Homework 4 additional problems

1. *Simple portfolio optimization.* We consider a portfolio optimization problem as described on pages 155 and 185–186 of *Convex Optimization*, with data that can be found in the file `simple_portfolio_data.m`.

   (a) Find minimum-risk portfolios with the same expected return as the uniform portfolio ($x = (1/n)\mathbf{1}$), with risk measured by portfolio return variance, and the following portfolio constraints (in addition to $\mathbf{1}^T x = 1$):

   - No (additional) constraints.
   - Long-only: $x \succeq 0$.
   - Limit on total short position: $\mathbf{1}^T(x_-) \le 0.5$, where $(x_-)_i = \max\{-x_i, 0\}$.

   Compare the optimal risk in these portfolios with each other and the uniform portfolio.

   (b) Plot the optimal risk-return trade-off curves for the long-only portfolio, and for total short-position limited to 0.5, in the same figure. Follow the style of figure 4.12 (top), with horizontal axis showing standard deviation of portfolio return, and vertical axis showing mean return.

2. *Minimum fuel optimal control.* Solve the minimum fuel optimal control problem described in exercise 4.16 of *Convex Optimization*, for the instance with problem data

$$A = \begin{bmatrix} -1 & 0.4 & 0.8 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 0 \\ 0.3 \end{bmatrix}, \quad x_{\mathrm{des}} = \begin{bmatrix} 7 \\ 2 \\ -6 \end{bmatrix}, \quad N = 30.$$

   You can do this by forming the LP you found in your solution of exercise 4.16, or more directly using `cvx`. Plot the actuator signal $u(t)$ as a function of time t.

3. *Numerical perturbation analysis example.* Consider the quadratic program

$$\begin{array}{ll} \text{minimize} & x_1^2 + 2x_2^2 - x_1x_2 - x_1 \\ \text{subject to} & x_1 + 2x_2 \le u_1 \\ & x_1 - 4x_2 \le u_2, \\ & 5x_1 + 76x_2 \le 1, \end{array}$$

   with variables x_1, x_2, and parameters u_1, u_2.

   (a) Solve this QP, for parameter values $u_1 = -2$, $u_2 = -3$, to find optimal primal variable values $x_1^\star$ and $x_2^\star$, and optimal dual variable values $\lambda_1^\star$, $\lambda_2^\star$ and $\lambda_3^\star$. Let $p^\star$ denote the optimal objective value. Verify that the KKT conditions hold for the optimal primal and dual variables you found (within reasonable numerical accuracy).

Hint: See §3.6 of the `CVX` users' guide to find out how to retrieve optimal dual variables. To specify the quadratic objective, use `quad_form()`.

(b) We will now solve some perturbed versions of the QP, with

$$u_1 = -2 + \delta_1, \qquad u_2 = -3 + \delta_2,$$

where δ_1 and δ_2 each take values from $\{-0.1, 0, 0.1\}$. (There are a total of nine such combinations, including the original problem with $\delta_1 = \delta_2 = 0$.) For each combination of δ_1 and δ_2, make a prediction $p^\star_{\rm pred}$ of the optimal value of the perturbed QP, and compare it to $p^\star_{\rm exact}$, the exact optimal value of the perturbed QP (obtained by solving the perturbed QP). Put your results in the two righthand columns in a table with the form shown below. Check that the inequality $p^\star_{\rm pred} \leq p^\star_{\rm exact}$ holds.

δ_1	δ_2	$p^\star_{\rm pred}$	$p^\star_{\rm exact}$
0	0		
0	-0.1		
0	0.1		
-0.1	0		
-0.1	-0.1		
-0.1	0.1		
0.1	0		
0.1	-0.1		
0.1	0.1		