

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

6.341 DISCRETE TIME SIGNAL PROCESSING
Fall 2004

BACKGROUND EXAM

September 30, 2004.

Full Name: _____

Note:

This exam is closed book. No notes or calculators permitted.

The exam will be graded on the basis of the answers only. Please don't put any explanations or work in the answer booklet.

Five pages of scratch paper are attached at the end of the exam. These are not to be handed in. Let us know if you need additional scratch paper.

Problem 1 [8%] Consider the discrete-time system described by the following equation:

$$y[n] = x[-n]$$

- 2% (a) Is this system linear?
 2% (b) Is this system time-invariant?
 2% (c) Is this system causal?
 2% (d) Is this system stable?

Problem 2 [4%] Consider an LTI discrete time system for which the input and output satisfy the following difference equation:

$$y[n] + \frac{1}{2}y[n-1] = x[n]$$

- 2% (a) Is this system causal?
 2% (b) Is this system stable?

Problem 3 [4%] Consider the discrete time LTI system described by the following frequency response:

$$H(e^{j\omega}) = \frac{1 - 2e^{-j\omega}}{(1 - \frac{1}{2}e^{-j\omega})(1 - 3e^{-j\omega})}$$

- 2% (a) Is this system causal?
 2% (b) Is this system stable?

Problem 4 [6%] Determine the transfer function $H_{xy}(z)$ from $x[n]$ to $y[n]$ and the transfer function $H_{ey}(z)$ from $e[n]$ to $y[n]$ for the following system:

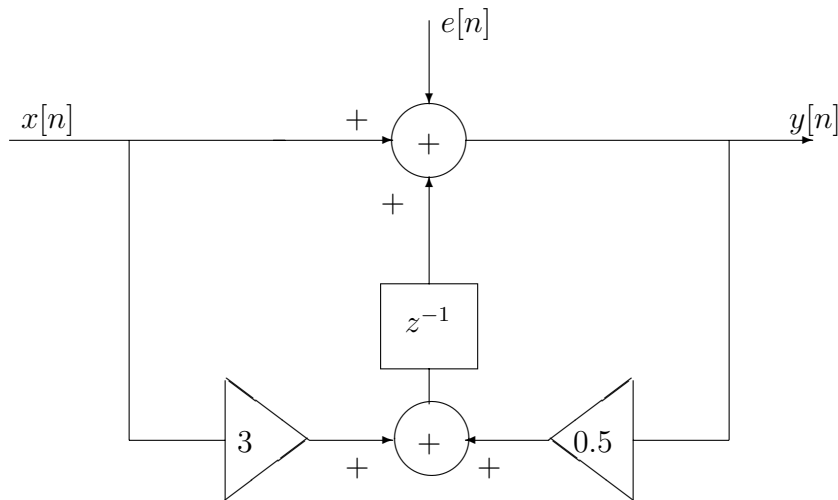


Figure 4-1:

Problem 5 [5%] Determine the frequency response $H(e^{j\omega})$ of the stable LTI system whose input and output satisfy the difference equation:

$$y[n] - \frac{1}{3}y[n-1] = x[n] + \frac{1}{2}x[n-1]$$

Problem 6 [9%] We generate a discrete time random process $x[n]$ by drawing a sequence of i.i.d. numbers from a random number generator with uniform distribution in $[-1, 1]$. $x[n]$ is then processed through an LTI system with impulse response $h[n] = \delta[n] + \delta[n-1]$ to obtain $y[n]$, i.e. $y[n] = x[n] + x[n-1]$.

3% (a) What is the mean $m_y[n]$ of the process?

3% (b) What is the autocorrelation $\phi_{yy}[m]$ of the process?

3% (c) What is the power spectral density $P_{yy}(e^{j\omega})$ of the process?

Problem 7 [9%] We process a zero-mean, unit variance, white, discrete-time process $x[n]$ through the stable LTI system with transfer function

$$H_1(z) = \frac{1 - \frac{1}{2}z^{-1}}{1 - \frac{1}{4}z^{-1}}$$

to get an output $y_1[n]$.

- 3%(a) Find a stable (possibly non-causal) system $H_2(z)$ such that if we pass $y_1[n]$ through $H_2(z)$, the output is white.
- 3% (b) Is your answer to (a) unique within a scalar factor?
- 3% (c) Assume we pass $y_1[n]$ through $H_2(z)$ to obtain $y_2[n]$. Are $x[n]$ and $y_2[n]$ uncorrelated? (i.e. is $\phi_{xy_2}[m]$ equal to 0?)

Problem 8 [8%] In the system shown below, two functions of time, $x_1(t)$ and $x_2(t)$, are multiplied together, and the product $w(t)$ is sampled by a periodic impulse train. $x_1(t)$ is bandlimited to Ω_1 , and $x_2(t)$ is bandlimited to Ω_2 ; that is,

$$X_1(j\Omega) = 0, \quad |\Omega| \geq \Omega_1$$

$$X_2(j\Omega) = 0, \quad |\Omega| \geq \Omega_2$$

Determine the *maximum* sampling interval T such that $w(t)$ is recoverable from $w_p(t)$ through the use of an ideal lowpass filter.

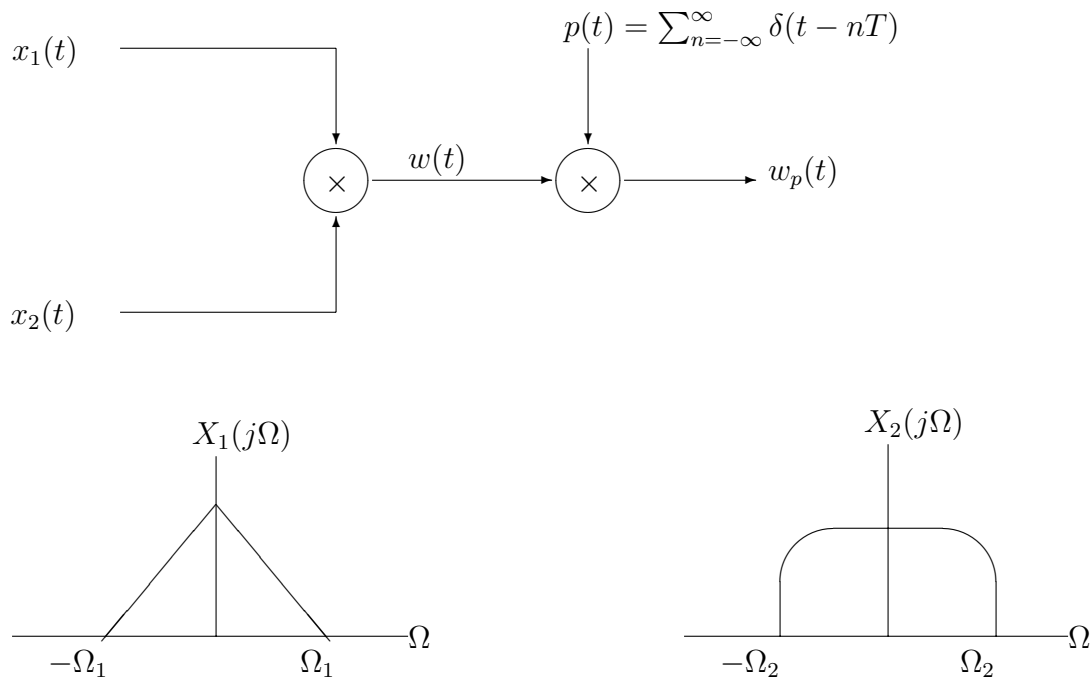


Figure 8-1:

Problem 9 [9%] Assume that the continuous-time signal $x_c(t)$ in the figure 9-1 has finite energy and is exactly bandlimited so that,

$$X_c(j\Omega) = 0 \quad \text{for } |\Omega| \geq 2\pi \times 10^4$$

The continuous-time signal $x_c(t)$ is sampled as indicated in the figure below to obtain the sequence $x[n]$, which we want to process to estimate the total area A under $x_c(t)$ as precisely as possible. Specifically, we define

$$A = \int_{-\infty}^{\infty} x_c(t) dt$$

The discrete-time signal calculates

$$\hat{A} = T \sum_{n=-\infty}^{\infty} x[n]$$

5%(a) Find the largest possible value of T which will result in as accurate an estimate as possible of A for any $x_c(t)$ consistent with the stated assumptions.

4%(b) Under the stated assumptions will your estimate of A be exact or approximate?

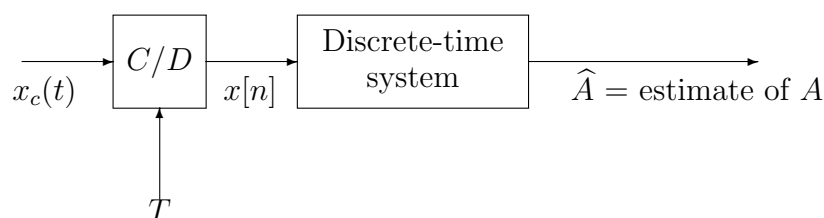
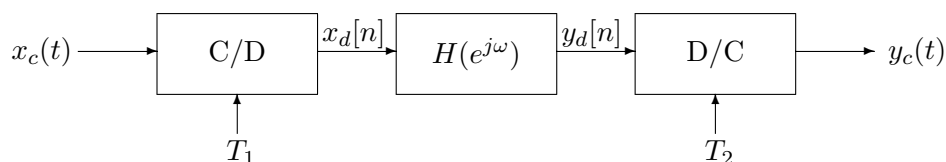


Figure 9-1:

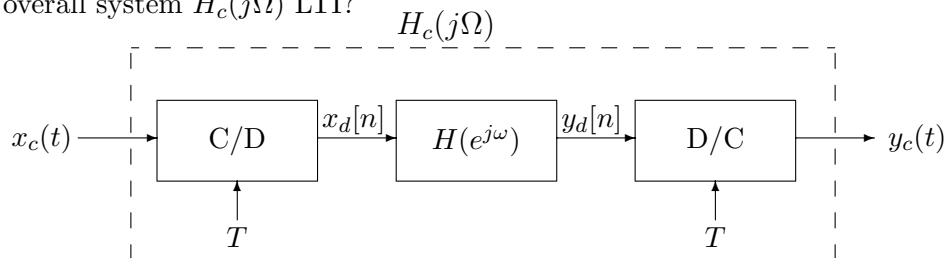
Problem 10 [8%] In the system below $H(e^{j\omega})$ is a discrete time all-pass filter with a constant group delay of 3.7 and continuous phase, i.e., $H(e^{j\omega}) = e^{-j\omega(3.7)}$, $|\omega| < \pi$.



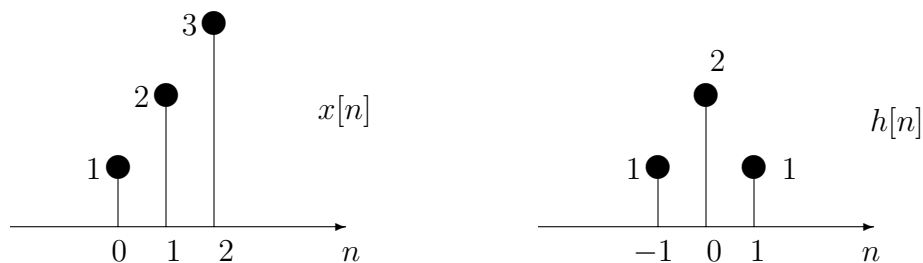
4% (a) Assuming no aliasing, describe in words (one sentence) the output of the overall system, $y_c(t)$, in terms of $x_c(t)$, $x_d[n]$, T_1 and T_2 .

4% (b) Assuming no aliasing, describe in words (one sentence) the output of the discrete time system $y_d[n]$ in terms of $x_c(t)$, $x_d[n]$, T_1 and T_2 .

Problem 11 [4%] For the system shown below, if the discrete-time system $H(e^{j\omega})$ is LTI, is the overall system $H_c(j\Omega)$ LTI?



Problem 12 [4%] Determine the output $y[n]$ for a system with the following input sequence $x[n]$ and impulse response $h[n]$.



Problem 13 [8%] For the pole-zero plot given below answer the following questions:

4%(a) If the ROC is $|z| > 2$:

2%(i) Is the system stable?

2%(ii) Is the system causal?

4%(b) If the ROC $|z| < 2$:

2%(i) Is the system stable?

2%(ii) Is the system causal?

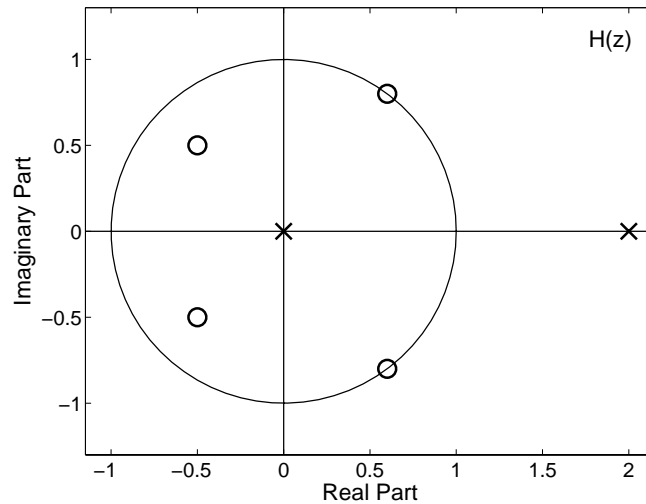
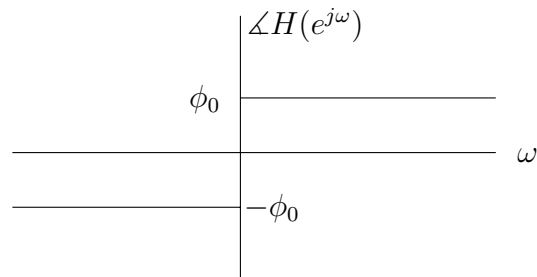


Figure 13-1: Pole-Zero Diagram

Problem 14 [6%] Consider a continuous-time LTI system for which the magnitude of the frequency response is 1 and the phase is as shown below. Determine the response of that system to the input $x(t) = s(t) \cos(\Omega_c t)$, where $S(j\Omega) = 0$ for $|\Omega| \geq \frac{\Omega_c}{2}$.



Problem 15 [8%] The first plot in Figure 15-1 shows a signal $x[n]$ that is the sum of three narrow-band pulses which do not overlap in time. Its transform magnitude $|X(e^{j\omega})|$ is shown in the second plot. The group delay and frequency response magnitude functions of Filter A , a discrete-time LTI system, are shown in the third and fourth plots, respectively. The remaining plots in Figures 15-2 and 15-3 show 8 possible output signals, $y_i[n]$ $i = 1, 2, \dots, 8$. Determine which of the possible output signals is the output of Filter A when the input is $x[n]$.

Problem 16 What is your best estimate of your grade on this exam?

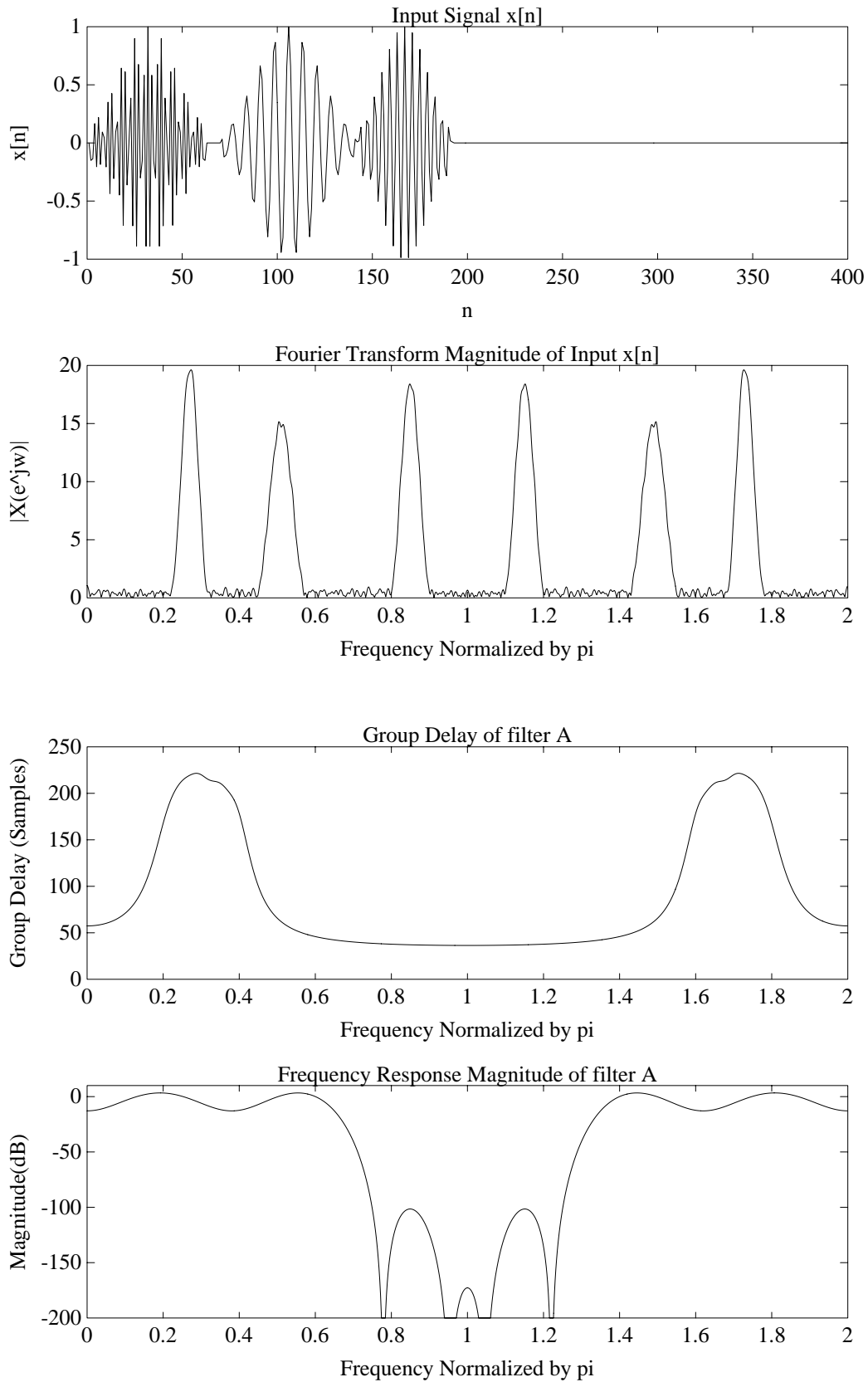
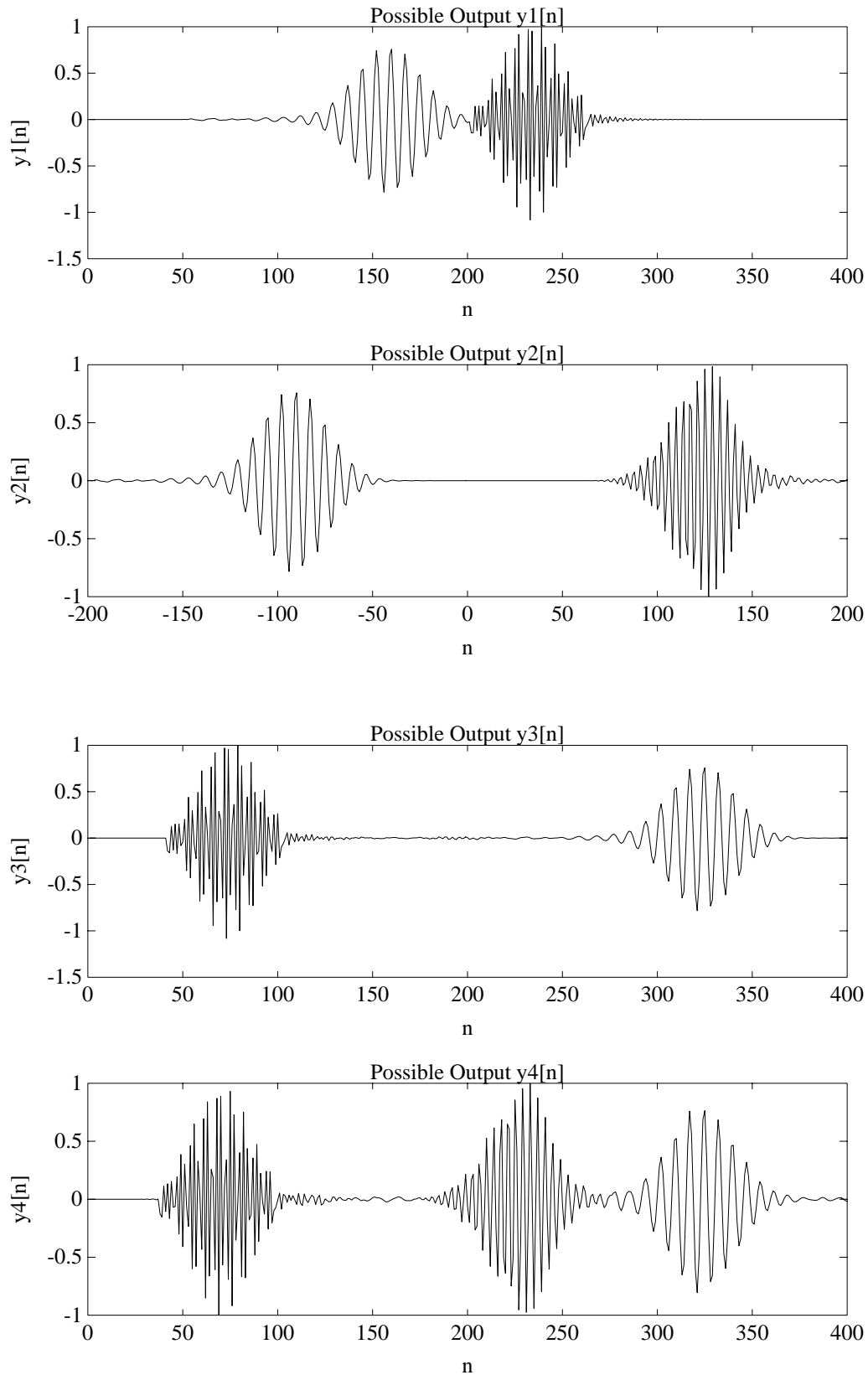
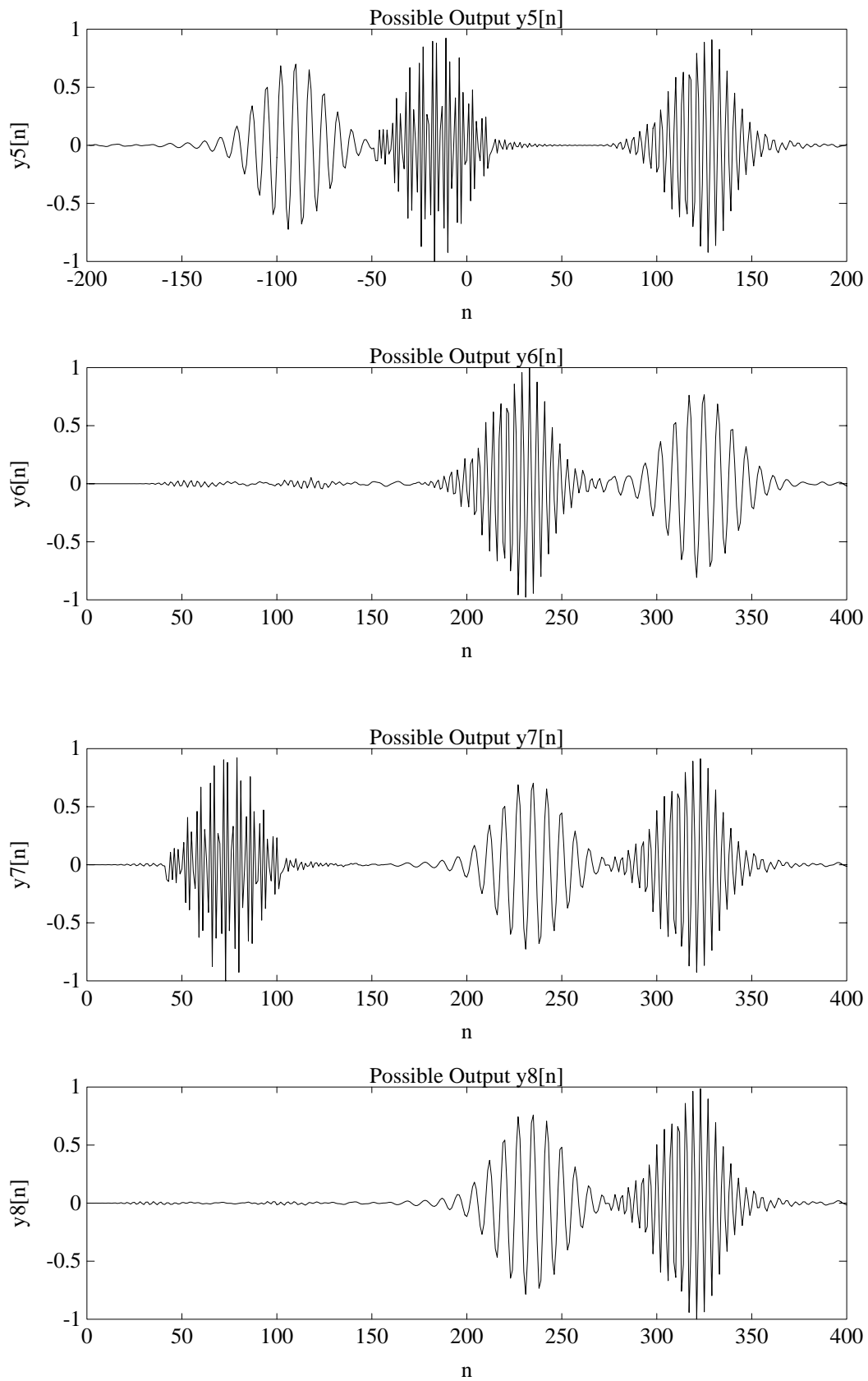


Figure 16-1: Input and Filter A information

Figure 16-2: Outputs $y_1 - y_4$

Figure 16-3: Outputs $y_5 - y_8$