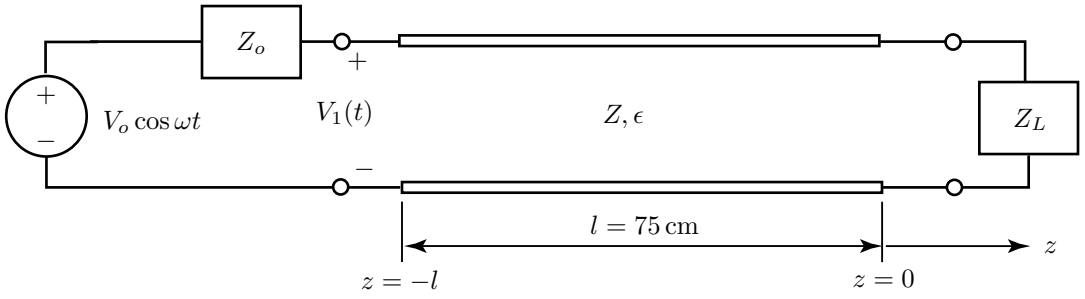
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

Problem Set No. 8 | 6.630 Electromagnetics | Issued: Week 9
Fall Term 2006 | Due: Week 10

Reading assignment: Section 2.4, 2.6; J. A. Kong, "*Electromagnetic Wave Theory*".

Problem P8.1

A TEM transmission line is driven by a voltage source $V_o \cos \omega t$ at 100 MHz. The characteristic impedance of the transmission line is $Z_o = 100\ \Omega$ for $\epsilon = \epsilon_o$. The transmission line length is $l = 75$ cm



(a) Let $\epsilon = \epsilon_o$, show that $l = \lambda/4$. If the voltage at $z = -l$ is

$$V_1(t) = \frac{1}{\sqrt{2}} V_o \cos\left(\omega t - \frac{\pi}{4}\right),$$

what is the load impedance Z_L?

(b) Let $\epsilon = 4\epsilon_o$, find the wavelength λ. What is the input impedance Z_{in} at $z = -l$? What is the voltage $V_1(t)$ at $z = -l$?

Problem P8.2

Convert the following time domain expressions into their complex equivalents in the frequency domain, where we have defined

$$A = \text{Re}\left[\underline{A} e^{j\omega t}\right]$$

Example : $A = \sin \omega t \qquad \underline{A} = -j$

(a)Find $\underline{A}$.

(i) $A = 3 \sin\left(\omega t - \frac{\pi}{4}\right)$

(ii) $A = \hat{x} \sin \omega t - \hat{y} 2 \cos \omega t$

(iii) $A = \cos \phi \cos \omega t$

(b) Find A.

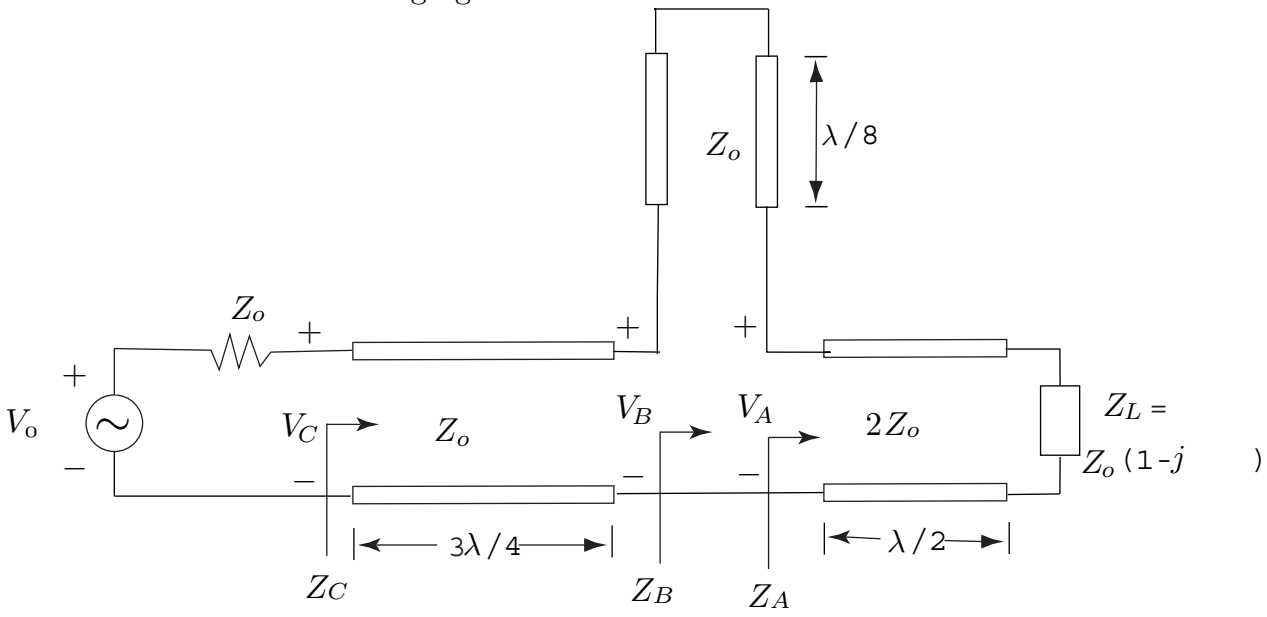
(i) $\underline{A} = j e^{j\pi/4}$

(ii) $\underline{A} = \hat{x} + \hat{y} 3j$

(iii) $\underline{A} = A_0 e^{j\phi} + j$

Problem P8.3

Consider the TEM transmission line system connected to a time-harmonic voltage source as shown in the following figure.



(a) Find the impedance Z_A in terms of Z_o.
(b) Find the impedance Z_B in terms of Z_o.
(c) Find the impedance Z_C in terms of Z_o.
(d) Show that the time average power dissipated in Z_C is $|V_o|^2/8Z_0$ .Assume Z_o is real.
(e) Find the voltage V_L across the load Z_L in terms of V_o and use V_L to calculate the time average power dissipated in the load Z_L in terms of V_o and Z_o. Assume Z_o is real.