MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

6.341 DISCRETE-TIME SIGNAL PROCESSING
Fall 2005

MIDTERM EXAM
Tuesday, November 8, 2005

- This is a closed book exam, but two $8\frac{1}{2}'' \times 11''$ sheets of notes (both sides) are allowed.

- Calculators are not allowed.

- Make sure you have all 14 numbered pages of this exam.

- There are 5 problems on the exam.

- The problems are not in order of difficulty. We recommend that you read through all the problems, then do the problems in whatever order suits you best. **Also note that very often the later parts of a problem can be done independently of earlier parts.**

- A correct answer does not guarantee full credit, and a wrong answer does not guarantee loss of credit. You should clearly but concisely indicate your reasoning and **show all relevant work**. Your grade on each problem will be based on our assessment of your level of understanding as reflected by what you have written in the space provided.

- Please be neat—we can't grade what we can't decipher.

- Only the booklet supplied is to be handed in. **No additional pages will be considered in the grading.** You may want to first work through the problems on the scratch paper provided and then neatly transfer the work you would like us to look at into the exam booklet. Let us know if you need additional scratch paper.

- We will be following a grading strategy that focuses on your level of understanding of the material associated with each problem. When we grade each part of a problem we will do our best to assess your level of understanding from what you have written.

  Based on our judgement, each part will receive one of the following grades: E (EXCELLENT UNDERSTANDING), G (GOOD), R (REASONABLE), M (MARGINAL) or U (UNSATISFACTORY). Where appropriate, we may also use grades of E/G, G/R, R/M, or M/U so that there is a finer scale.

  For each part of each exam question we indicate the allotted percentage of the total exam grade. Your numerical score on that part will then be calculated accordingly. (For example, if a part represents 16% of the exam grade, then E=16, E/G=14, G=12, G/R=10, R=8, R/M=6, M=4, M/U=2, U=0.)

  This system (which we've used in the course many times in the past) is designed to focus your attention and ours on your level of understanding of the material, rather than on what calculations and miscalculations receive or lose points. Consequently, in looking at your graded exam, we ask that you focus on our judgement of your level of understanding. If you honestly feel that we've misjudged it based on what you've written in the exam booklet, then we'd be happy to discuss it with you.

THIS PAGE IS INTENTIONALLY LEFT BLANK. YOU CAN USE IT AS SCRATCH PAPER BUT NOTHING ON THIS PAGE WILL BE CONSIDERED DURING GRADING.

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NAME: _____

Problem	Grade	Points	Grader
1		/15	
2 (a)		/12	
2 (b)		/6	
2 (c)		/12	
3 (a)		/10	
3 (b)		/10	
4		/15	
5 (a)		/10	
5 (b)		/10	
Total		/100	

Problem 1   (15%)

A stable system with system function $H(z)$ has the pole-zero diagram shown in Figure 1-1. It can be represented as the cascade of a stable minimum-phase system $H_{min}(z)$ and a stable all-pass system $H_{ap}(z)$.
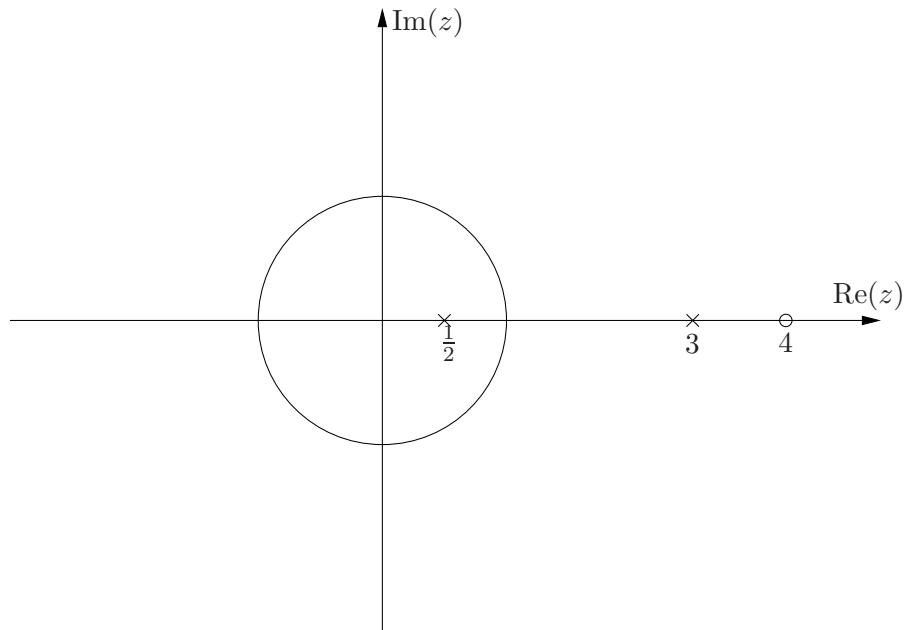

Figure 1-1: Pole-zero diagram for $H(z)$.

Determine a choice for $H_{min}(z)$ and $H_{ap}(z)$ (up to a scale factor) and draw their corresponding pole-zero plots. Indicate whether your decomposition is unique up to a scale factor.
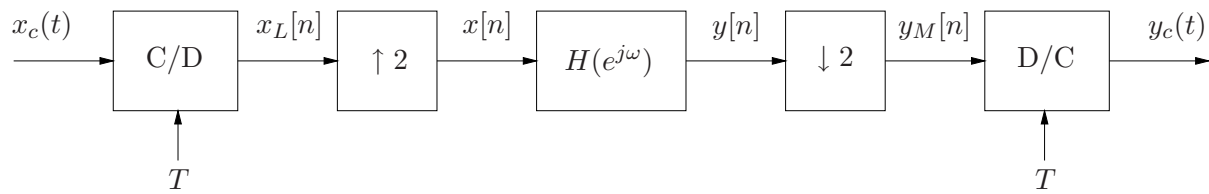
Problem 2 (30%)

Figure 2-1:

For parts (a) and (b) only, $X_c(j\Omega) = 0$ for $|\Omega| > 2\pi \times 10^3$ and $H(e^{j\omega})$ is as shown in Figure 2-2 (and of course periodically repeats).
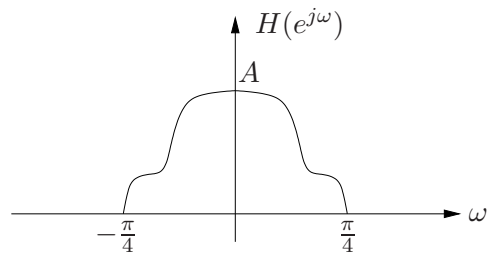

Figure 2-2:

(12%) (a) Determine the most general condition on T, if any, so that the overall continuous-time system from $x_c(t)$ to $y_c(t)$ is LTI.

(6%) (b) Sketch and clearly label the overall equivalent continuous-time frequency response $H_{\text{eff}}(j\Omega)$ that results when the condition determined in (a) holds.

(12%) (c) **For this part only** assume that $X_c(j\Omega)$ in Figure 2-1 is bandlimited to avoid aliasing, i.e. $X_c(j\Omega) = 0$ for $|\Omega| \geq \frac{\pi}{T}$. For a general sampling period T, we would like to choose the system $H(e^{j\omega})$ in Figure 2-1 so that the overall continuous-time system from $x_c(t)$ to $y_c(t)$ is LTI for any input $x_c(t)$ bandlimited as above.

Determine the most general conditions on $H(e^{j\omega})$, if any, so that the overall CT system is LTI. Assuming that these conditions hold, also specify in terms of $H(e^{j\omega})$ the overall equivalent continuous-time frequency response $H_{\text{eff}}(j\Omega)$.

Problem 3   (20%)

For this problem you may find the information on page 14 useful.

Consider the LTI system represented by the FIR lattice structure in Figure 3-1.

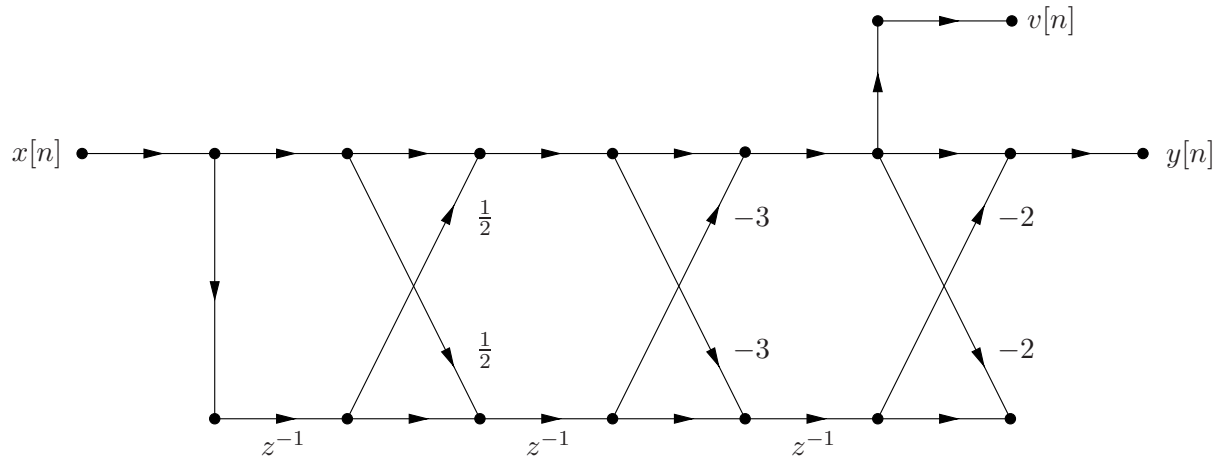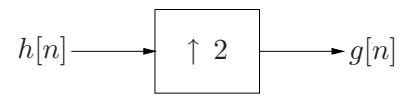Figure 3-1:

(10%) (a) Determine the system function from the input $x[n]$ to the output $v[n]$ (NOT $y[n]$).

(10%) (b) Let $H(z)$ be the system function from the input $x[n]$ to the output $y[n]$, and let $g[n]$ be the result of expanding the associated impulse response $h[n]$ by 2:

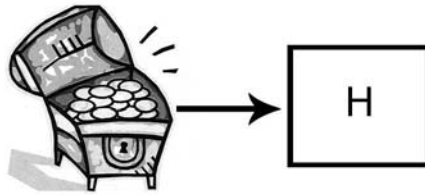$$h[n] \longrightarrow \boxed{\uparrow 2} \longrightarrow g[n]$$

The impulse response $g[n]$ defines a new system with system function $G(z)$.

We would like to implement $G(z)$ using an FIR lattice structure as defined by the figure on page 14. Determine the k-parameters necessary for an FIR lattice implementation of $G(z)$.

Note: You should think carefully before diving into a long calculation.

Problem 4   (15%)

We find in a treasure chest an even-symmetric FIR filter $h[n]$ of length $2L + 1$, *i.e.*

$$h[n] = 0 \text{ for } |n| > L,$$

$$h[n] = h[-n].$$

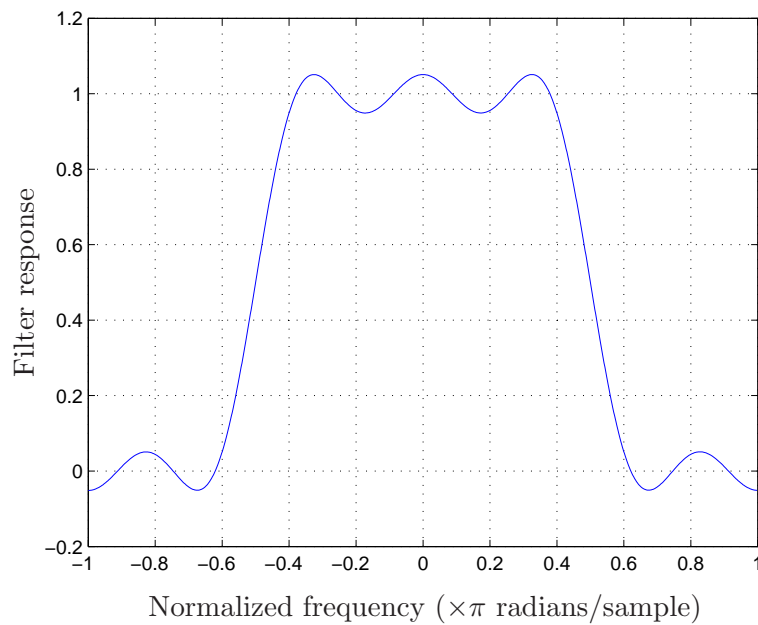$H(e^{j\omega})$, the DTFT of $h[n]$, is plotted over $-\pi \leq \omega \leq \pi$ in Figure 4-1.

Figure 4-1: Plot of $H(e^{j\omega})$ over $-\pi \leq \omega \leq \pi$.

What can be inferred from Figure 4-1 about the possible range of values of L? Clearly explain the reason(s) for your answer. Do not make any assumptions about the design procedure that might have been used to obtain $h[n]$.

Problem 5   (20%)

In Figure 5-1, $x[n]$ is a finite sequence of length 1024. The sequence $R[k]$ is obtained by taking the 1024-point DFT of $x[n]$ and compressing the result by 2.
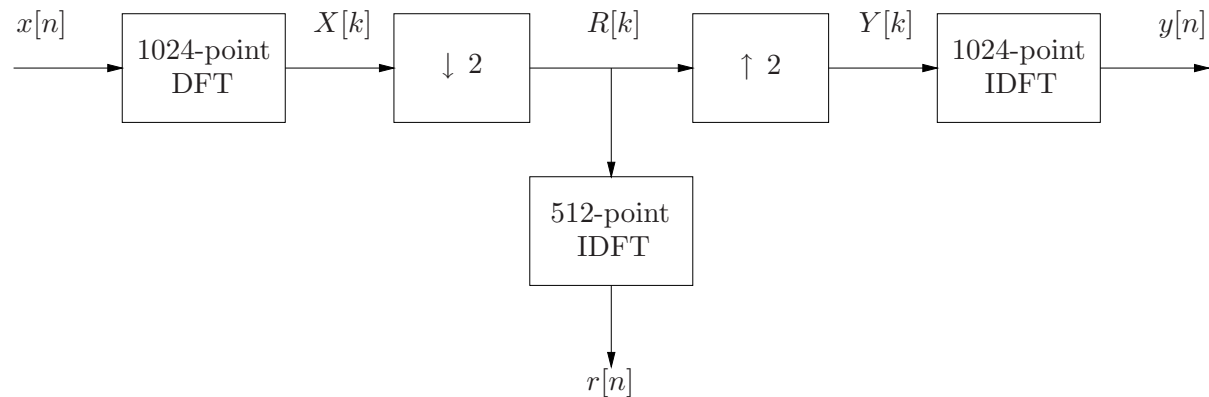

Figure 5-1:

(10%) (a) Choose the most accurate statement for $r[n]$, the 512-point inverse DFT of $R[k]$. Justify your choice in a few concise sentences.

    A. $r[n] = x[n],\ 0 \leq n \leq 511$

    B. $r[n] = x[2n],\ 0 \leq n \leq 511$

    C. $r[n] = x[n] + x[n + 512],\ 0 \leq n \leq 511$

    D. $r[n] = x[n] + x[-n + 512],\ 0 \leq n \leq 511$

    E. $r[n] = x[n] + x[1023 - n],\ 0 \leq n \leq 511$

In all cases $r[n] = 0$ outside $0 \leq n \leq 511$.

(10%) (b) The sequence $Y[k]$ is obtained by expanding $R[k]$ by 2. Choose the most accurate statement for $y[n]$, the 1024-point inverse DFT of $Y[k]$. Justify your choice in a few concise sentences.

A. $y[n] = \begin{cases} \frac{1}{2}\left(x[n] + x[n+512]\right), & 0 \le n \le 511 \\ \frac{1}{2}\left(x[n] + x[n-512]\right), & 512 \le n \le 1023 \end{cases}$

B. $y[n] = \begin{cases} x[n], & 0 \le n \le 511 \\ x[n-512], & 512 \le n \le 1023 \end{cases}$

C. $y[n] = \begin{cases} x[n], & n \text{ even} \\ 0, & n \text{ odd} \end{cases}$

D. $y[n] = \begin{cases} x[2n], & 0 \le n \le 511 \\ x[2(n-512)], & 512 \le n \le 1023 \end{cases}$

E. $y[n] = \frac{1}{2}\left(x[n] + x[1023-n]\right), \ 0 \le n \le 1023$

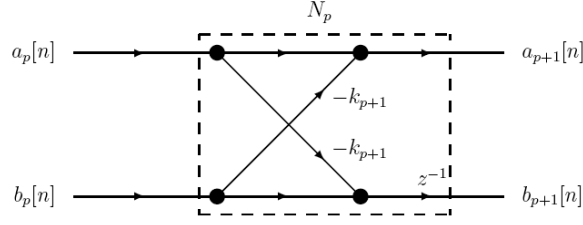In all cases $y[n] = 0$ outside $0 \le n \le 1023$.

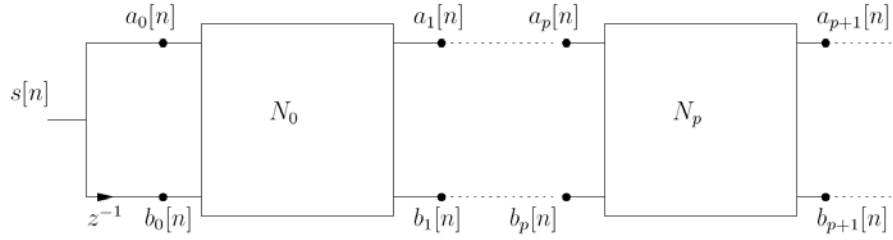**END OF EXAM**

Figure 1: One Section of Lattice Structure

Figure 2:

$$A_{p+1}(z) = A_p(z) - k_{p+1}B_p(z) \tag{1a}$$

$$B_p(z) = z^{-(p+1)}A_p(1/z) \tag{3a}$$

$$A_0(z) = 1 \quad \text{and} \quad B_0(z) = z^{-1} \tag{2}$$

$$a_k^{(p+1)} = \left[a_k^{(p)} - k_{p+1}a_{p-k+1}^{(p)}\right], \qquad k = 1, \dots, p \tag{8a}$$

$$a_{p+1}^{(p+1)} = k_{p+1} \tag{8b}$$

Define:

$$\underline{\alpha}_p = \left[a_1^{(p)} \; a_2^{(p)} \; \cdots \; a_p^{(p)}\right]^T$$

$$\underline{\check{\alpha}}_p = \left[a_p^{(p)} \; a_{p-1}^{(p)} \; \cdots \; a_1^{(p)}\right]^T$$

Then equations (8) become:

$$\underline{\alpha}_{p+1} = \begin{bmatrix} \underline{\alpha}_p \\ \cdots \\ 0 \end{bmatrix} - k_{p+1} \begin{bmatrix} \underline{\check{\alpha}}_p \\ \cdots \\ -1 \end{bmatrix} \tag{9}$$

Reverse recursion:

$$a_k^{(M-1)} = \frac{a_k^{(M)} + k_M a_{M-k}^{(M)}}{1 - k_M^2}, \qquad k = 1, \dots, M-1 \tag{11}$$

with $k_M = a_M^{(M)}$, $k_{M-1} = a_{M-1}^{(M-1)}$.