Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science
6.061/6.690 Introduction to Power Systems

Problem Set 4 Solutions February 28, 2007

Problem 1: Part 1: Chapter 3, Problem 3

The speed of light is $c = \frac{1}{\sqrt{LC}} = 3 \times 18^8$m/S and line characteristic impedance is $Z_0 = \sqrt{\frac{L}{C}} = 250\Omega$, so that

$$\begin{aligned} L &= \sqrt{\frac{L}{C}}\sqrt{LC} = \frac{Z_0}{c} = \frac{250}{3 \times 10^8} \approx 8.333 \times 10^{-7} \\ C &= \sqrt{\frac{C}{L}}\sqrt{LC} = \frac{1}{Z_0 c} = \frac{1}{250 \times 3 \times 10^8} \approx 1.333 \times 10^{-11} \end{aligned}$$

The 20 kA pulse splits in two, making a pulse that propagates in the +x direction and a second pulse that propagates in the negative x direction as shown in Figure 1

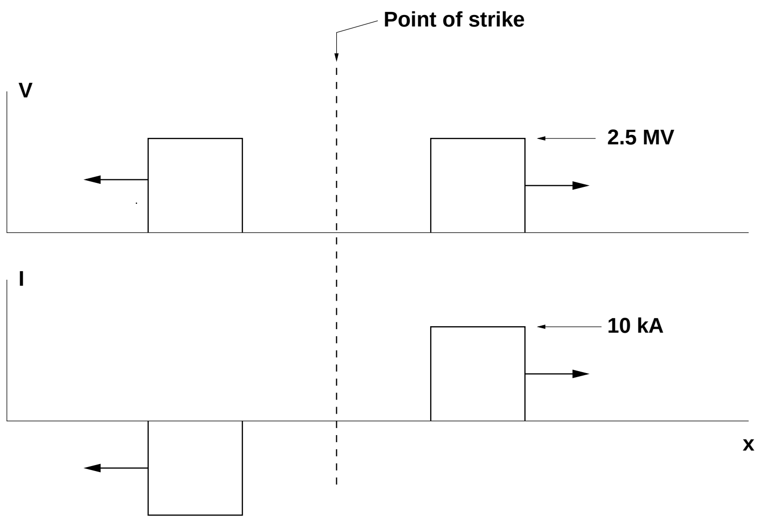

Figure 1: Pulses at about 40 microseconds

Transit time from the point of the fault to either end is $T = \frac{1.5 \times 10^5 \text{m}}{3 \times 10^8 \text{m/s}} \approx 500\mu S$. When the current pulse gets to the shorted end, since the voltage is zero, a reflected pulse is launched with opposite voltage sign. This is a positive going pulse, so the current is doubled. At the matched end, the first voltage pulse arrives after $150\mu S$. The reflected pulse arrives 1 mS after that. This is the end of the story as both pulses are absorbed by the matched resistor. This is shown in Figure 2.

Problem 1: Part 2: Chapter 3, Problem 4

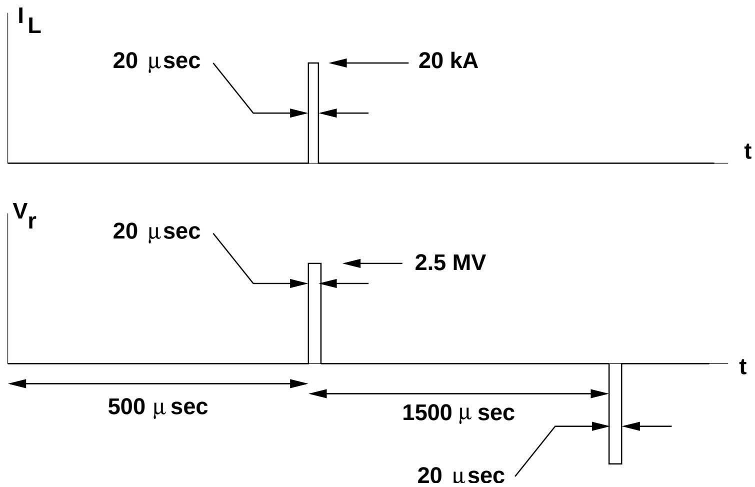

Figure 2: Shorted end current and matched end voltage

Wavelength is $\lambda = \frac{3\times10^8}{60} = 5 \times 10^6$m, so for a 300 kilometer line, $kl = 2\pi \times \frac{300}{5000} \approx .377$. Then open circuit sending end voltage is $V_r = \frac{V_s}{\cos kl} = \frac{500}{.93} \approx 537.8$kV. Sending end current is $I_s = \frac{V_s}{Z_0} \tan kl = \frac{500\text{kV}}{250\Omega} \times \tan 0.37 \approx 792$A

For source impedance of zero, voltage and current along the line are:

$$\begin{aligned}\underline{V}(x) &= V_s \frac{\frac{Z_L}{Z_0}\cos kx - j\sin kx}{j\sin kl + \frac{Z_L}{Z_0}\cos kl} \\ \underline{I}(x=-l) &= \frac{V_s}{Z_0}\frac{\cos kx + j\frac{Z_L}{Z_0}\sin kx}{\frac{Z_L}{Z_0}\cos kl + j\sin kl}\end{aligned}$$

Evaluated for $\frac{Z_L}{Z_0} = \frac{1}{0.8}$, 1.0 and $\frac{1}{1.2}$,and with source voltage of 500 kV, receiving end voltage (at $x = 0$), sending end voltage and current and sending end current, real and reactive power are evaluated by the attached script. The results are:

```
Chapter 3, Problem 4
Receiving end open:
Receiving end Voltage = 537764 V, RMS
Sending end Current = 791.856 A, RMS
R = 312.5   Vr = 512662  Is = 1701.9
Pr = 8.4103e+08  Ps = 8.4103e+08   Qs = -1.29538e+08
R = 250   Vr = 500000  Is = 2000
Pr = 1e+09  Ps = 1e+09   Qs = 0
R = 208.333   Vr = 485728  Is = 2282.72
Pr = 1.13247e+09  Ps = 1.13247e+09   Qs = 1.42126e+08
```

Using the same formulae, with varying receiving end resistance, voltage is plotted in Figure 3.

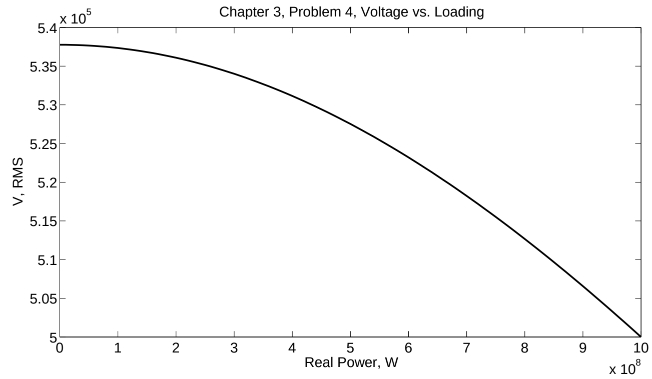

Figure 3: Receiving End Voltage

To estimate the effect of compensation, we assume a capacitance in parallel with the receiving end, with a capacitive admittance of $Y_c = \frac{2Q}{V^2}$. This is placed in parallel with the receiving end resistance. The voltage at the receiving end is calculated in the normal way and is shown in Figure 4. Note there are three curves, corresponding to the three levels of real load. Note also that the case of surge impedance loading (2,000 A) has nominal voltage with zero compensation.

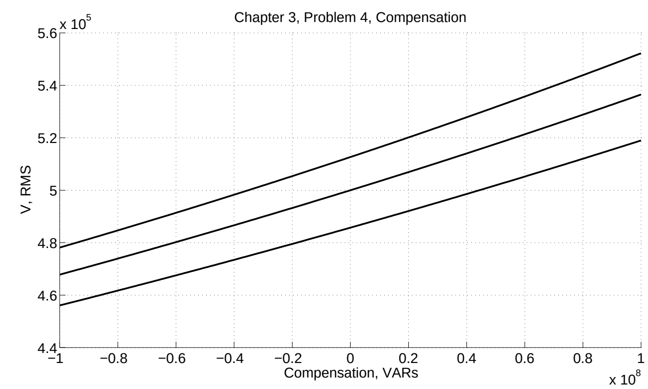

Figure 4: Receiving End Voltage

Problem 2: We take the easy way out here and let Matlab do all the heavy lifting. Noting that the admittance of the line itself is:

$$Y_c = \frac{1}{R + jX_L}$$

we can build up the admittance matrix quite easily:

$$\underline{\underline{Y}} = \begin{bmatrix} Y_c - \frac{j}{X_c} & -Y_c \\ -Y_c & Y_c - \frac{j}{X_c} \end{bmatrix}$$

Current in the line is

$$I_L = V_s(e^{j\delta} - 1)Y_c$$

So sending and receiving end complex powers are:

$$
\begin{aligned}
(P+jQ)_S &= |V_s|^2\left(Y_c^* - \frac{j}{X_c}\right) - |V_s|^2 Y_c^* e^{j\delta} \\
(P+jQ)_R &= -|V_s|^2\left(Y_c^* - \frac{j}{X_c}\right) + |V_s|^2 Y_c^* e^{-j\delta}
\end{aligned}
$$

This is programmed up and shown in Figure 5. Also shown in this figure are the vectors related to the condition described in the next part, with 7.5 MW real power at the receiving end.Vectors from the centers of the power circle to the sending and receiving complex power points are plotted in Figure 5.

The problem of finding the angle for a defined power flow involves solving a transcendental function. We could do this in a variety of ways, but MATLAB gives us lazy people a clever way of doing it. The function fzero(FOO(X), X0) returns a value of X that makes FOO(X) equal to zero. See the script attached for details. The answers are:

```
For 7500.0 kW at receiving end
Angle = 0.922506 radians = 52.8557 degrees
Sending end Power = 8.28451e+06
Sending end Reactive = 1.46664e+06
Receiving end Reactive = -3.04509e+06
```

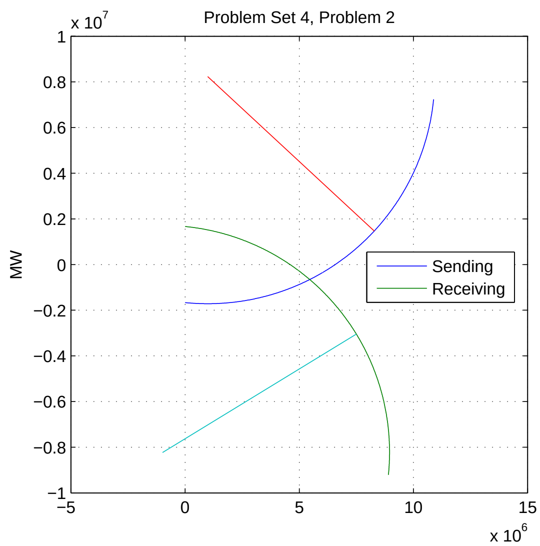

Figure 5: Power Circles

Problem 3: The three phase voltages are:

$$
\begin{aligned}
v_a &= \sqrt{2}\cdot 120\cos(\omega t) \\
v_b &= \sqrt{2}\cdot 120\cos(\omega t - \frac{2\pi}{3}) \\
v_c &= \sqrt{2}\cdot 120\cos(\omega t + \frac{2\pi}{3})
\end{aligned}
$$

and the center point of this source is grounded.

B We take this one out of order as it is the easiest. The voltages across each of the resistances is defined by the matching source, so that:

$$\begin{aligned} i_a &= \sqrt{2} \cdot \cos(\omega t) \\ i_b &= \sqrt{2} \cdot \cos(\omega t - \frac{2\pi}{3}) \\ i_c &= \sqrt{2} \cdot \cos(\omega t + \frac{2\pi}{3}) \end{aligned}$$

A Noting that in part B, the sum of the three currents is zero, the neutral point at the junction of the three resistors can (and in fact will be) at zero potential and so the currents are exactly the same.

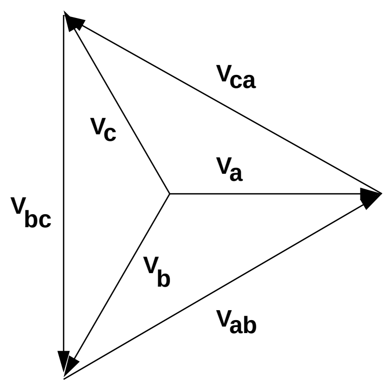

Figure 6: Three-Phase Voltages

C Refer to Figure 6. The voltage across the resistor is:

$$v_{ab} = v_a - v_b = \sqrt{2} \times 120 \left(\cos \omega t - \cos(\omega t - \frac{2\pi}{3}) = \sqrt{2} \times \sqrt{3} \times 277 \cos(\omega t + \frac{\pi}{6}) \right)$$

and of course $\sqrt{3} \times 120 \approx 208$. So

$$i_a = -i_b = \sqrt{2} \times \frac{208}{360} \left(\cos \omega t + \frac{\pi}{6} \right) = \sqrt{2} \times .578 \cos \left(\omega t + \frac{\pi}{6} \right) \approx .817 \times \cos \left(\omega t + \frac{\pi}{6} \right)$$

D Since $3 \times 120 = 360$ this load is equivalent to that of Part A and so the currents are the same. If you want you can do this the hard way by following the recipe for Part E and adding the two resistor currents at each node.

E This one involves computing the two resistor voltages:

$$\begin{aligned} v_{ab} &= v_a - v_b = & \sqrt{2} \times 120 \left(\cos \omega t - \cos(\omega t - \frac{2\pi}{3}) \right) = \sqrt{2} \times \sqrt{3} \times 120 \cos(\omega t + \frac{\pi}{6}) \\ v_{ca} &= v_c - v_a = & \sqrt{2} \times 120 \left(\cos(\omega t + \frac{4\pi}{3}) - \cos \omega t \right) = \sqrt{2} \times \sqrt{3} \times 120 \cos(\omega t + \frac{5\pi}{6}) \end{aligned}$$

Noting that $\cos\left(\omega t + \frac{5\pi}{6}\right) = -\cos\left(\omega t - \frac{\pi}{6}\right)$, we may use the identity:

$$\cos(\omega t - \frac{\pi}{6}) + \cos(\omega t + \frac{\pi}{6}) = 2\cos\omega t \cos\frac{\pi}{6} = \sqrt{3}\cos\omega t$$

and, using the results obtained in Part C,

$$\begin{aligned} i_a &= \sqrt{2}\cos\omega t \\ i_b &= -\sqrt{2} \times .578\cos(\omega t + \frac{\pi}{6}) \\ i_c &= -\sqrt{2} \times .578\cos(\omega t - \frac{\pi}{6}) \end{aligned}$$

F This is just like case B, except for phase C is not connected:

$$\begin{aligned} i_a &= \sqrt{2} \times \cos\omega t \\ i_b &= \sqrt{2} \times \cos\left(\omega t - \frac{2\pi}{3}\right) \\ i_c &= 0 \end{aligned}$$

Problem 4: This one is best done graphically. Note that the current through the ground resistor is just the sum of the three phase currents. Shown in Figure 7 is the same figure that established the currents, but with this summation shown.

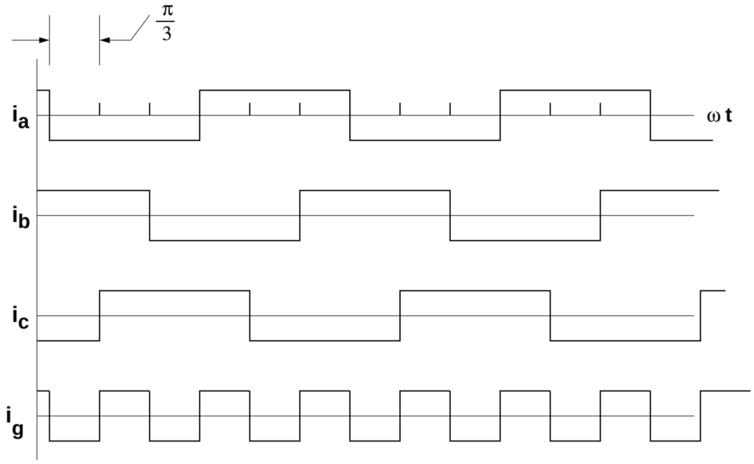

Figure 7: Currents

Now, the voltages in the individual resistors will be just the current sources times the ten ohm resistance. The voltage across the ground resistance will, similarly, be just the bottom trace times fifty ohms. This is shown in Figure 8.

Problem 5: For 6.690

To start, note the wye connected resistors are equivalent to a delta connection as shown in Figure 9

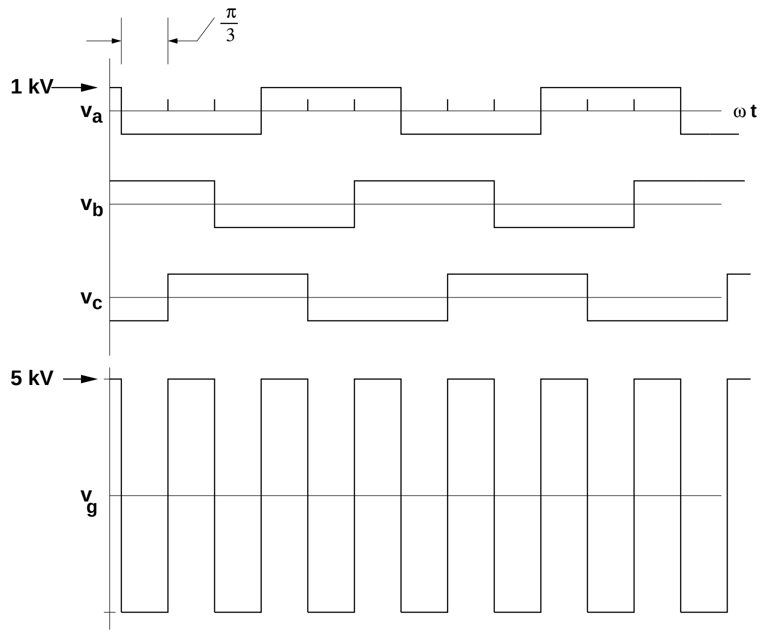

Figure 8: Voltages

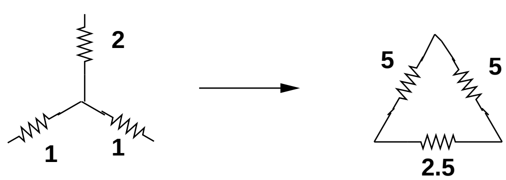

Figure 9: Wye-Delta Equivalent

Currents through the resistors of the delta are, noting that $\frac{120}{5} = 24$:

$$\begin{aligned} I_{ab} &= 24\sqrt{3}e^{j\frac{\pi}{6}} \\ I_{bc} &= -j48\sqrt{3} \\ I_{ca} &= 24\sqrt{3}e^{j\frac{5\pi}{6}} \end{aligned}$$

Then currents from the three elements of the voltage source are:

$$\begin{aligned} I_a &= I_{ab} - I_{ca} &&= 24\sqrt{3}\left(e^{j\frac{\pi}{6}} - e^{j\frac{5\pi}{6}}\right) = 3 \times 24 = 72 \\ I_b &= I_{bc} - I_{ab} &&= 24\sqrt{3}\left(-2j - e^{j\frac{\pi}{6}}\right) = 24\sqrt{3}\left(-2.5j - \frac{\sqrt{3}}{2}\right) \approx -104j - 36 \\ I_c &= I_{ca} - I_{bc} &&= 24\sqrt{3}\left(2j + e^{j\frac{5\pi}{6}}\right) = 24\sqrt{3}\left(2.5j - \frac{\sqrt{3}}{2}\right) \approx 104j - 36 \end{aligned}$$

The phasor diagram is shown in Figure 10

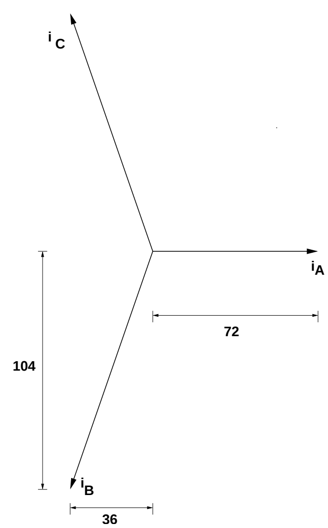

Figure 10: Phasor Diagram

Appendix: MATLAB scripts:

```
% Chapter 3, Problem 4
% transmission line problem

f = 60;              % electrical frequency
Z0 = 250;            % characteristic impedance
L = 300e3;           % line length
vp = 3e8;            % speed of light
k =2*pi*f/vp;        % wavenumber at frequency
kl = k*L;            % to be used enough
Vs = 500e3;          % RMS voltage

% part a)
Vrz = Vs/cos(kl);    % receiving end voltage, open
Isz = (Vs/Z0) * tan(kl);  % sending end current

fprintf('Chapter 3, Problem 4\n')
fprintf('Receiving end open:\n')
fprintf('Receiving end Voltage = %g V, RMS\n', Vrz)
fprintf('Sending end Current = %g A, RMS\n', Isz)

Rl = [Z0/.8 Z0 Z0/1.2];   % gonna do this for these values

for k = 1:length(Rl);
    R = Rl(k);
    vr = (R/Z0) / ((R/Z0)*cos(kl) + j*sin(kl));
    Vr = Vs * abs(vr);
    I_s = abs((Vs/Z0)*(cos(kl)-j*(R/Z0)*sin(kl))/((R/Z0)*cos(kl)-j*sin(kl)));
    fprintf('R = %g   Vr = %g  Is = %g\n', R, Vr, I_s)
end

P_sil = Vs^2/Z0      % surge impedance loading

P_r = P_sil/100:P_sil/100:P_sil;

R = Vs^2 ./ P_r;
V_l = Vs .* R ./ Z0 ./ ((R ./ Z0) .* cos(kl) + j * sin(kl));
Vl = abs(V_l);

P_rec = [0 P_r];
V_rec = [Vrz Vl];

figure(1)
plot(P_rec, V_rec)
```

```
title('Chapter 3, Problem 4, Voltage vs. Loading')
ylabel('V, RMS')
xlabel('Real Power, W')
grid on

% now look at compensation

Qcm = 100e6;                  % maximum reactive compensation
Qc = -Qcm:Qcm/100:Qcm;        % this range of compensation
Ycm = (j*2/Vs^2) .* Qc;       % reactive admittance

figure(2)
clf
hold on

for k = 1:length(Rl)
    Yr = 1/Rl(k) + Ycm;
    vr = (1 ./(Z0 .* Yr)) ./ ((1 ./(Z0 .* Yr) .*cos(kl) + j*sin(kl)));
    Vr = abs(vr) .* Vs;
    plot(Qc, Vr)
end

hold off
title('Chapter 3, Problem 4, Compensation')
ylabel('V, RMS')
xlabel('Compensation, VARs')
grid on
% Chapter 3, Problem 4
% transmission line problem

f = 60;             % electrical frequency
Z0 = 250;           % characteristic impedance
L = 300e3;          % line length
vp = 3e8;           % speed of light
k =2*pi*f/vp;       % wavenumber at frequency
kl = k*L;           % to be used enough
Vs = 500e3;         % RMS voltage

% part a)
Vrz = Vs/cos(kl);    % receiving end voltage, open
Isz = (Vs/Z0) * tan(kl);  % sending end current

fprintf('Chapter 3, Problem 4\n')
fprintf('Receiving end open:\n')
fprintf('Receiving end Voltage = %g V, RMS\n', Vrz)
```

```
fprintf('Sending end Current = %g A, RMS\n', Isz)

Rl = [Z0/.8 Z0 Z0/1.2];   % gonna do this for these values

for k = 1:length(Rl);
    R = Rl(k);
    vr = (R/Z0) / ((R/Z0)*cos(kl) + j*sin(kl));
    Vr = Vs * abs(vr);
    I_s = (Vs/Z0)*(cos(kl)+j*(R/Z0)*sin(kl))/((R/Z0)*cos(kl)+j*sin(kl));
    P_s = real(Vs*conj(I_s));
    Q_s = imag(Vs*conj(I_s));
    P_r = Vr^2/Rl(k);
    fprintf('R = %g   Vr = %g  Is = %g\n', R, Vr, abs(I_s))
    fprintf('Pr = %g  Ps = %g   Qs = %g\n', P_r, P_s, Q_s)
end

P_sil = Vs^2/Z0     % surge impedance loading

P_r = P_sil/100:P_sil/100:P_sil;

R = Vs^2 ./ P_r;
V_l = Vs .* R ./ Z0 ./ ((R ./ Z0) .* cos(kl) + j * sin(kl));
Vl = abs(V_l);

P_rec = [0 P_r];
V_rec = [Vrz Vl];

figure(1)
plot(P_rec, V_rec)
title('Chapter 3, Problem 4, Voltage vs. Loading')
ylabel('V, RMS')
xlabel('Real Power, W')
grid on

% now look at compensation

Qcm = 100e6;                  % maximum reactive compensation
Qc = -Qcm:Qcm/100:Qcm;        % this range of compensation
Ycm = (j*2/Vs^2) .* Qc;       % reactive admittance

figure(2)
clf
hold on

for k = 1:length(Rl)
```

```
    Yr = 1/Rl(k) + Ycm;
    vr = (1 ./(Z0 .* Yr)) ./ ((1 ./(Z0 .* Yr) .*cos(kl) + j*sin(kl)));
    Vr = abs(vr) .* Vs;
    plot(Qc, Vr)
end

hold off
title('Chapter 3, Problem 4, Compensation')
ylabel('V, RMS')
xlabel('Compensation, VARs')
grid on



---------------
% Problem Set 4, Problem 2

global Vs R Xl Xc Yc P0

% parameters
Vs = 1e4;                % voltage
R = 1;                   % line resistance
Xl = 10;                 % line reactance
Xc = 60;                 % compensating reactance
delt = 0:pi/1000:pi/2;   % run over this range of angles
P0 = 7.5e6;

Yc = 1/(R-j*Xl);          % conjugate of line admittance

Ss = Vs^2*(Yc-j/Xc) - Vs^2*Yc .* exp(j .* delt);
Sr = -Vs^2*(Yc-j/Xc) + Vs^2*Yc .* exp(-j .* delt);

% now find the angle for P0 at receiving end

delta = fzero('fz', .2);

Ssp = Vs^2*(Yc-j/Xc) - Vs^2*Yc * exp(j * delta);
Srp = -Vs^2*(Yc-j/Xc) + Vs^2*Yc * exp(-j * delta);

Psp = real(Ssp);
Prp = real(Srp);
Qsp = imag(Ssp);
Qrp = imag(Srp);

Ps = real(Ss);
```

```
Qs = imag(Ss);
Pr = real(Sr);
Qr = imag(Sr);

Psz = real(Vs^2*(Yc-j/Xc));
Qsz = imag(Vs^2*(Yc-j/Xc));
Prz = -Psz;
Qrz = -Qsz;

figure(2)
plot(Ps, Qs, Pr, Qr, [Psz Psp], [Qsz Qsp], [Prz Prp], [Qrz Qrp])
title('Problem Set 4, Problem 2')
ylabel('MVAR')
ylabel('MW')
axis ([-5e6 15e6 -10e6 10e6])
legend('Sending', 'Receiving')
axis square
grid on

fprintf('For %4.1f kW at receiving end\n', P0/1000)
fprintf('Angle = %g radians = %g degrees\n', delta, (180/pi)*delta)
fprintf('Sending end Power = %g\n', Psp)
fprintf('Sending end Reactive = %g\n', Qsp)
fprintf('Receiving end Reactive = %g\n', Qrp)
```