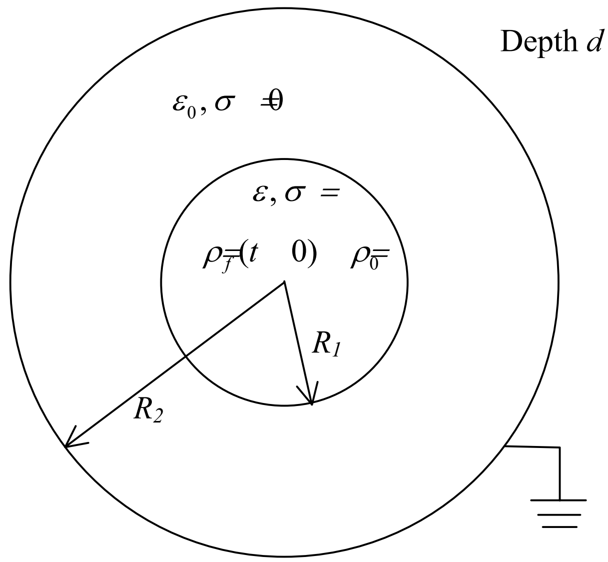
6.641 Electromagnetic Fields, Forces, and Motion, Spring 2005

Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science
6.641 Electromagnetic Fields, Forces, and Motion

Final Exam May 16, 2005
Spring 2005, 1:30-4:30PM

6.641 Formula Sheet Attached in the study materials section. You are also allowed to prepare one 8 ½ ” x 11” formula sheet (both sides) for use at the Final Exam.

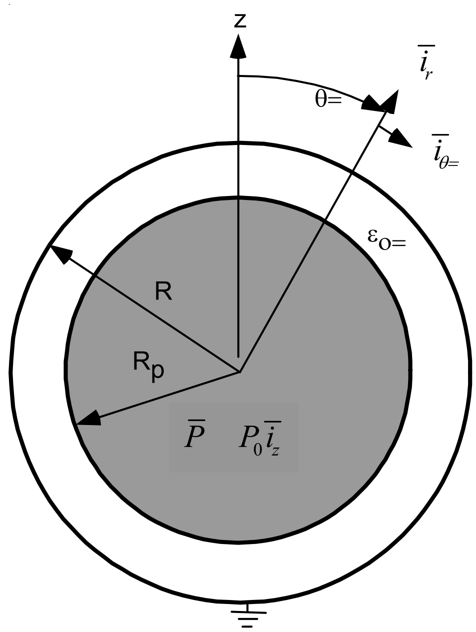
Problem 1. (15 points)



A lossy dielectric cylinder of radius R_1, dielectric permittivity ε, and ohmic conductivity σ is uniformly charged at time $t=0$ with free volume charge density $\rho_f(t \quad 0) \quad \rho_0$. The charged dielectric cylinder is surrounded by free space with permittivity ε_0 and zero conductivity, and the free space is surrounded by a grounded coaxial perfectly conducting cylinder of radius R_2. The electric field for $r>R_2$ is zero. The depth d of the cylinder is very large so that fringing fields can be neglected.

a) What are the volume charge density and electric field within the inner cylinder as a function of time?
b) What is the electric field in the free space region, $R_1<r<R_2$, as a function of time?
c) What are the surface charge densities on the interfaces at $r=R_1$ and $r=R_2$ as a function of time?

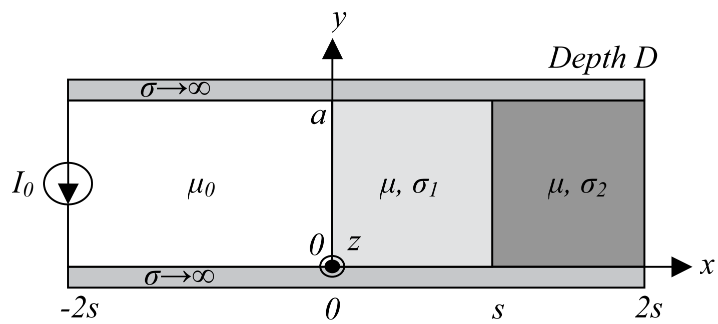
Problem 2. (15 points)



A sphere with radius R_p is comprised of permanently polarized material uniformly polarized in the z direction, $P_0\bar{i}_z$. The sphere is placed concentric with a second, grounded, perfectly-conducting sphere of radius R as shown in the cross-sectional drawing in the figure above. The region $R_p<r<R$ is filled with free space with dielectric permittivity ε_0. There is no free surface charge on the $r=R_p$ interface.

a) Express the permanent polarization in spherical coordinates.
b) Clearly write the boundary conditions at all surfaces required to determine the electric fields for $r \leq R$.
c) Find the electric field everywhere for $r<R$.
d) Find the free surface charge density at $r=R$.

Problem 3 (20 point)



A DC current source I_0 is connected to two perfectly conducting planes with spacing *a* and length 4*s*. Two conducting blocks, each with the same width *s* and the same magnetic permeability μ, have different respective ohmic conductivities σ_1 and σ_2, and are placed side by side in the region $0<x<2s$ as shown in the above figure.

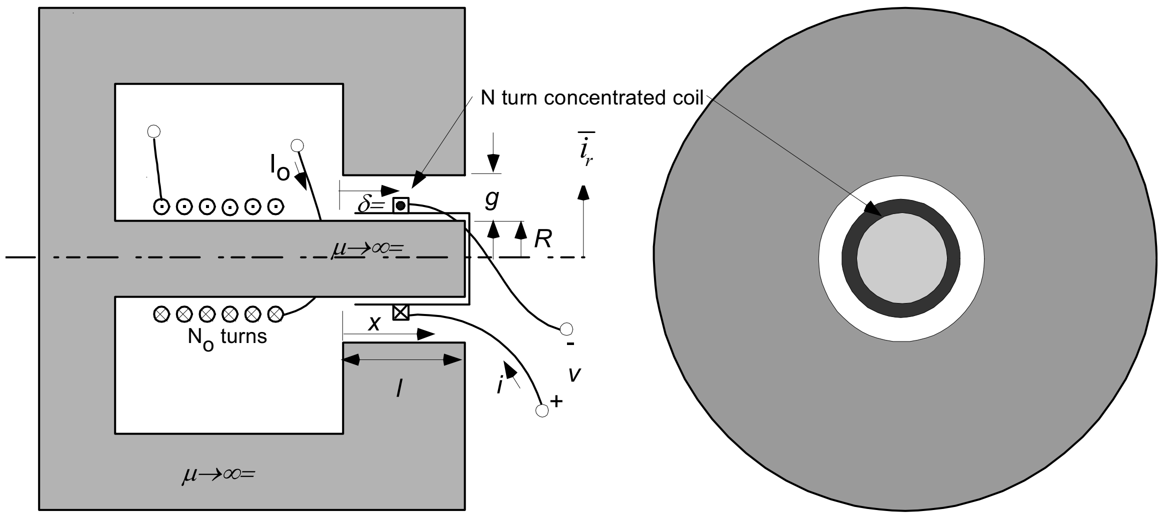
The region $-2s < x < 0$ between the plates is free space with magnetic permeability μ_0. The whole system has a depth *D*. Neglect fringing field effects; assume the current I_0 has been on for a long period of time so that the system is in the DC steady state; and assume that this system obeys the magnetoquasistatic (MQS) approximation.

a) What is the magnetic field $\bar{H}$ for $-2s < x < 0$? (Assume the magnetic field is uniform in the free space region.)
b) What are the necessary governing DC steady state equations for magnetic fields $\bar{H}_1$ and $\bar{H}_2$ in the two conducting materials for $0 < x < s$ and $s < x < 2s$?
c) Solve (b) for the general form of solution for $\bar{H}$, $\bar{J}$, and $\bar{E}$ in each conducting block assuming that the field only depend on the *x* coordinate and not on the *y* or *z* coordinates.
d) What are the boundary conditions on $\bar{H}$ and $\bar{E}$ necessary to find $\bar{H}$ and $\bar{E}$.
e) Solve for $\bar{H}$, $\bar{J}$, and $\bar{E}$ in each block.
f) Use the MQS Maxwell stress tensor to find the *x*-directed total force on the conducting material that extends over the interval $s < x < 2s$.

Problem 4 (15 points)

Tall buildings with a steel structure often vibrate under wind loadings and means must be used to control the displacements in the structure. It is proposed to use an electromagnetic device to control these vibrations.

A simple version of such a device is shown in the figure below. A highly permeable **cylindrical** magnetic structure with permeability $\mu \to \infty$ has a coil of N_0 turns with a DC current I_0 that produces a magnetic field in the gap of width *g*. A concentrated coil of N turns carrying a current *i* is mounted on a sliding mechanism that allows it to move with displacement δ as shown. For purposes of this analysis assume that the free-space gap is small, $g \ll R$, and that the field in the gap is purely in the radial direction but does not significantly vary with the radial coordinate *r*.

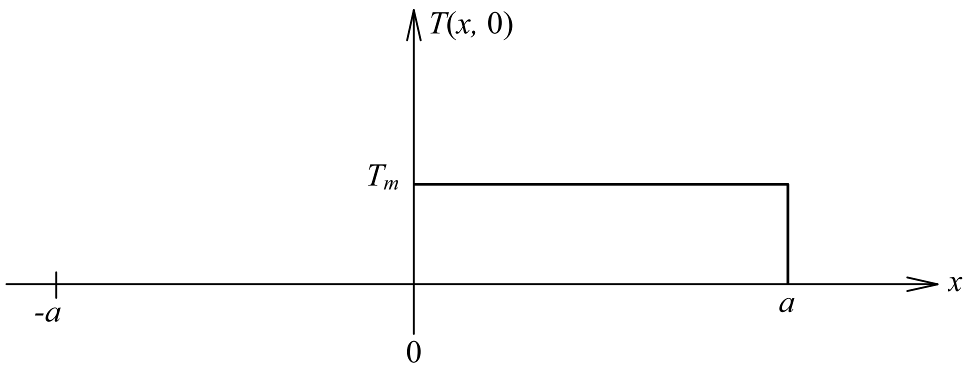


Side cross-Section View

End view

a) Find the magnetic field in the gap due to both the coil with current I_0 and the concentrated coil with current *i*. Plot this field as a function of *x,* the distance along the gap, carefully labeling key values and axes.
b) Find the flux linking each of the coils as a function of δ and other relevant parameters.
c) Find the open circuit voltage *v* at the terminals for concentrated coil displacement $\delta \quad \delta_0 \cos(\omega t)$.
d) Find the force on the concentrated coil due to currents I_0 and *i* and other relevant parameters.

Problem 5 (15 points)



A thin elastic rod of length $2a$ is placed on the x axis and extends from $-a<x<a$. The initial velocity and stress distributions at $t=0$ are:

$$T(x,t=0)=\begin{cases}T_m & 0<x<a\\ 0 & -a<x<0\end{cases}$$

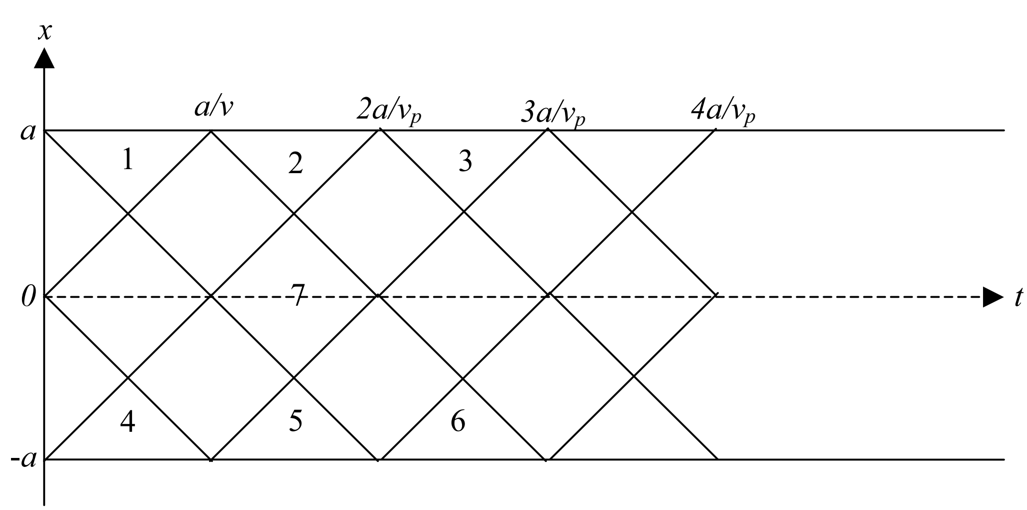
$$v(x,t=0)=0 \qquad -a<x<a$$

The rod has a fixed end at $x=a$ and a free end at $x=-a$. The rod has Young's modulus E, mass density ρ, and cross-sectional area A.

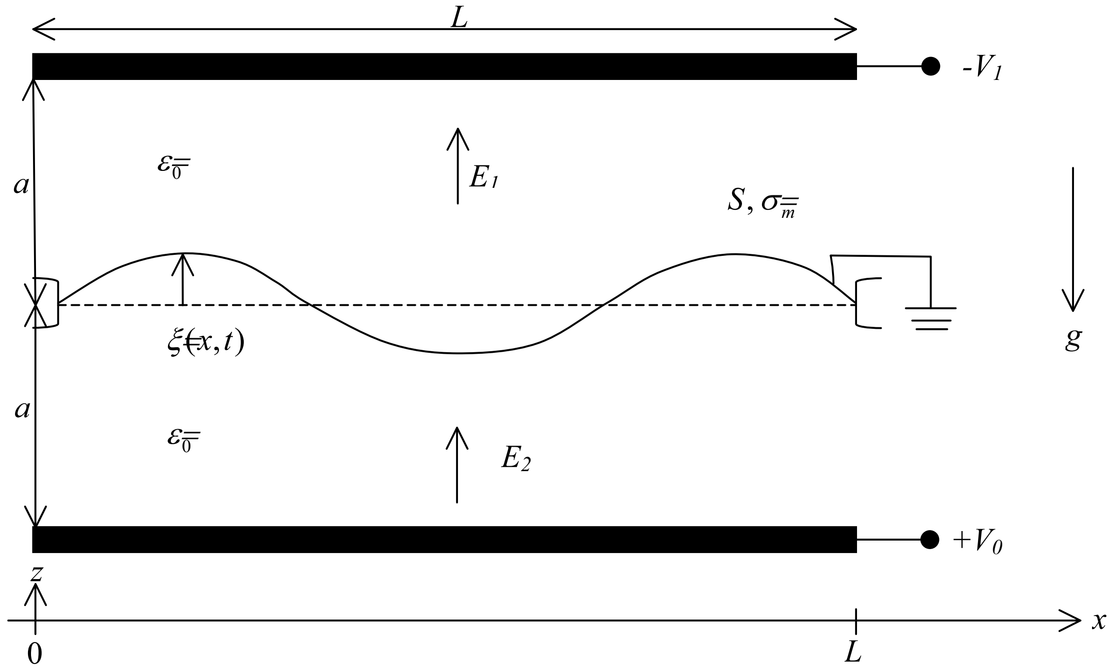
a) Find the solution of $T(x, t)$ and $v(x, t)$ in terms of T_m, E, ρ, and A by the method of characteristics within the regions 1-7 shown below.

b) Plot $T(x=a, t)$ and $v(x=-a, t)$. for $0<t<3a/v_p$, where $v_p = \sqrt{E/\rho}$ is the speed of longitudinal elastic waves in the rod.

c) Plot $T(x, t=3a/(2v_p))$ and $v(x, t=3a/(2v_p))$ for $-a<x<a$.



Problem 6 (20 points)



A perfectly conducting membrane with equilibrium tension per unit depth S and mass per unit area σ_m is stretched horizontally and fixed at two rigid supports a distance L apart. The membrane is grounded so that its voltage is zero. Parallel plate electrodes are placed a distance a above and below the membrane. The upper electrode is put at voltage $-V_1$ and the lower electrode is put at voltage $+V_0$. The region surrounding the membrane is free space with permittivity ε_0. Assume that transverse displacements of the membrane only depend on x and not y. Membrane displacements are in the "long-wave limit" so that the electric fields above and below the membrane are essentially in the z direction. Gravity is downwards with acceleration g.

a) To linear terms in membrane displacement $\xi(x,t)$, find the electric fields E_1 and E_2 in terms of V_0, V_1, a, and $\xi(x,t)$.
b) Find the z directed electric force per unit area on the membrane to linear terms in $\xi(x,t)$.
c) What is the governing linearized differential equation of motion of the membrane?
d) What must be V_1 be in terms of V_0, σ_m, and other relevant parameters so that the membrane is in static equilibrium with $\xi(x,t) \quad 0$.
e) For small signal membrane deflections of the form $\xi(x,t) \quad \mathrm{Re}\left[\hat{\xi} e^{j(\omega t - kx)}\right]$, find and plot the $\omega - k$ dispersion relation. The plot should be well labeled showing significant intercepts on the axes and slope asymptotes.
f) What are the allowed values of k that satisfy the zero deflection boundary conditions at $x=0$ and $x=L$?
g) For what value of $V_1^2 + V_0^2$ will the membrane equilibrium with $\xi(x,t) \quad 0$ first become unstable?