

Review of basics of acoustics of tubes

Short tubes ($l \ll \lambda/4$)

Acoustic mass

$$\frac{l_1}{A_1} \quad M_A = \frac{\rho l_1}{A_1} \quad U_{in} = U_{out}$$

M_A remains the same if l_1/A_1 remains the same.

Acoustic compliance

$$\frac{l_2}{A_2} \quad C_A = \frac{l_2 A_2}{\rho c^2} \quad p \text{ constant throughout the tube}$$

C_A remains the same if $l_2 A_2$ remains the same.

Helmholtz resonator

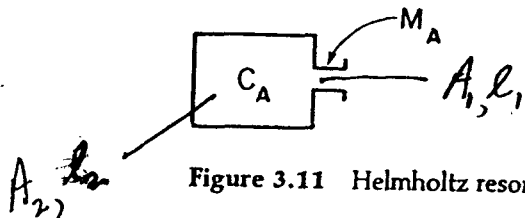
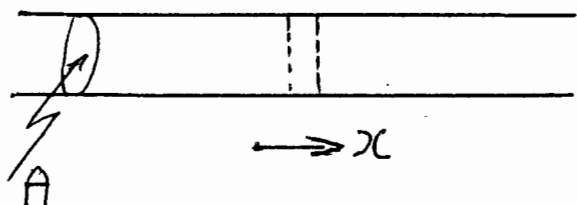


Figure 3.11 Helmholtz resonator.

Natural frequency

$$F_1 = \frac{1}{2\pi \sqrt{M_A C_A}} = \frac{c}{2\pi} \sqrt{\frac{A_1}{A_2 l_1 l_2}}$$

Review of one-dimensional sound propagation in tubes



$p(x, t)$ sound pressure
 $U(x, t)$ volume velocity

Basic equations:

$$\frac{\partial p}{\partial x} = - \frac{\rho_0}{A} \frac{\partial U}{\partial t} \quad (1)$$

$$\frac{\partial U}{\partial x} = - \frac{A}{\rho_0} \frac{\partial p}{\partial t} \quad (2)$$

(1) If there is a gradient in pressure, i.e., if the pressure is greater on one side of a small element of length than on the other, then there is a net force on the element that will accelerate it (i.e. $f = ma$).

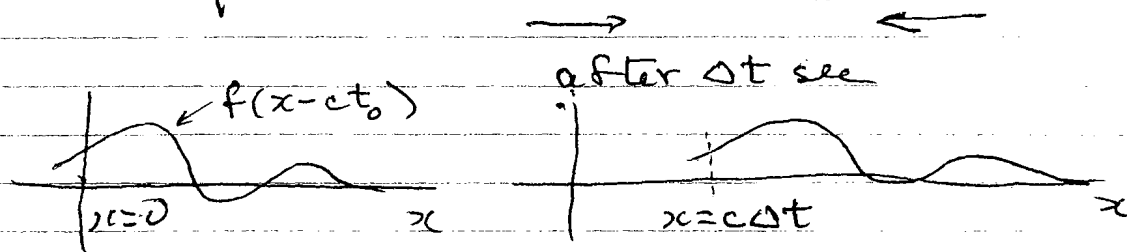
(2) If there is a gradient in volume velocity, i.e., if the volume velocity is greater on one side of a small element than on the other, then there is a net rate of expansion of the volume, and consequently a decrease in pressure in the element over time (from $PV = \text{constant}$).

Combine equations (1) and (2)

$$\frac{\partial^2 p}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} \quad c = \sqrt{\frac{\gamma P_0}{\rho}}$$

General solution:

$$p(x, t) = f_1(x - ct) + f_2(x + ct)$$



Waveform moves distance ct in time t
 $c = \text{Velocity of sound.}$

For a sinusoidal signal

$$p(x, t) = A_1 \sin \omega \left(t - \frac{x}{c} \right) + A_2 \sin \omega \left(t + \frac{x}{c} \right)$$

For wave moving in $+x$ direction

if $\frac{\omega x}{c}$ is increased by $\frac{2\pi}{f}$:

Call this change in x as λ

$$\frac{2\pi f \lambda}{c} = 2\pi \quad \lambda = \frac{c}{f} = \text{wavelength}$$

Express $p(x, t)$ and $U(x, t)$ with sinusoidal (exponential) time dependence

$$p(x, t) = \text{Re}[p(x) e^{j\omega t}]$$

$$U(x, t) = \text{Re}[U(x) e^{j\omega t}]$$

Equations (1) and (2) become

$$\frac{d p(x)}{dx} = - \frac{P_0}{A} \cdot j\omega U(x) \quad (3)$$

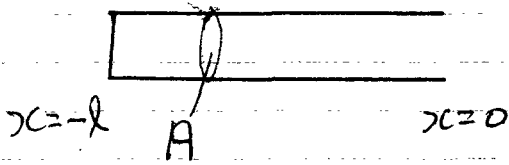
$$\frac{d U(x)}{dx} = - \frac{A}{r P_0} j\omega p(x) \quad (4)$$

From these, eliminate U to get

$$\frac{d^2 p}{dx^2} + k^2 p = 0 \quad k = \frac{\omega}{c} \quad (5)$$

$$c = \sqrt{\frac{r P_0}{\rho}}$$

$$p(x) = \frac{\sin}{\cos} kx$$



At $x=0$, assume $p=0$

A solution satisfying this boundary condition is

$$p(x) = P_m \sin kx$$

From eq. (3) can derive

$$\underline{v}(x) = j \frac{A}{\rho_0 c} P_m \cos kx$$

Apply boundary condition at $x=-l$.

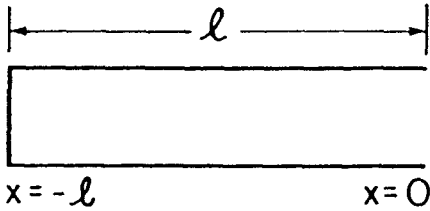
$$v(-l) = 0$$

$$\therefore \cos kl = 0 \quad kl = \frac{2\pi f l}{c} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \dots$$

$$f = \frac{c}{4l}, \frac{3c}{4l}, \frac{5c}{4l}, \dots$$

i.e., only a set of discrete frequencies can satisfy boundary conditions at the two ends of the tube.

Uniform Tubes



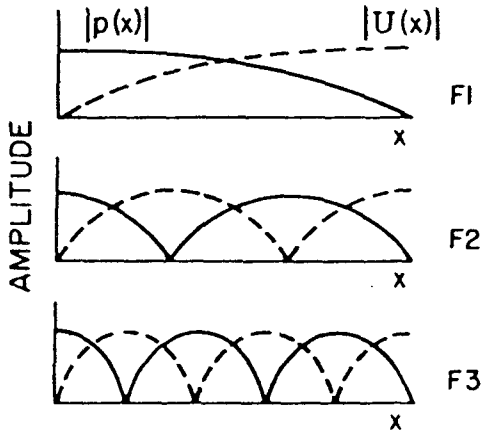
$$F_1 = \frac{c}{4l}$$

$$F_2 = \frac{3c}{4l}$$

$$F_3 = \frac{5c}{4l}$$

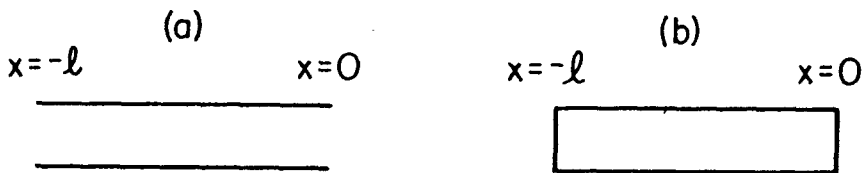
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Figure 3.8 Uniform tube of length l , closed at one end and open at the other.



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Figure 3.9 Distribution of sound pressure amplitude $|p(x)|$ and volume velocity amplitude $|U(x)|$ in a uniform tube for the first three natural frequencies F_1 , F_2 , and F_3 . Tube is closed at the left-hand end and open at the right-hand end.



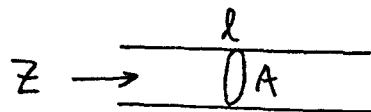
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Figure 3.10 (a) Uniform tube open at both ends. (b) Uniform tube closed at both ends.

$$F_n = n \cdot \frac{c}{2l} \quad n = 0, 1, 2, \dots$$

$$Z = -j \frac{\rho c}{A} \cot \frac{\omega l}{c}$$

$$Z = j \frac{\rho c}{A} \tan \frac{\omega l}{c}$$



Effect of radiation impedance at open end of tube.

$$\underline{Z}_r = R_r + jX_r \quad (R_r \text{ discussed later})$$

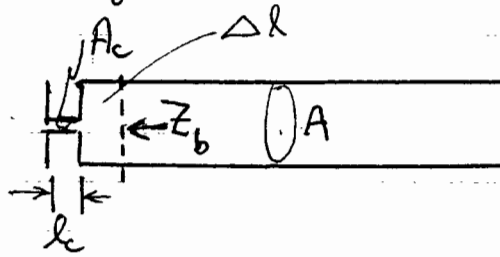
For a circular tube, radius a , area A_m

$$X_r = 2\pi f \cdot \frac{\rho(0.8a)}{A_m}$$

In effect, an acoustic mass of $\frac{\rho(0.8a)}{A_m}$ is

added at end of tube, adding an effective length of $0.8a$, i.e., an "end correction". Thus the natural frequencies are a bit lower than those calculated on the assumption of zero radiation impedance (i.e., a boundary condition of $p=0$ at the open end).

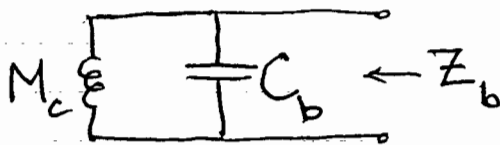
Effect of a small opening at closed end of tube.



Impedance of opening is $Z_c = j\omega \frac{\rho l_c}{A_c} = j\omega M_c$

Consider a point near the end of the tube shown by the dashed line, distance Δl from end

Impedance Z_b looking back from this point is



$$C_b = \frac{\Delta l \cdot A}{\rho c^2}$$

This impedance becomes infinite when

$$j\omega C_b + \frac{1}{j\omega M_c} = 0$$

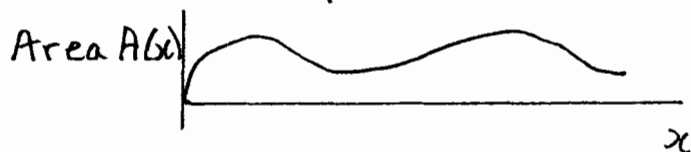
$$C_b = \frac{1}{\omega^2 M_c}$$

$$\text{or } \frac{\Delta l \cdot A}{\rho c^2} = \frac{A_c}{\omega^2 \rho l_c}$$

$$\text{or } \Delta l = \left(\frac{c}{\omega}\right)^2 \frac{A_c}{A} \frac{1}{l_c}$$

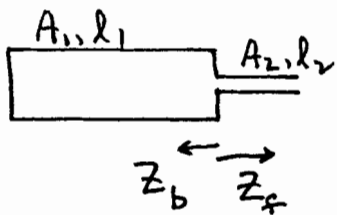
Thus we can consider that the "effective" closed end of the tube is Δl from the closed end. So the natural frequency of the tube is slightly higher than it would be if there were no narrow opening at the left.

Estimating natural frequencies of more complex tube shapes.



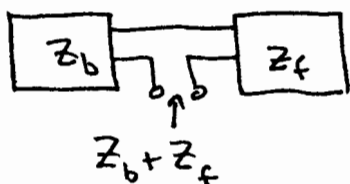
(1.) Use computer program: enter values of $A(x)$, and it calculates natural frequencies.

(2.)



find frequencies for which

$$Z_b + Z_f = 0$$



$$Z = P/U$$

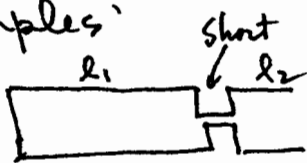
When $Z_b + Z_f = 0$, the frequency is such that $P = 0$

$$-j \frac{\rho c}{A_1} \cot \frac{\omega l_1}{c} + j \frac{\rho c}{A_2} \tan \frac{\omega l_2}{c} = 0$$

Solve for $\omega = 2\pi f$

(3.) Divide tube into sections, each with a uniform area, but tubes must be basically uncoupled, i.e., areas of adjacent tubes must be very different.

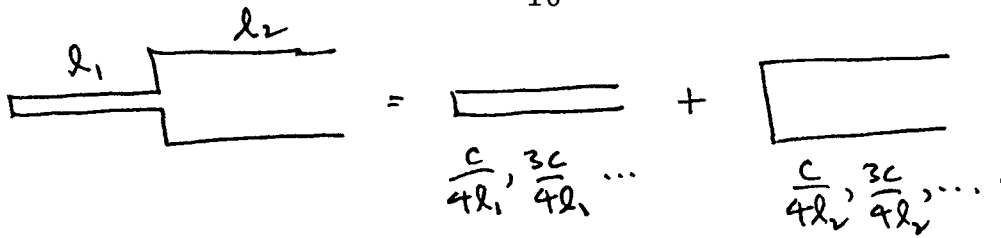
Examples:



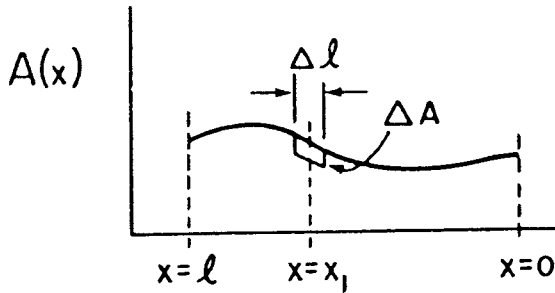
$$= \boxed{} + \boxed{} + \boxed{}$$

$$\frac{c}{2l_1}, \frac{c}{l_1}, \dots \quad \frac{c}{4l_2}, \frac{3c}{4l_2}, \dots$$

+ Helmholtz resonance

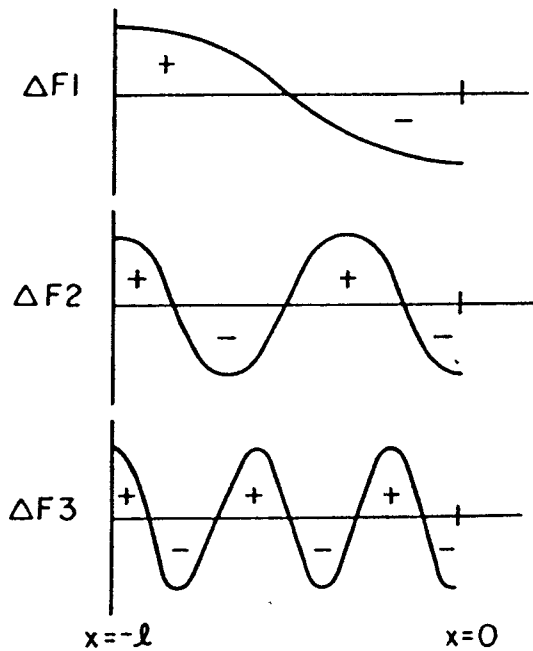


(4.) Perturbation method



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Figure 3.18 Illustrating a perturbation ΔA in the area of an acoustic tube at a short segment of length Δl centered at point $x = x_1$.

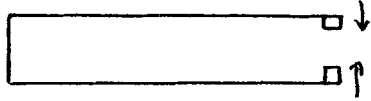


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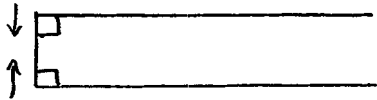
Figure 3.19 Curves showing the relative magnitude and direction of the shift ΔF_n in formant frequency F_n for a uniform tube when the cross-sectional area is decreased at some point along the length of the tube. The abscissa represents the point at which the area perturbation is made. The minus sign represents a decrease in formant frequency and the plus sign an increase.

In order to estimate how a natural frequency changes when you make a perturbation in area ΔA at a point $x = x_1$, you need to look at the distribution of sound pressure and volume velocity at that natural frequency.

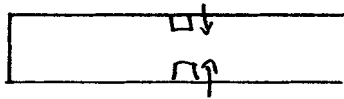
Some examples



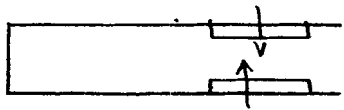
$U(x)$ is a maximum at opening for all formants.
All formant frequencies decrease.



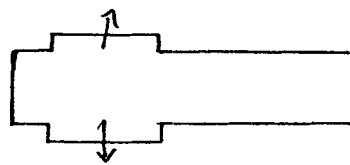
$p(x)$ is a maximum at closed end for all formants.
All formant frequencies increase.



ΔF is zero for all formants.



ΔF_1 is	positive <input type="checkbox"/>	negative <input type="checkbox"/>
ΔF_2	<input type="checkbox"/>	<input type="checkbox"/>
ΔF_3	<input type="checkbox"/>	<input type="checkbox"/>



ΔF_1 is	positive <input type="checkbox"/>	negative <input type="checkbox"/>
ΔF_2	<input type="checkbox"/>	<input type="checkbox"/>
ΔF_3	<input type="checkbox"/>	<input type="checkbox"/>