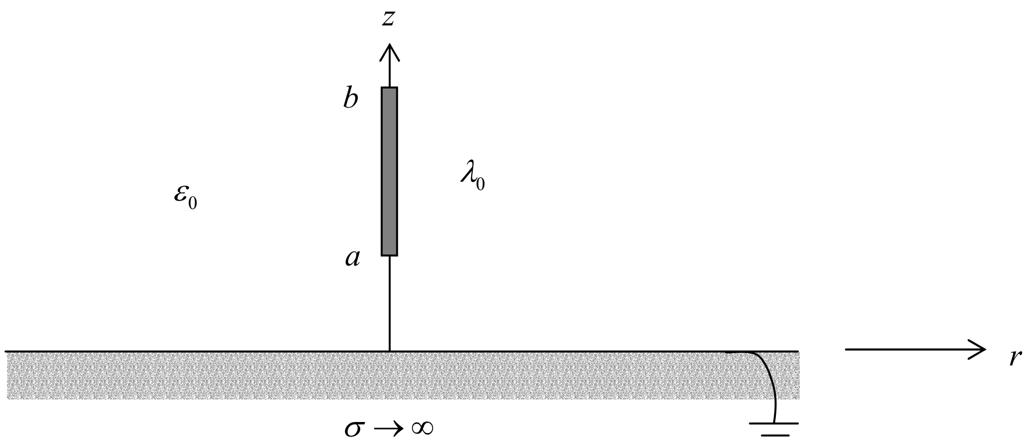
6.641 Electromagnetic Fields, Forces, and Motion, Spring 2005

Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science
6.641 Electromagnetic Fields, Forces, and Motion

Quiz 1 March 12, 2003
Spring Term 2003, 7:30-9:30PM

6.641 Formula Sheet Attached in the study materials section. You are also allowed to use one 8 ½" x 11" formula sheet (both sides) that you have personally prepared.

Problem 1 (35 points)



A finite length line charge that has charge per unit length λ_0 is placed in free space along the z axis and extends over the range $a<z<b$. There is a perfectly conducting ground plane at zero potential for $z \le 0$.

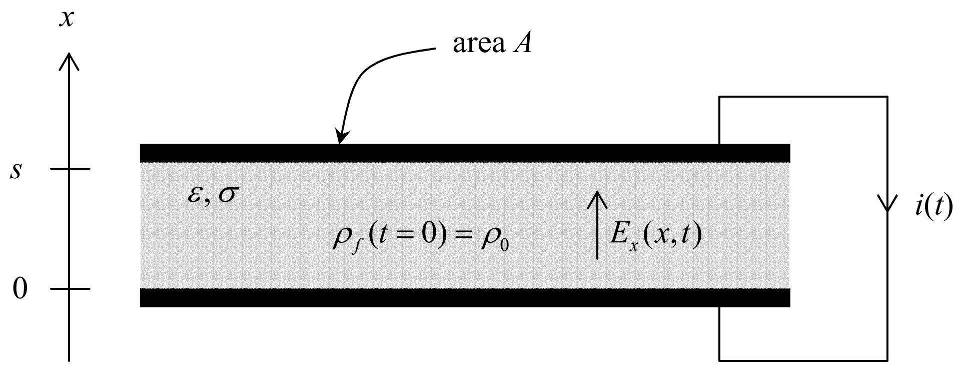
(a) What is the potential distribution, $\Phi(r=0, z)$, along the z axis for $z>b$?
(b) What is the electric field, $\bar{E}(r=0, z)$, along the z axis for $z>b$?
(c) What is the free surface charge density, $\sigma_s(r, z=0)$, as a function of radial distance r on the $z=0$ ground plane?

Hint: One or more of these integrals may be useful for solving this problem.

$$\int \frac{x\,dx}{[x^2+c^2]^{3/2}} = -\frac{1}{\sqrt{x^2+c^2}}; \quad \int \frac{dx}{[x^2+c^2]^{3/2}} = \frac{x}{c^2\sqrt{x^2+c^2}}$$

$$\int \frac{x\,dx}{\sqrt{x^2+c^2}} = \sqrt{x^2+c^2}; \quad \int \frac{dx}{x\sqrt{x^2+c^2}} = -\frac{1}{c}\ln\left[\frac{c+\sqrt{x^2+c^2}}{x}\right]$$

Problem 2 (30 points)



Short circuited parallel plate electrodes of area *A* enclose a lossy dielectric of thickness *s* with dielectric permittivity ε and ohmic conductivity σ. The lossy dielectric at time *t=0* has a uniformly distributed free volume charge density ρ_0. Neglect fringing field effects.

(a) What is the volume charge distribution for $0<x<s$ as a function of time?
(b) What is the electric field $E_x(x,t)$?
(c) What are the surface charge densities as a function of time at $x=0$ and $x=s$?
(d) What is the current $i(t)$ flowing through the short circuit?

Problem 3 (35 points)
This problem is concerned with computing the magnetic fields in the two systems shown below.

(a) Consider first the system shown in Figure 3.1. In this system, a surface current exists only in the plane *z=0* and is everywhere surrounded by free space. For *x>0*, the surface current is given by $\bar{K} = K_0 \bar{i}_y$. For *x<0*, the surface current is given by $\bar{K} = -K_0 \bar{i}_y$. Which components of the magnetic field $\bar{H}$ do you expect to be non-zero in this system? On which coordinates (*x* and/or *y* and/or *z*) do you expect the non-zero components to depend? Explain briefly.

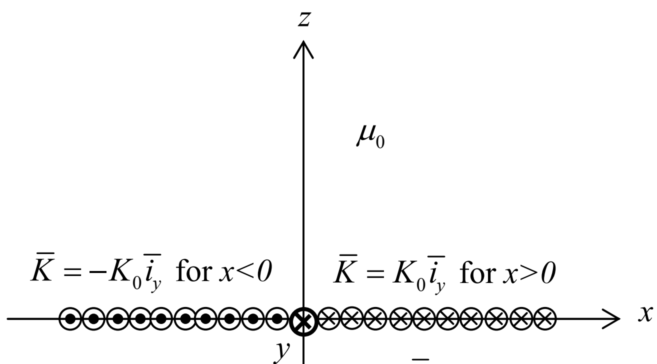


Figure 3.1

(b) Derive an integral expression for the magnetic field $\bar{H}$ but do not evaluate it.

(c) Consider now the system shown in Figure 3.2. In this system, a surface current exists only in the plane *z=d* above a perfect conductor that occupies the infinite half space *z<0*. Free space occupies the infinite half-space *z>0*. For *x>L*, the surface current is given by $\bar{K} = K_0 \bar{i}_y$. For *x<-L*, the surface current is given by $\bar{K} = -K_0 \bar{i}_y$. Writing the answer to Part (b), as $\bar{H}^*(x, y, z)$ which of the possible answers given below is the solution $\bar{H}(x, y, z)$ for the configuration shown in Fig. 3.2?

i) $\bar{H}(x,y,z) = \left[\bar{H}^*(x-L,y,z-d) + \bar{H}^*(x+L,y,z-d) + \bar{H}^*(x-L,y,z+d) + \bar{H}^*(x+L,y,z+d)\right]$

ii) $\bar{H}(x,y,z) = \frac{1}{2}\left[\bar{H}^*(x-L,y,z-d) + \bar{H}^*(x+L,y,z-d) - \bar{H}^*(x-L,y,z+d) - \bar{H}^*(x+L,y,z+d)\right]$

iii) $\bar{H}(x,y,z) = \frac{1}{2}\left[\bar{H}^*(x-L,y,z-d) - \bar{H}^*(x+L,y,z+d)\right]$

iv) $\bar{H}(x,y,z) = \left[\bar{H}^*(x-L,y,z+d) + \bar{H}^*(x+L,y,z-d)\right]$

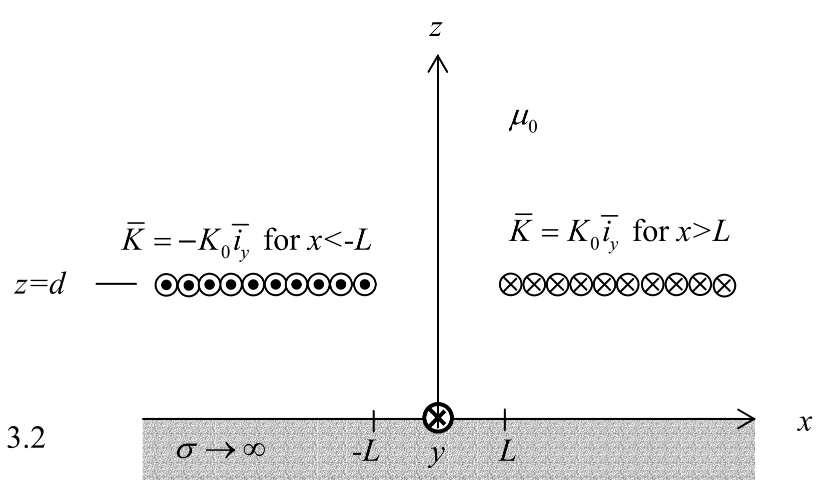


Figure 3.2