6.453 Quantum Optical Communication
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6.453 *Quantum Optical Communication* Lecture 2

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6.453 *Quantum Optical Communication* — Lecture 2

- Handouts
  - Lecture notes, slides
  - if you missed Lecture 1, see instructor after class
- Fundamentals of Dirac-Notation Quantum Mechanics
  - Quantum systems
  - States as ket vectors
  - State evolution via Schrödinger's equation
  - Quantum measurements — observables

Quantum Systems and Quantum States

- *Definition 1*:
  A quantum-mechanical system $\mathcal{S}$ is a physical system governed by the laws of quantum mechanics.

- *Definition 2*:
  The state of a quantum mechanical system at a particular time t is the sum total of all information that can be known about the system at time t. It is a ket vector $|\psi(t)\rangle$ in an appropriate Hilbert space $\mathcal{H}_\mathcal{S}$ of possible states. Finite energy states have unit length ket vectors, i.e., $\langle\psi(t)|\psi(t)\rangle = 1$.

Time Evolution via the Schrödinger Equation

- *Axiom 1:*
  For $t \geq 0$, an isolated system with initial state $|\psi(0)\rangle$ will reach state

$$|\psi(t)\rangle = \hat{U}(t,0)|\psi(0)\rangle$$

where $\hat{U}(t,0)$ is the unitary time-evolution operator for the system $\mathcal{S}$. $\hat{U}(t,0)$ is obtained by solving

$$j\hbar\frac{\mathrm{d}\hat{U}(t,0)}{\mathrm{d}t} = \hat{H}\hat{U}(t,0), \quad \text{for } t \geq 0, \text{ with } \hat{U}(0,0) = \hat{I}$$

where $\hat{H}$ is the Hamiltonian (energy) operator for $\mathcal{S}$. Equivalently, we have the Schrödinger equation

$$j\hbar\frac{\mathrm{d}|\psi(t)\rangle}{\mathrm{d}t} = \hat{H}|\psi(t)\rangle, \quad \text{for } t \geq 0, \text{ with } |\psi(0)\rangle \text{ initial condition}$$

Quantum Measurements: Observables

- *Axiom 2:*

  An observable is a measurable dynamical variable of the quantum system $\mathcal{S}$. It is represented by an Hermitian operator which has a complete set of eigenkets.

- *Axiom 3:*

  For a quantum system $\mathcal{S}$ that is in state $|\psi(t)\rangle$ at time t, measurement of the observable

  $$\hat{O} \equiv \sum_n o_n |o_n\rangle\langle o_n|$$

  yields an outcome that is one of the eigenvalues, $\{o_n\}$, with

  $$\Pr(\text{outcome } = o_n) = |\langle o_n|\psi(t)\rangle|^2$$

Quantum Measurements: Observables

- *Projection postulate:*

  Immediately after a measurement of an observable $\hat{O}$, with distinct eigenvalues, yields outcome o_n the state of the system becomes $|o_n\rangle$.

- *Axiom 3a:*

  For a quantum system $\mathcal{S}$ that is in state $|\psi(t)\rangle$ at time t, measurement of the observable

  $$\hat{O} = \int_{-\infty}^{\infty} \mathrm{d}o\, o|o\rangle\langle o|$$

  yields an outcome that is one of the eigenvalues, o, with

  $$p(o) = |\langle o|\psi(t)\rangle|^2$$

Coming Attractions: Lectures 3 and 4

- Lecture 3:
  Fundamentals of Dirac-Notation Quantum Mechanics
  - Quantum measurements — statistics
  - Schrödinger picture versus Heisenberg picture
  - Heisenberg uncertainty principle

- Lecture 4:
  Quantum Harmonic Oscillator
  - Quantization of a classical *LC* circuit
  - Annihilation and creation operators
  - Energy eigenstates — number-state kets