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6.641 Electromagnetic Fields, Forces, and Motion, Spring 2005

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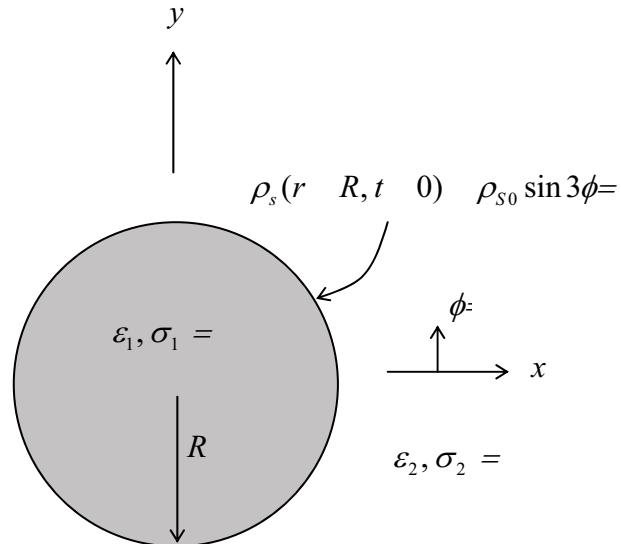
Massachusetts Institute of Technology
 Department of Electrical Engineering and Computer Science
 6.641 Electromagnetic Fields, Forces, and Motion

Quiz 2
 Spring Term 2003, 7:30-9:30PM

April 23, 2003

6.641 Formula Sheet Attached in the study materials section. You are also allowed to use the formula sheet that you prepared for Quiz 1 plus an additional 8 1/2" x 11" formula sheet (both sides) that you have prepared for Quiz 2.

Problem 1 (35 points)

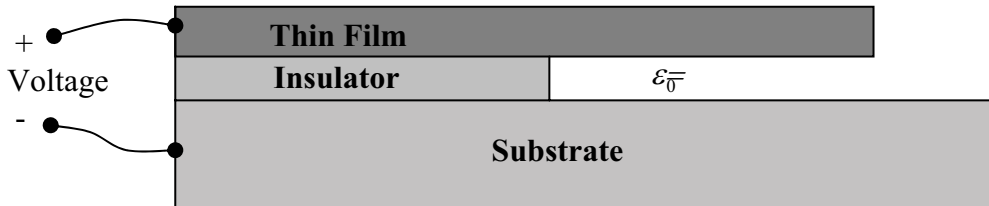


A cylinder of radius R and infinite extent in the z direction with dielectric permittivity ϵ_1 and ohmic conductivity σ_1 is placed within a medium of infinite extent that has permittivity ϵ_2 and ohmic conductivity σ_2 . At time $t=0$ the free surface charge density on the $r=R$ interface is $\rho_s(r=R, t=0) = \rho_{s0} \sin 3\phi$.

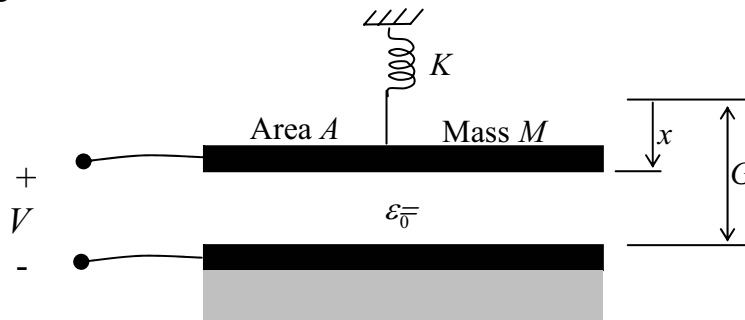
- a) Calculate the electric scalar potential inside and outside the cylinder at time $t=0$.
- b) Calculate the electric scalar potential inside and outside the cylinder for $t \geq 0$.
- c) What is the free surface charge density $\rho_s(r=R, t)$ for $t \geq 0$?

Problem 2 (30 points)

The structure shown below is often used to determine the bending stiffness of thin films deposited during the fabrication of a MEMS device. The structure is a thin film cantilever supported above a silicon substrate by an insulator. When a voltage is applied between the cantilever and the substrate, the cantilever is attracted to the substrate and deflects downward. Eventually, at a critical voltage, the deflection becomes unstable and the cantilever snaps down. The stiffness of the cantilever is determined from this voltage.

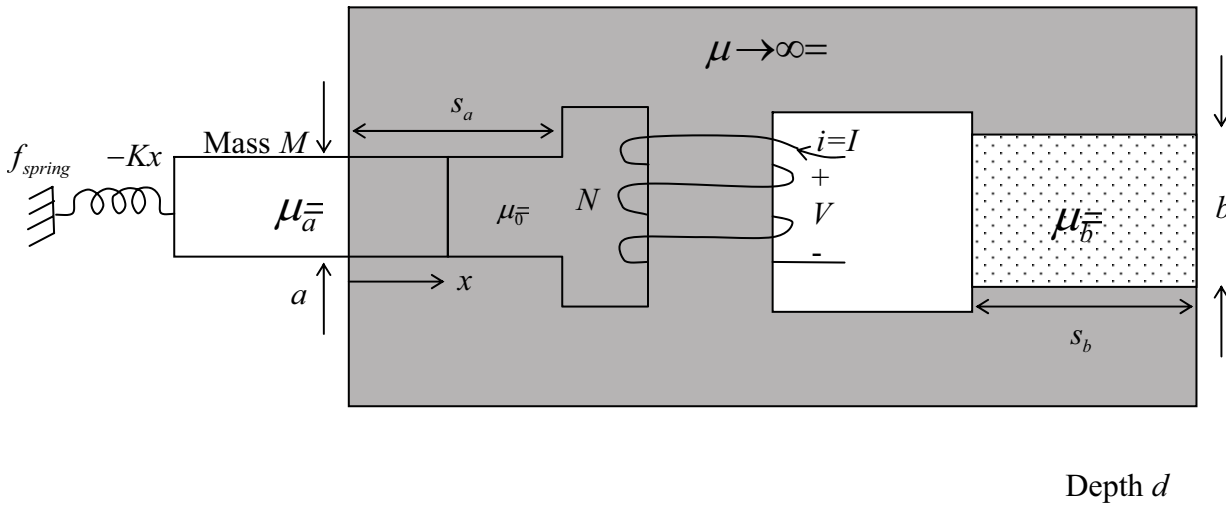


To understand the behavior of the stiffness tester, model it as shown below. Let the cantilever and substrate be modeled as a free-space parallel-plate capacitor with cross sectional area A . The upper electrode has mass M . Let the stiffness of the cantilever be modeled as a spring with spring constant K . When the capacitor is unexcited, and the spring is unstretched, the capacitor gap is G . When voltage is applied across the capacitor, its upper plate moves toward its lower plate with displacement x as it stretches the spring.



- Determine the force of electric origin that acts on the upper capacitor plate in the direction of increasing x , as a function of the applied voltage V , the plate deflection x , and the parameters of the model.
- Determine a differential equation that describes the time response of the deflection x as the upper capacitor plate is deflected by the force of electric origin. Neglect gravity.
- Assume that the deflection of the upper capacitor plate is in static equilibrium for a given voltage V . Determine a relation between the equilibrium deflection $x=X$, the applied voltage V , and the parameters of the model. You need not explicitly solve for X .
- We wish to determine the voltage at which the equilibrium found in Part c becomes unstable. To do so, let $x = X + x'$ and linearize the differential equation found in Part b for small displacements x' from the equilibrium X , where $x' \ll X$.
- Combine the answers to Parts c and d to show that the deflection of the upper plate becomes unstable (as determined by the dynamics of the linearized differential equation found in Part d) when the upper plate deflects a fraction of the gap G . Also, determine this fraction.
- Determine the voltage in terms of the parameters of the model at the onset of instability determined in Part e.

Problem 3 (35 points)



The above magnetic circuit of depth d has an infinitely magnetically permeable core ($\mu \rightarrow \infty$) with two gaps of height a and b . The right gap of height b and width s_b is filled with a non-moving and non-conducting magnetic material with magnetic permeability μ_b . The left gap of height a and width s_a is partially filled with a non-conducting movable block of mass M and magnetic permeability μ_a and with an air gap with magnetic permeability μ_0 . The distance that the μ_a material penetrates into the left air-gap is x . The μ_a movable block is also connected on its left hand side with a spring with spring constant K , that is fixed at its left end. The spring exerts no force on the μ_a block when $x=0$. The magnetic circuit has an N turn coil on its middle leg excited by a constant current I .

- Find the magnetic field \vec{H} in each gap within the magnetic circuit. Neglect fringing field effects.
- Find the self-inductance $L(x)$ of the coil as a function of block position x .
- Find the magnetic force on the movable block.
- What is the governing differential equation for the position of the movable block?
- Find the equilibrium position $x = x_{eq}$ of the movable block assuming $0 < x < s_a$.
- Is this equilibrium stable or unstable?
- The movable block of mass M is slightly displaced from its equilibrium at x_{eq} by an amount Δx and released with velocity $\left. \frac{dx}{dt} \right|_{t=0} = v_0$. To first order calculate $x'(t)$ where $x(t) = x_{eq} + x'(t)$ with $x'(t) \ll x_{eq}$. Neglect friction.