Massachusetts Institute of Technology

Department of Electrical Engineering and Computer Science

6.245: MULTIVARIABLE CONTROL SYSTEMS

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Problem Set 3 (due February 25, 2004) [1]

Problem 3.1

Consider a control system described by

$$\ddot{q}(t) - a^2 q(t) = v(t) + f_1(t), \quad g(t) = q(t) + f_2(t),$$

where $f = [f_1; f_2]$ is a normalized white noise, v is the control signal, g is the sensor measurement, and $a > 0$ is a parameter. The objective is to find a dynamic feedback controller (with input g and output v) which stabilizes the system while using a minimum of control effort (defined as the asymptotic variance of $v(t)$ as $t \to \infty$).

(a) Find the coefficients of the auxiliary abstract H2 optimization problems associated with the original task.

(b) Write analytically the associated Hamiltonian matrices, bases of their stable invariant subspaces, stabilizing solutions of the Riccati equations, and optimal controller and observer gains.

(c) Derive an analytical expression for the transfer function of the optimal dynamic feedback controller, and verify it using numerical calculations with `h2syn.m`.

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[1] Version of February 18, 2004

Problem 3.2

Random signal $q = q(t)$ is assumed to be a "bandlimited white noise" of a given bandwidth B (i.e. the result of passing the true white noise $v_1(t)$ through an ideal low-pass filter of bandwidth w_0 rad/sec). A high quality sensor is assumed to measure $q(t)$ accurately, except for a white additive noise, with the signal-to-noise ratio of 10.

(a) Use `h2syn.m` to design a 10-th order linear filter which inputs the sensor output, and outputs an estimate of $\dot{q}(t)$ which makes the mean square estimation error as small as possible.

(b) Test your design by comparing the *simulated* performance of filters you have designed for $B = 10$ rad/sec and $B = 1$ rad/sec on signals $q(\cdot)$ of bandwidths of $B = 10$ and $B = 1$ rad/sec. (One expects that the filter optimized for $B = 10$ rad/sec will be better on the $q(\cdot)$ with bandwidth $B = 10$ rad/sec than the filter optimized for $B = 1$ rad/sec, and vice versa.) Use the generator of bandlimited white noise supplied with the SIMULINK to perform the simulations.