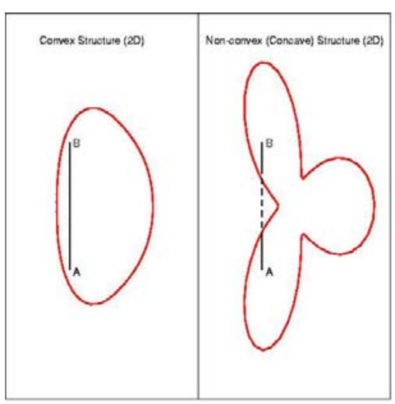
15.081/6.251 Fall 2009
Introduction to Mathematical Programming
Recitation #1

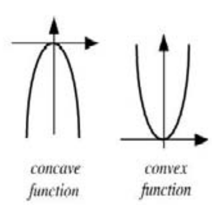
1 Outline

- Convex Sets and Convex Functions
- Linear Programs - Formulating good LPs
- Using CPLEX and AMPL

2 Convex Objects and Convex functions

- Convex Object (Set) : In *Euclidean space* ($\mathbb{R}^n$) , an object is **convex** if for every pair of points within the object, every point on the straight line segment that joins them is also within the object. Note that, for other spaces the definition is not simple.





- Convex Function (Alternate definition) : A function is said to be convex if its epigraph is a convex set. (The epigraph of a function is the set of points lying on or above its graph)

3 Linear Programs

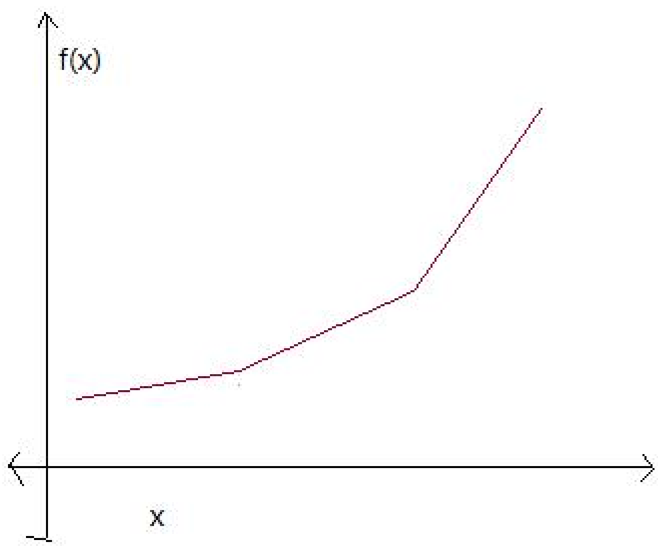
A convex program is an optimization problem where we aim to minimize a convex function over a convex set. A linear program is an instance of a convex optimization problem where we minimize a linear function (which is convex) over a polyhedron (which is a convex set).

3.1 Examples - Modeling the minimization of piecewise linear functions

Consider the following minimization problem.

$$\begin{aligned} \textbf{min} \quad & f(\mathbf{x}) \\ \textbf{s.t} \quad & \mathbf{Ax} = \mathbf{b} \end{aligned}$$

where $f(\mathbf{x})$ is as shown in the figure



This can be formulated as a linear optimization problem by observing that

$$f(\mathbf{x}) = \max_{j=1\ldots n} \left(\mathbf{c_j}'\mathbf{x} + d_j \right)$$

for appropriate $\{\mathbf{c_j}, d_j\}_{j=1\ldots n}$
Then the problem can be reformulated as

$$\begin{array}{rl} \textbf{min} & z \\ \textbf{s.t} & \mathbf{Ax} = \mathbf{b} \\ & z \geq \mathbf{c_j'x} + d_j \quad \forall j = 1 \ldots n \end{array}$$

3.2 Absolute Value Function

Minimizing $\sum_{j \in \{1,\ldots,n\}} c_j |x_j|$, where $c_j \geq 0,\ \forall j \in \{1,\ldots,n\}$, subject to $Ax = b,\ x \geq 0$ can be written as the following LP:

$$\begin{array}{rlcl} \min & c'(x^+ + x^-) & & \\ \text{s/t} & A(x^+ - x^-) & = & b \\ & x^+, x^- & \geq & 0. \end{array}$$

Check for correctness: when $x_j^{+*} > 0,\ x_j^{-*} = 0$ and vice-versa.

Alternatively, we can write

$$\begin{array}{rrcl} \min & c'z & & \\ \text{s/t} & z_j & \geq & x_j \\ & z_j & \geq & -x_j \\ & Ax & = & b \\ & x & \geq & 0. \end{array}$$

4 Linear Algebra review (for Lecture 2)

1. Linear independence
2. Subspace
3. Span
4. Basis
5. Dimension
6. Rank of a matrix
7. Affine Subspace

5 AMPL and CPLEX review

AMPL (A Mathematical Programming Language) is a high-level programming language for writing and solving mathematical programs (linear and non-linear, in continuous and discrete variables). AMPL itself does not solve the problems, instead it calls an external solver (such as CPLEX) to solve the optimization problem.

An AMPL program consists of two parts:

- a model file (.mod file), and
- a data file (.dat file)

The model file writes the linear program using the grammar of AMPL and it defines various sets, parameters, variables, objective and constraints. The data file provides data for the model file.

AMPL tutorial available online

Go to athena and do the following.
athena% add oplstudio
athena% ampl (starts the interactive development environment)
AMPL can be run with a built-in CPLEX solver (the default) or using one of two optional solvers, LOQO, or SNOPT.
To select LOQO as solver, type option solver loqo; at the ampl: prompt.
To select SNOPT as solver, type option solver snopt; at the ampl: prompt.
To exit, type quit at the ampl: prompt.