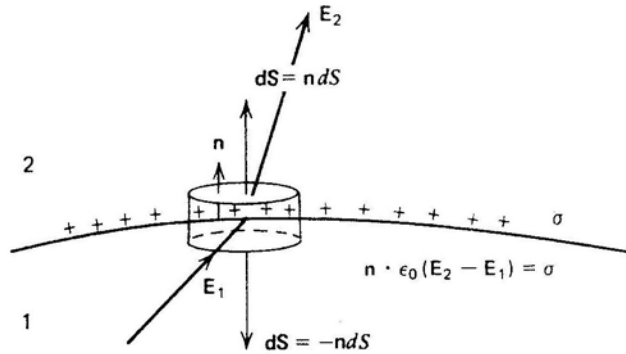
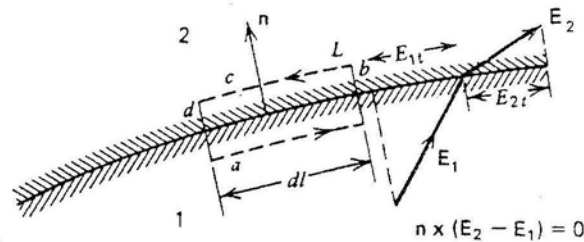


Figure 2-19 Gauss's law applied to a differential sized pill-box surface enclosing some surface charge shows that the normal component of $\epsilon_0 \mathbf{E}$ is discontinuous in the surface charge density.

$$\oint_S \epsilon_0 \bar{\mathbf{E}} \cdot \overline{d\mathbf{a}} = \int_S \sigma_s dS \Rightarrow \epsilon_0 (E_{2n} - E_{1n}) dS = \sigma_s dS$$

$$\epsilon_0 (E_{2n} - E_{1n}) = \sigma_s \Rightarrow \bar{\mathbf{n}} \cdot [\epsilon_0 (\bar{\mathbf{E}}_2 - \bar{\mathbf{E}}_1)] = \sigma_s$$

2. Continuity of Tangential $\bar{\mathbf{E}}$



(a)

Figure 3-12 (a) Stokes' law applied to a line integral about an interface of discontinuity shows that the tangential component of electric field is continuous across the boundary.

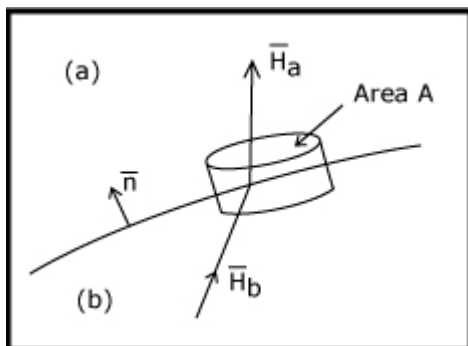
$$\oint_C \bar{\mathbf{E}} \cdot \overline{d\mathbf{s}} = (E_{1t} - E_{2t}) dl = 0 \Rightarrow E_{1t} - E_{2t} = 0$$

$$\bar{\mathbf{n}} \times (\bar{\mathbf{E}}_1 - \bar{\mathbf{E}}_2) = 0$$

Equivalent to $\Phi_1 = \Phi_2$ along boundary

3. Normal H

$$\nabla \cdot \mu_0 \bar{H} = 0 \Rightarrow \oint_S \mu_0 \bar{H} \cdot \bar{d}a = 0$$



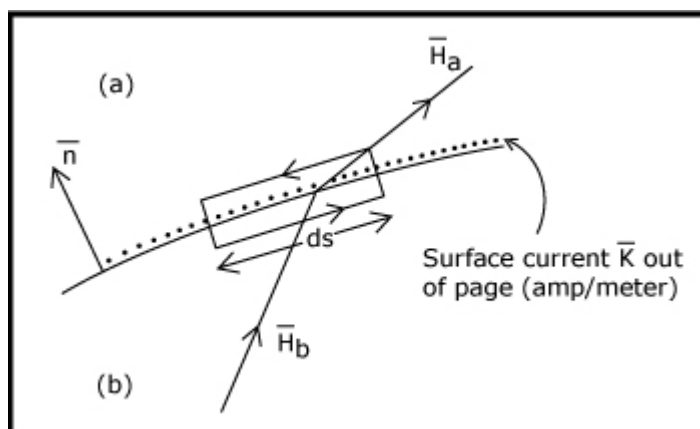
$$\mu_0 (H_{an} - H_{bn}) A = 0$$

$$H_{an} = H_{bn}$$

$$\bar{n} \cdot [\bar{H}_a - \bar{H}_b] = 0$$

4. Tangential H

$$\nabla \times \bar{H} = \bar{J} \Rightarrow \oint_C \bar{H} \cdot \bar{d}s = \int_S \bar{J} \cdot \bar{d}a$$

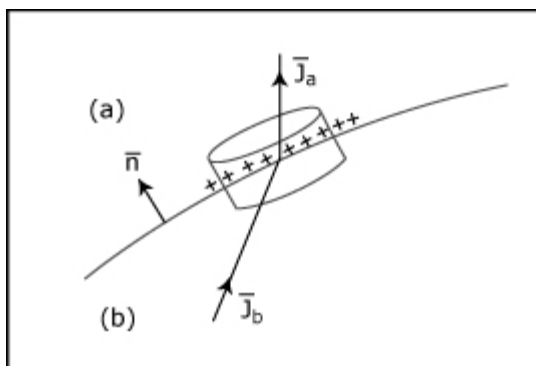


$$H_{bt} ds - H_{at} ds = K ds$$

$$H_{bt} - H_{at} = K$$

$$\bar{n} \times [\bar{H}_a - \bar{H}_b] = \bar{K}$$

5. Conservation of Charge Boundary Condition



$$\nabla \cdot \bar{J} + \frac{\partial \rho}{\partial t} = 0$$

$$\oint_S \bar{J} \cdot \bar{d}a + \frac{d}{dt} \int_V \rho dV = 0$$

$$\bar{n} \cdot [\bar{J}_a - \bar{J}_b] + \frac{\partial}{\partial t} \sigma_s = 0$$