15.081 Fall 2009
Recitation
for Lectures {5,6,7,8}
Simplex Method

Note: In what follows, I will present the material that was covered in the recitations corresponding to the lectures 5, 6, 7 and 8 which dealt with the geometry of linear programming. The material will be somewhat a superset of what I covered in the recitations. These material are intended to cover the most important principles of Chapter 3 from the book. These notes should be viewed as a summary of the chapter and not as a reading material (for the exam).

1 Definitions

1. Basis
2. Feasible direction
3. Reduced cost

2 Important Theorems

This set of lectures develops the simplex method. Theorem 3.1 and its proof are most interesting.

3 Algorithms

The simplex method, revised simplex method and the corresponding space and time complexities should be studied. The techniques used in reducing the space complexity of simplex method are standard in various other algorithms involving matrix computations. This techique should be well studied.

The algorithm used to drive put the artificial variables in the phase 1 of the simplex algorithm is also interesting.

4 Column Geometry

Column geometry view of the simplex method allows one to expect good performance from the simplex algorithm. The view helps us to see how, if a good pivoting rule is used, simplex will take $O(n)$ iterations to achieve optimality. The "good pivoting rule" is still open.

5 Problems and Exercises

The following problems were covered in the recitation.

Problem 3.19

The problem is an exercise to internalise the simplex algorithm. One should keep in mind the various "if-else" considitons in the simplex algorithm.

Problem 3.27 (The linear scaling method)

(a)

the key here is to identify that if $\exists x$ s.t. $x_i > 0$ then $\exists \alpha > 0$ where $z + y = \alpha x$ and $y_i = 1$. Therefore maximizing $\sum y_i$ is equivalent to maximizing the number of positive components of x.

(b)

The same scaling method should be used here. But since the right hand side is not zero, the previous LP cannot be used directly. $\therefore$ we should make the right hand zero. This can be accomplished as follows.Consider the following LP

$$
\begin{aligned}
\textit{maximize} \quad & \sum y_i \\
s.t \quad & A(z + y) = \alpha b \\
& y_i \leq 1 \\
& z, y \geq 0 \\
& \alpha \geq 1
\end{aligned}
$$

Here, α is also a decision variable. The key again here is to notice that if $\exists x$ s.t. $x_i > 0$ then $\exists \alpha, y, z > 0$ where $z + y = \alpha x$ and $y_i = 1$. And the result follows. By making α a decision variable, we made the problem invariant under scaling. This is a technique that should be treasured!

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