MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

6.302 Feedback Systems

Spring Term 2007 — Issued : February 27, 2007
Problem Set 4 — Due : Tuesday, March 13, 2007

Directions: For problems 1, 2, 3, and 4, you are asked to draw the root locus. In each problem, be sure to include

- Separate plots of the root locus for positive K and negative K.
- Centroid locations and angles of the asymptotes for large $|K|$.
- Departure angles and entry angles for complex-conjugate poles and zeros.
- Locations of all real axis breakaway and entry points and the associated values of K at these points.

After you have tried these by hand, feel free to verify your work with MATLAB or similar software. You do not need to turn in your MATLAB plots.

Problem 1: Draw the root locus

$$L(s) = \frac{K}{(\frac{2}{3}s+1)(s^2+6s+10)}$$

Problem 2: Draw the root locus

$$L(s) = \frac{K}{s(s+2)(s+3)(s+100)}$$

Problem 3: Draw the root locus

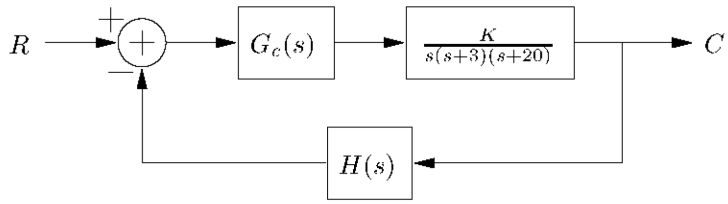
$$L(s) = \frac{K(s+100)(s+200)}{(s+1)(s+2)(s+3)}$$

Problem 4: Draw the root locus

$$L(s) = \frac{K(s^2+s+1)}{s^3}$$

Problem 5: Root Locus Under A Microscope

This problem will explore some of the nuances of root locus in systems with non-unity feedback.



(a) As a matter of familiarizing you with the system shown above, set $G_c(s) = 1$ and $H(s) = 1$. Plot the root locus of the loop transfer function $L(s)$, and specify the range of positive K for which the system is stable.

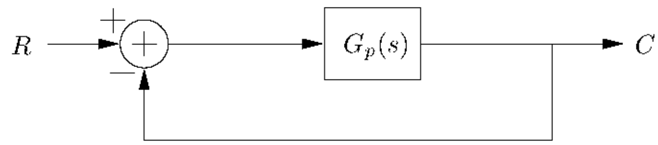
(b) Since this system is stable for an unacceptably small range of K, your friend Edward suggests one method of compensation; you follow his suggestion and set $G_c(s) = (s + 4)$ while leaving $H(s) = 1$. Plot the root locus of $L(s)$ with this compensator in place, and plot the locations of the closed-loop poles and zeros for $K = 200$. Additionally, use the `step` command in MATLAB to plot the step response of the closed-loop system with K still fixed at 200.

(c) As you are boasting about Ed's clever method of compensation, your other friend Harold mentions that he has thought of another, more attractive, way of compensating the system. He suggests setting $G_c(s) = 1$, but making $H(s) = (s+4)$. You are intrigued, and decide to compare Harold's scheme to Ed's configuration. Plot the root locus of $L(s)$ with this compensator in place, and plot the locations of the closed-loop poles and zeros for $K = 200$. Plot the step response of this system in MATLAB as well with the same value of K as you were previously using.

(d) Are the step responses different? If so, provide a short explanation for why this might be so.

Problem 6: Root Locus and Root Contours

You are given the following dual simple lag plant (as shown below). It could represent a two-tank level control, a temperature control with two heat capacitances in series, or a motor speed control.



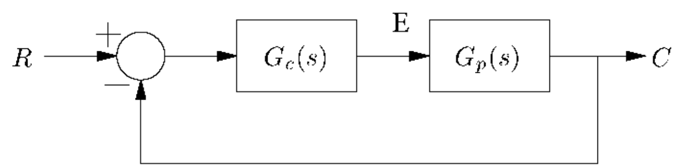
$$G_p(s) = \frac{K}{(s+1)(s+2)}$$

(a) Sketch the root locus of the uncompensated plant $G(s)$ for positive values of gain K. Find the value of K which makes $\zeta = 0.707$. What is the time constant of the resulting system?

(b) A proportional-derivative (PD) compensator of the form

$$G_c(s) = K_c + K_d s$$

is placed in series with the plant as shown below



Using the K which you found in part (a), derive the transfer function of the forward path for the system above. Write the characteristic equation $P(s)$ and compare it to the root locus form

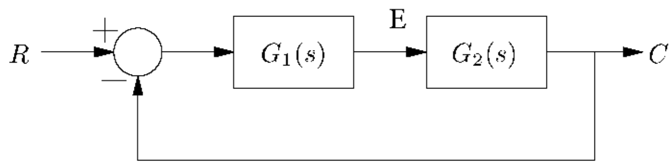
$$P(s) = d(s) + Kn(s)$$

Assume that the zero is to the left of the poles and sketch the root locus for positive values of the gain K_d. (Since we are changing the parameter K_d instead of the gain K, this plot is called a "root contour.")

(c) You now want to use the PD controller to decrease the time constant by a factor of 2 while retaining $\zeta = 0.707$. Where would you place the zero? What are the values of K_c and K_d? If more than one possibility exists, give an explanation of the reason for those possibilities.

Problem 7: Strange response.

You are sitting in the final exam for 6.302, and upon turning to the second page, you are confronted with the block diagram illustrated below



where

$$G_1(s) = \frac{0.9(s + \frac{8}{9})}{(s + \frac{8}{10})} \qquad G_2(s) = \frac{K}{(s + 0.8)(s^2 + 0.6s + 64.09)}$$

In order to maximize your suffering, the exam asks you to answer the following questions:

(a) Sketch (by hand) the step response of $G_1(s)$. What is this type of behavior called, and why is the practice of avoiding this type of behavior recommended?

(b) Sketch the root locus of $L(s) = G_1(s)G_2(s)$.

(c) Use MATLAB to draw the step response of $\frac{C}{R}(s)$ with $K = 1$. Provide an explanation for the atypical characteristics of this plot.

Problem 8: Introduction to the Nyquist Criterion.

For each of the three loop transfer functions $L(s)$ below, draw the Nyquist plot. In each case, determine the range (or ranges) of the gain K for stability.

$$L_a(s) = \frac{K}{s + 1} \qquad L_b(s) = \frac{K}{(s + 1)^2} \qquad L_c(s) = \frac{K}{(s + 1)^3}$$