# 6.301 Solid-State Circuits

Recitation 18: "Pythagorators," and other circuits
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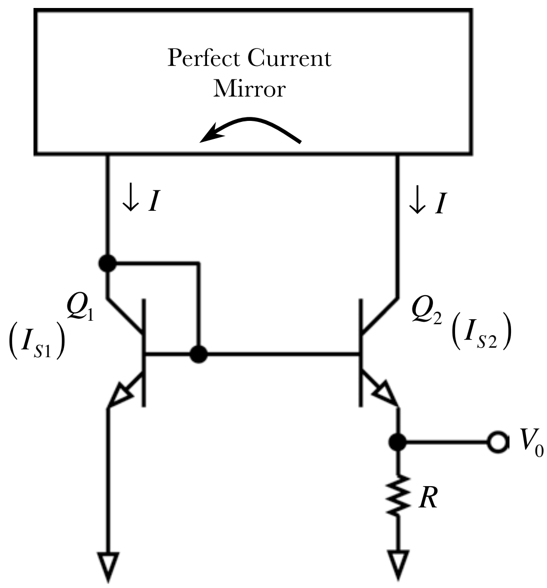
Way back at the beginning of the term, in Recitation 2, we talked about what it means to do engineering design. Quoting from those notes:

> "In engineering design, we make use of nature's laws to build useful machines."

The Gilbert Principle is a classic example of how we use the exponential $I_C$ vs. $V_{BE}$ relationship to build analog computation elements. Before examining this in more detail, let's use the class exercise to look at another intriguing case.

<u>CLASS EXERCISE</u>: Consider the following




$$Let\ \frac{I_{S2}}{I_{S1}} > 1$$

Using KVL, write $V_0$ as a function of $I$, $I_{S1}$, and $I_{S2}$. Can you think of any useful function served by $V_0$?

(Workspace)

How about <u>that</u>?! Note that while the temperature dependence of $V_{BE}$ is complicated, the temperature dependence of a $\Delta V_{BE}$ is simple:

$$\Delta V_{BE} = \frac{kT}{q}\ln\left(\frac{I_{C1}}{I_S}\right) - \frac{kT}{q}\ln\left(\frac{I_{C2}}{I_S}\right) = \frac{kT}{q}\ln\left(\frac{I_{C1}}{I_{C2}}\right)$$

If $\frac{I_{C1}}{I_{C2}}$ is independent of temperature, we say that $\Delta V_{BE}$ is "PTAT," or

<u>P</u>roportional <u>t</u>o <u>A</u>bsolute <u>T</u>emperature.

So…we took advantage of our detailed knowledge of bipolar transistors to build a thermometer. The Gilbert Principle takes advantage of our knowledge in a different way, and for a different end.

Final Note: Do not use circuits like the class exercise without some sort of start-up circuitry. Note that $I=0$ is a valid state.

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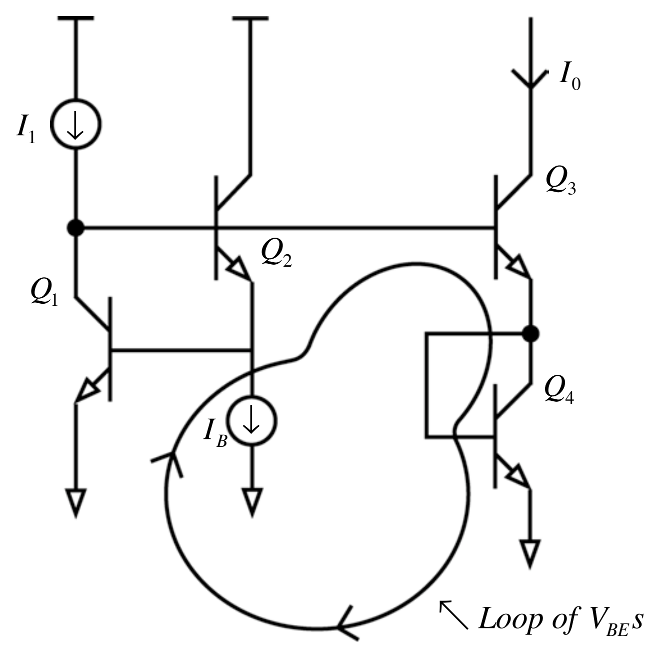
As we saw in lecture yesterday, the Gilbert Principle is a kind of mathematical shorthand for KVL. Let's review.

Current square-root circuit:



Write out KVL:

$$V_{BE1} + V_{BE2} - V_{BE3} - V_{BE4} = 0$$

$$V_T \ln\left(\frac{I_1}{I_S}\right) + V_T \ln\left(\frac{I_B}{I_S}\right) - V_T \ln\left(\frac{I_0}{I_S}\right) - V_T \ln\left(\frac{I_0}{I_S}\right) = 0$$

$$\cancel{V_T} \ln\left(\frac{I_1 I_B}{I_S^2}\right) = \cancel{V_T} \ln\left(\frac{I_0^2}{I_S^2}\right)$$

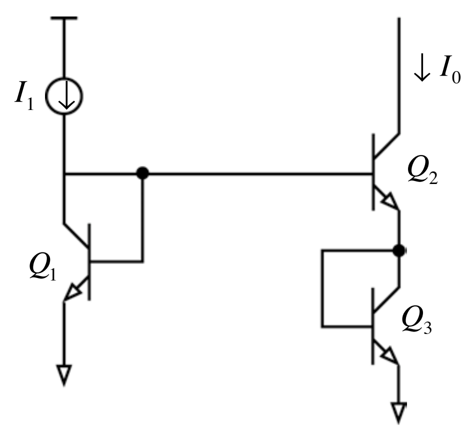
$$\boxed{I_0 = \sqrt{I_B I_1} = k\sqrt{I_1}}$$

Pretty neat. From this example, simple though it is, we can draw a couple of very general conclusions.

(1) Translinear circuits are <u>fast</u>.
(2) Translinear circuits always involve an even number of $V_{BE}s$.

To understand (1), look at all of the $C_\mu$ s. None of them get Miller multiplied. $C_{\mu 1}$ Even gets bootstrapped, while $C_{\mu 4}$ gets shorted out altogether.

To understand (2), consider a tempting but <u>extremely wrong</u> way to implement the square root function.




Translinear principle:

$$I_1 = I_0^2$$
$$I_0 = \sqrt{I_1}$$

NO!!

(The units don't even work.)

KVL:

$$\cancel{V_T}\ln\left(\frac{I_1}{I_S}\right) = 2\cancel{V_T}\ln\left(\frac{I_0}{I_S}\right)$$

$$\ln\left(\frac{I_1}{I_S}\right) = \ln\left(\frac{I_0^{\,2}}{I_S^{\,2}}\right)$$

$$I_0 = \sqrt{I_S}\,\sqrt{I_1}$$

> Vitally important that all of the $I_S$ s cancel out. This is only possible if the number of CW $V_{BE}$ s equals the number of CCW $V_{BE}$ s.

To finish off the basics, we recall that we sometimes have the freedom to change emitter areas. If we write

$$I_S = A_E J_S$$

We have

$$\sum_{CW} V_{BEm} = \sum_{CCW} V_{BEn}$$

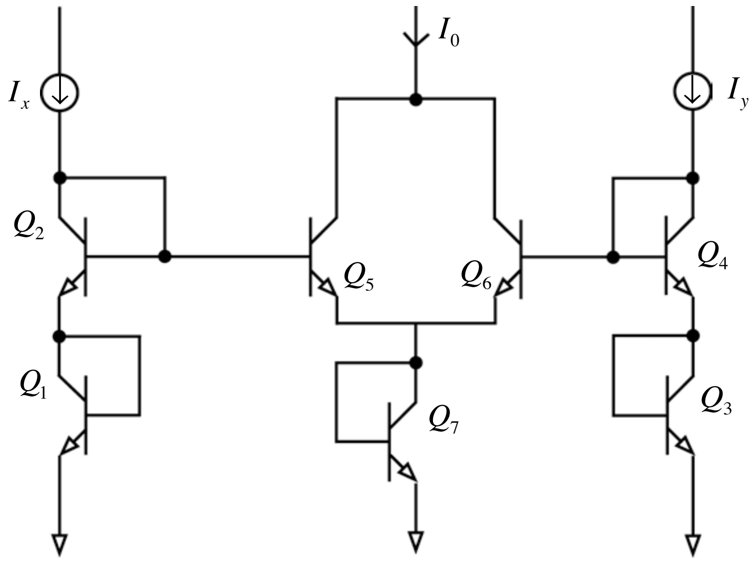
$$\prod_{CW} \frac{I_{Cm}}{I_{Sm}} = \prod_{CCW} \frac{I_{Cn}}{I_{Sn}}$$

$$\prod_{CW} \frac{I_{Cm}}{A_{En} J_S} = \prod_{CCW} \frac{I_{Cn}}{A_{En} J_S}$$

$$\boxed{\prod_{CW} \frac{I_{Cm}}{A_{Em}} = \prod_{CCW} \frac{I_{Cn}}{A_{En}}}$$

This is the most general translinear principle.

Now, at last, a Pythagorator. One such cell was shown to you in lecture yesterday. Here is a seven-transitor version.




How to analyze? Start with

$$I_0 = I_5 + I_6$$

And

$$I_7 = I_0$$

The left Gilbert loop gives:

$$I_1 I_2 = I_5 I_7 = I_5 I_0$$

$$I_x^2 = I_5 I_0 \Rightarrow I_5 = \frac{I_x^2}{I_0}$$

Right Gilbert loop gives:

$$I_6 = \frac{I_y^2}{I_0}$$

So for the output:

$$I_0 = I_5 + I_6 = \frac{I_x^2}{I_0} + \frac{I_y^2}{I_0}$$

$$I_0^2 = I_x^2 + I_y^2$$

$$I_0 = \sqrt{I_x^2 + I_y^2}$$

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