Massachusetts Institute of Technology

6.435 System Identification

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Problem Set No. 5 — Due 04/25/1994

Reading: Chapters 9,10,11.

Do the following problems from Ljung's book:

a. 7G.3

b. 7E.6

c. 7D.1

d. 7D.5

e. 7C.1 (a,b,c)

f. 7C.2

g. 8G.2

h. 8E.2

i. 8E.3

h. 8E.4

i. 9G.3

k. 9E.2

Problem 2

Let the output of some physical process be given by:

$$y(t) = \theta_0 u(t) e(t)$$

where u is a known input, e is a white gaussian noise with variance $= 1$, and θ_0 is a real unknown parameter. Let the predictor have the form:

$$\hat{y} = \theta u(t)$$

1. Let $\hat{\theta}_N^{LS}$ be the estimate of θ_0 that minimizes the prediction error

   $$V_N(\theta, Z^N) = \frac{1}{N} \sum_1^N \frac{1}{2} \epsilon^2(t, \theta)$$

   Compute the limit of $\hat{\theta}_N^{LS}$. Justify your method.

2. Compute $\hat{\theta}_N^{ML}$, the maximum likelihood estimate of θ_0. Can the sign of $\hat{\theta}_N^{ML}$ be determined?

3. What is the limit of $\hat{\theta}_N^{ML}$ as N goes to infinity. Compute the asymptotic covariance of $\hat{\theta}_N^{ML}$? Does the covariance go to zero? Explain your results.

Problem 3

Given the model structure :

$$y(t) = Gf(u(t)) + e(t)$$

where

$$G = \frac{B(q)}{A(q)},$$

with the leading coefficient of the polynomial A equal to 1. f is a memoryless nonlinearity of the form

$$f(u) = \alpha_1 u + \alpha_2 u^2,$$

and e is a noise process. Assume that the real system has the above structure.

1. Show how to identify the above system. (Hint: It may be useful to think of this system as a multi-input system).

2. Is the model structure globally identifiable? If not, under what conditions on G will it be?

3. What is the minimal number of sinusoids that are needed to guarantee that the data is informative enough with respect to the above model structure when the noise $e = 0$?

4. Repeat (3) with e white noise.

5. Comment on how to identify a system with a dead zone at the input, i.e.,

$$f(u) = \begin{cases} 0 & \text{if } -a \leq u(t) \leq a \\ u - a & \text{if } u(t) \geq a \\ u + a & \text{if } u(t) \leq -a \end{cases}$$

Problem 4

Let the output of a process be given by

$$y(t) = G_0 u(t) + v(t)$$

where G_0 is linear time-invariant, with an impulse response satisfying $|g_0(k)| \leq A\rho^k, A > 0, \rho < 1$, and v is the noise process. We would like to identify G_0 such that the estimate satisfies

$$E\|G_0 - \hat{G}\|_2^2 = E\left[\sum_{k=0}^{\infty} |g_0(k) - \hat{g}(k)|^2\right] \leq \delta$$

for some δ. The inputs are deterministic, bounded, quasi-stationary signals, with a bounded correlation function.

1. Show that the model structures

   $$M_n = \{G | g(k) = 0, \ \forall k > n\}$$

   are appropriate for identifying the system G_0.

2. Asssume that v is a white noise process with variance λ uncorrelated with the input and that the input is sufficiently rich. The input and output are observed for $0 \leq t \leq N$, compute the LS estimate in M_n, i.e., the impulse response in M_n that minimizes the quadratic prediction error criterion. Find a lower and upper bounds on $E\|G_0 - \hat{G}\|_2^2$ such that their difference goes to zero as n goes to infinity.

3. Assume now that the noise v is not stochastic but is modelled as an unknown signal with energy less than or equal to one, i.e. $\|v\|_2 \leq 1$. Using the estimate of (2), compute the worst case error:

   $$\max_v \|G_0 - \hat{G}\|_2^2$$

   (The expected value operator is not necessary since all the quantities are deterministic). Compare this quantity with the error derived in the previous part.

4. What is the minimum order of excitation that the input has to have in order to obtain an informative set of data. Answer for both models of the noise.

5. Discuss the choice of the above model structures to estimate the system. What are the advantages and disadvantages of the above structure over a rational output-error model structure in terms of input richness, computational burden, and number of parameters?

6. If ρ were unknown, propose a procedure to estimate G_0 in the model structures M_n.

7. I claim that if I use the model structure M_r to identify the plant, I can immediately calculate the increase in the value of the quadratic prediction error for any smaller structure $M_i, i \leq r$. Prove or disprove my claim.

8. Show how to estimate the plant using the correlation method. Compare with the above approach.

Problem 5

It is desired to identify the unstable plant G shown in the figure below. First, a controller K was implemented that stabilized G and then a reference input r was applied. The values of both u, y were recorded for $N = 1000$. The reference input is given by

$$r = \cos\frac{20\pi}{1000}t + \cos\frac{200\pi}{1000}t.$$

The data r, u, y are in a file named *data.mat* that you can copy onto your current MIT Server directory as follows.

```
%attach srv
%cp /mit/srv/Public/data.mat .
```

The "." in the UNIX command line designates your current directory. In order to load this file into your matlab working space, simply type while inside matlab

```
>> load data.mat
```

Identify the plant G.

Make your solution concise and brief. I want to see the logic you followed in arriving to your answer, and your analysis of the answer (whether it makes sense or not ...). A correct answer for this problem does not necessarily mean $\hat{G} = G$, and certainly it has nothing to do with how long your answer is, or the number of plots, matlab routines, ... that you turn in.