Massachusetts Institute of Technology

6.435 System Identification

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Problem Set No. 4 — Due 3/28/1994

Reading: Chapters 7,8.

Do the following problems from Ljung's book:

a. 7G.1

b. 7G.2 (part a)

c. 7G.7

d. 7E.2

e. 7E.3

f. 7E.4

g. 7C.1 (a,b)

(Continued on next page..)

Problem 2: The aim of this problem is to derive a discrete state space representation starting from input output data for SISO systems.

1. Given an impulse response function $\{h_k\}_{k=1}^{\infty}$ (notice $h_0 = 0$) which are alternatively known as markov parameters, and two integers α, β, the Hankel matrix is defined as:

$$H(k-1) = \begin{bmatrix} h_k & h_{k+1} & \cdots & h_{k+\beta-1} \\ h_{k+1} & h_{k+2} & \cdots & \cdots \\ \vdots & \vdots & \vdots & \vdots \\ h_{k+\alpha-1} & \cdots & \cdots & \cdots \end{bmatrix}$$

   (a) Prove that if the underlying system is of degree n then the rank of the hankel matrix cannot exceed n. Also show that for some α and β the hankel matrix will have rank n. Note that we are talking about the noise free case here.

   (b) Let $H(0) = R\Sigma S^T$ be the s.v.d of $H(0)$. Suppose $\Sigma = \begin{bmatrix} \Sigma_n & 0 \\ 0 & 0 \end{bmatrix}$, we can rewrite $H(0) = R_n \Sigma_n S_n^T$, where R_n and S_n are formed by first n columns of R and S respectively. Prove that the a state space realization of the above system has:

$$\hat{A} = \Sigma_n^{-1/2} R_n^T H(1) S_n \Sigma_n^{-1/2} \quad (1)$$
$$\hat{B} = \Sigma_n^{1/2} S_n^T E_r \quad (2)$$
$$\hat{C} = E_m^T R_n \Sigma_n^{1/2} \quad (3)$$

   where r, m are number of inputs and outputs respectively and $E_j^T = [I_j, O_j, \ldots]$ where I_j is the identity matrix of order j and O_j is a null matrix of order j. Note that in the case of noisy data the matrix H(0) in general will have full rank in which case we can remove small singular values to obtain a lower order system. (Hint: There exists some realization which generates the impulse response.)

2. Let $y = Gu + e$, where $G = \frac{q^{-1}}{1-0.6q^{-1}}$, $e \equiv N(0,1)$ and $u = \sum \cos(k\pi/4)$.

   (a) Spectral estimation generally generates a smooth frequency response estimate. To obtain a transfer function from these estimates we may have to use the results from part 1. From the correlation analysis $R_{yu} = GR_u$.

i. Obtain an estimate of the Markov parameters.

ii. Use part 1 to estimate a low order transfer function.

iii. Compare it with spa.

(b) Another approach is to directly estimate the parameters of the numerator and denominator of the transfer function

i. Express the estimated transfer function as

$$\hat{G}(z_k) = Q^{-1}(z_k)R(z_k) + \epsilon(k) \tag{4}$$

where

$$\begin{aligned} Q(z_k) &= 1 + q_1 z_k^{-1} + \ldots + q_p z_k^{-p} \\ R(z_k) &= r_0 + r_1 z_k^{-1} + \ldots + r_p z_k^{-p} \end{aligned}$$

and p is some integer. Find the least squares solution which minimizes the error index $J = \sum \|Q(z_k)\epsilon(k)\|_2$ by rewriting Equation(4).

ii. Derive a recursive expression for impulse response in terms of the coefficients of R and Q.

iii. Obtain the state space representation using equations (1)-(3). Use your judgement in truncation of Σ.