6.453 Quantum Optical Communication
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6.453 *Quantum Optical Communication* Lecture 9

Jeffrey H. Shapiro

Optical and Quantum Communications Group

6.453 *Quantum Optical Communication* — Lecture 9

- Announcements
  - Turn in problem set 4
  - Pick up problem set 4 solution, problem set 5, lecture notes, slides
- Single-Mode Photodetection
  - Direct Detection — reprise
  - Homodyne Detection — reprise
  - Heterodyne Detection — semiclassical versus quantum
  - Realizing the $\hat{a}$ measurement

Single-Mode Quantized Electromagnetic Field

- Photon-Units Field Operator on Constant-z Plane:

$$\hat{E}_z(x,y,t) = \underbrace{\frac{\hat{a}e^{-j\omega t}}{\sqrt{AT}}}_{\text{excited mode}} + \underbrace{\text{other terms}}_{\text{unexcited modes}}$$

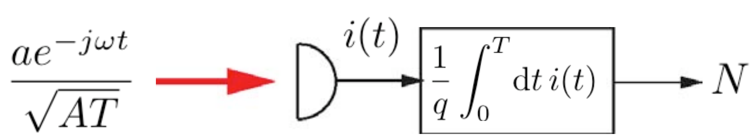
for $(x,y) \in \mathcal{A}, 0 \le t \le T$

- Photon Annihilation and Creation Operators: $\hat{a}, \hat{a}^\dagger$

with canonical commutation relation $\left[\hat{a}, \hat{a}^\dagger\right] = 1$

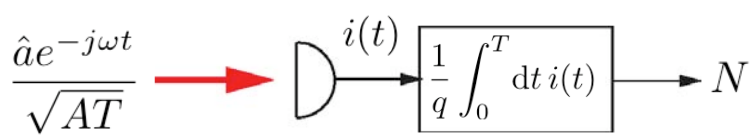
Direct Detection: Semiclassical versus Quantum

- Single-Mode Photon Counter: Semiclassical Description



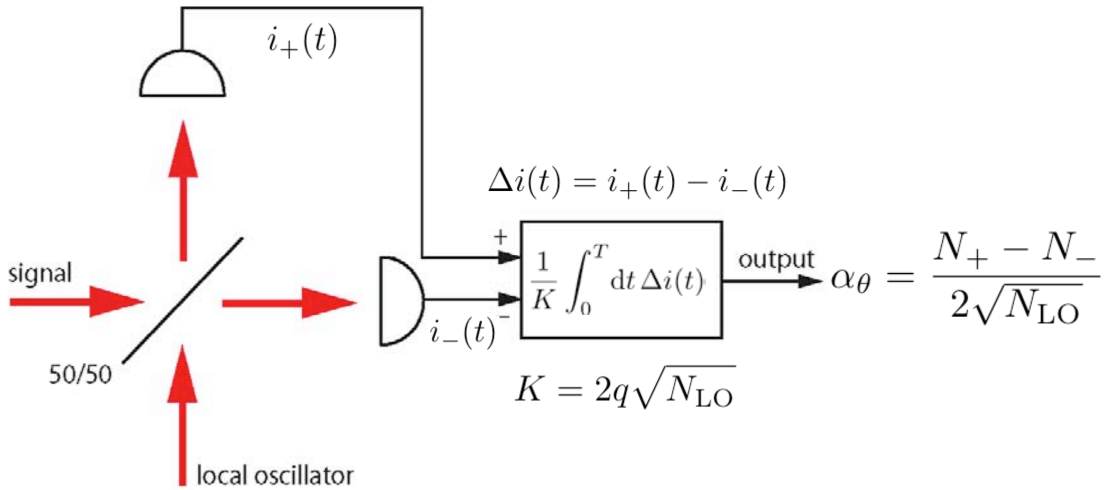
$$\Pr(\, N = n \mid a = \alpha \,) = \frac{|\alpha|^{2n}}{n!} e^{-|\alpha|^2}$$

- Single-Mode Photon Counter: Quantum Description



$$\Pr(\, N = n \mid \text{state} = |\psi\rangle \,) = |\langle n|\psi\rangle|^2$$

Single-Mode Balanced Homodyne Receiver



- Semiclassical Description: $\alpha_\theta \sim N(\mathrm{Re}(a_S e^{-j\theta}), 1/4)$
- Quantum Description: $\alpha_\theta \longleftrightarrow \hat{a}_{S_\theta} \equiv \mathrm{Re}(\hat{a}_S e^{-j\theta})$

Homodyne Detection: Semiclassical Theory

- Signal and Local Oscillator Fields:

$$E_{\mathrm{S}}(x,y,t) = \frac{a_S e^{-j\omega t}}{\sqrt{AT}}, \quad E_{\mathrm{LO}}(x,y,t) = \frac{a_{\mathrm{LO}} e^{-j\omega t}}{\sqrt{AT}}$$

- Strong Local-Oscillator Condition:

$$a_{\mathrm{LO}} = \sqrt{N_{\mathrm{LO}}}e^{j\theta}, \quad N_{\mathrm{LO}} \to \infty$$

- Characteristic Function Derivation:

$$M_{\alpha_\theta}(jv) = \lim_{N_{\mathrm{LO}}\to\infty} M_{N_+}\left(\frac{jv}{2\sqrt{N_{\mathrm{LO}}}}\right) M_{N_-}\left(-\frac{jv}{2\sqrt{N_{\mathrm{LO}}}}\right)$$

$$= e^{jv\mathrm{Re}(a_S e^{-j\theta}) - v^2/8}$$

Homodyne Detection: Quantum Theory

- Signal and Local Oscillator Field Operators:

$$\hat{E}_S(x,y,t) = \underbrace{\frac{\hat{a}_S e^{-j\omega t}}{\sqrt{AT}}}_{\text{excited mode}} + \underbrace{\text{other terms}}_{\text{unexcited modes}}$$

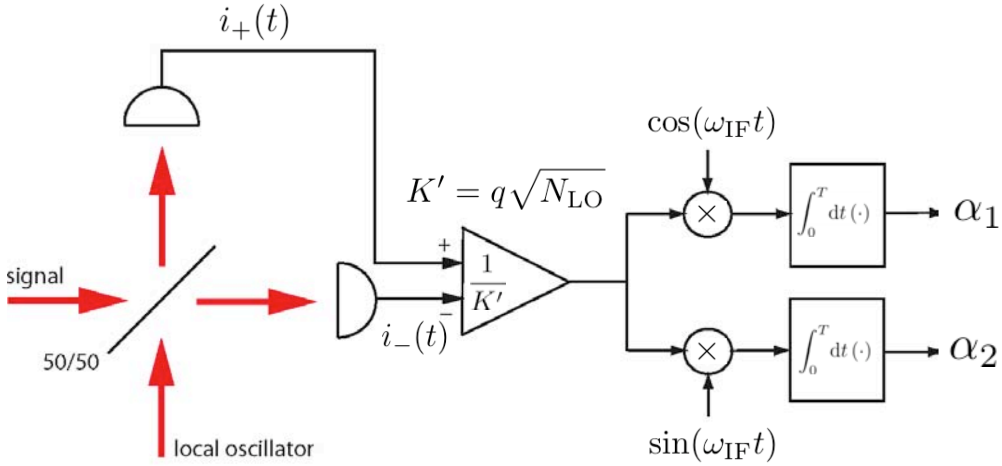
$$\hat{E}_{\rm LO}(x,y,t) = \underbrace{\frac{\hat{a}_{\rm LO} e^{-j\omega t}}{\sqrt{AT}}}_{\text{excited mode}} + \underbrace{\text{other terms}}_{\text{unexcited modes}}$$

- Local-Oscillator State: $|\sqrt{N_{\rm LO}}e^{j\theta}\rangle, \quad N_{\rm LO} \to \infty$

- Measurement Operator:

$$\frac{\hat{a}_+^\dagger \hat{a}_+ - \hat{a}_-^\dagger \hat{a}_-}{2\sqrt{N_{\rm LO}}} = \frac{\text{Re}(\hat{a}_S \hat{a}_{\rm LO}^\dagger)}{\sqrt{N_{\rm LO}}} \longrightarrow \text{Re}(\hat{a}_S e^{-j\theta})$$

Single-Mode Balanced Heterodyne Receiver



- Semiclassical Description: $\{\alpha_1, \alpha_2\}$ SI, $\quad \alpha_i \sim N(a_{S_i}, 1/2)$

- Quantum Description: $\alpha \longleftrightarrow \hat{a}_S$

Heterodyne Detection: Semiclassical Theory

- Signal and Local Oscillator Fields:

$$E_S(x,y,t) = \frac{a_S e^{-j\omega t}}{\sqrt{AT}}, \quad E_{\mathrm{LO}}(x,y,t) = \frac{a_{\mathrm{LO}} e^{-j(\omega-\omega_{\mathrm{IF}})t}}{\sqrt{AT}}$$

- Strong Local-Oscillator Condition:

$$a_{\mathrm{LO}} = \sqrt{N_{\mathrm{LO}}}, \quad N_{\mathrm{LO}} \rightarrow \infty$$

- Characteristic Function Derivation: random process theory

Heterodyne Detection: Quantum Theory

- Signal and Local Oscillator Field Operators:

$$\hat{E}(x,y,t) = \underbrace{\frac{\hat{a}_S e^{-j\omega t}}{\sqrt{AT}}}_{\text{excited mode}} + \underbrace{\frac{\hat{a}_{\mathrm{I}} e^{-j(\omega-2\omega_{\mathrm{IF}})t}}{\sqrt{AT}}}_{\text{unexcited mode}} + \underbrace{\text{other terms}}_{\text{unexcited modes}}$$

$$\hat{E}_{\mathrm{LO}}(x,y,t) = \underbrace{\frac{\hat{a}_{\mathrm{LO}} e^{-j(\omega-\omega_{\mathrm{IF}})t}}{\sqrt{AT}}}_{\text{excited mode}} + \underbrace{\text{other terms}}_{\text{unexcited modes}}$$

- Local-Oscillator State: $|\sqrt{N_{\mathrm{LO}}}\rangle, \quad N_{\mathrm{LO}} \rightarrow \infty$

- Simultaneous Measurement of Commuting Observables:

$$\alpha \longleftrightarrow \hat{a}_S + \hat{a}_I^\dagger, \quad \text{for state} = |\psi\rangle_S \otimes |0\rangle_I$$

Coming Attractions: Lecture 10

- Lecture 10:
  Single-Mode Photodetection
  - Signatures of non-classical light
  - Squeezed-state waveguide tap