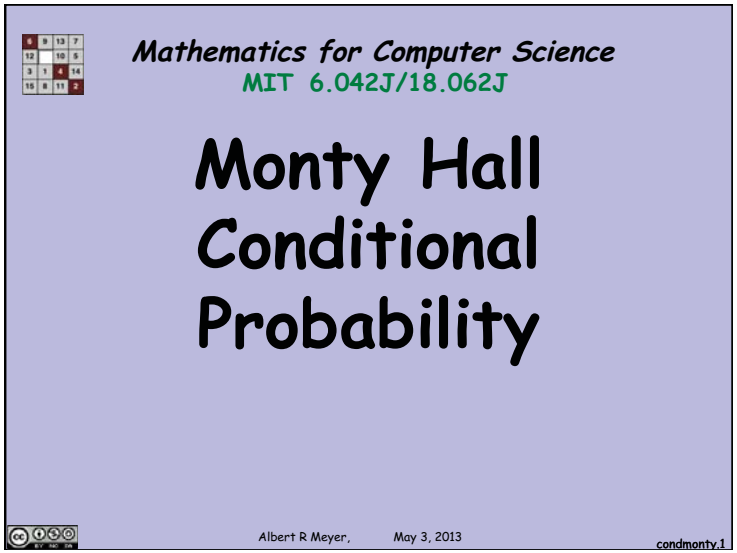
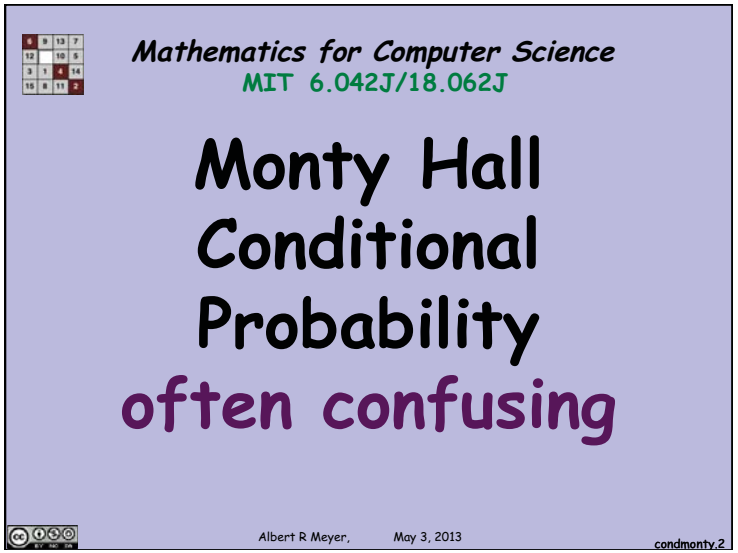
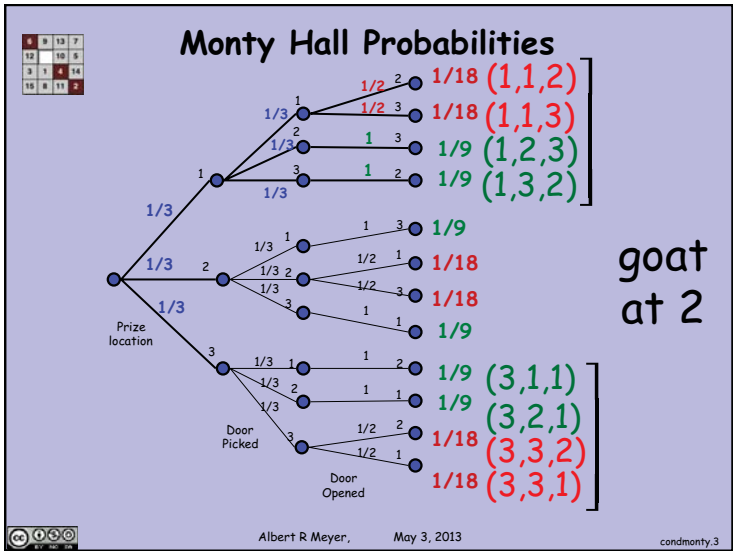
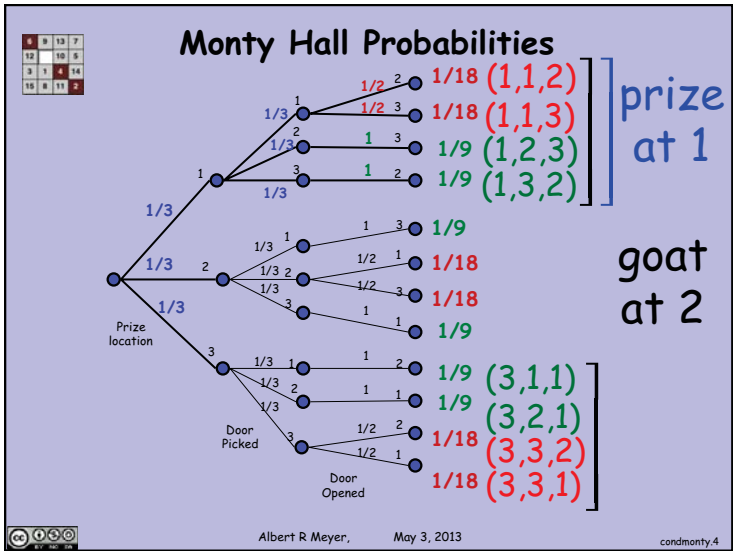

Mathematics for Computer Science
MIT 6.042J/18.062J
Monty Hall Conditional Probability
Albert R Meyer, May 3, 2013
condmonty.1


Mathematics for Computer Science
MIT 6.042J/18.062J
Monty Hall Conditional Probability
often confusing
Albert R Meyer, May 3, 2013
condmonty.2


Monty Hall Probabilities
1/18 (1,1,2)
1/18 (1,1,3)
1/9 (1,2,3)
1/9 (1,3,2)
1/9
1/18
1/18
1/9
1/9 (3,1,1)
1/9 (3,2,1)
1/18 (3,3,2)
1/18 (3,3,1)
goat at 2
Prize location
Door Picked
Door Opened
Albert R Meyer, May 3, 2013
condmonty.3

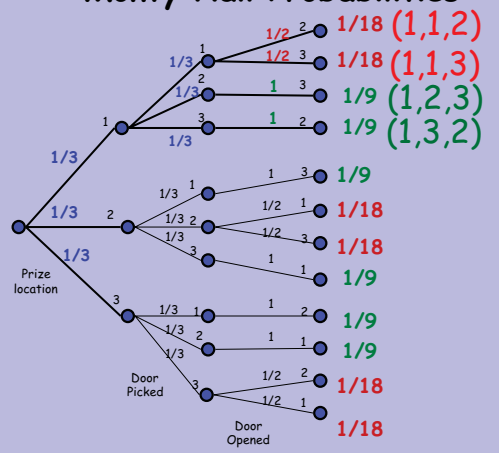

Monty Hall Probabilities
1/18 (1,1,2)
1/18 (1,1,3)
1/9 (1,2,3)
1/9 (1,3,2)
prize at 1
1/9
1/18
1/18
1/9
1/9 (3,1,1)
1/9 (3,2,1)
1/18 (3,3,2)
1/18 (3,3,1)
goat at 2
Prize location
Door Picked
Door Opened
Albert R Meyer, May 3, 2013
condmonty.4

Conditional Probability: Monty Hall

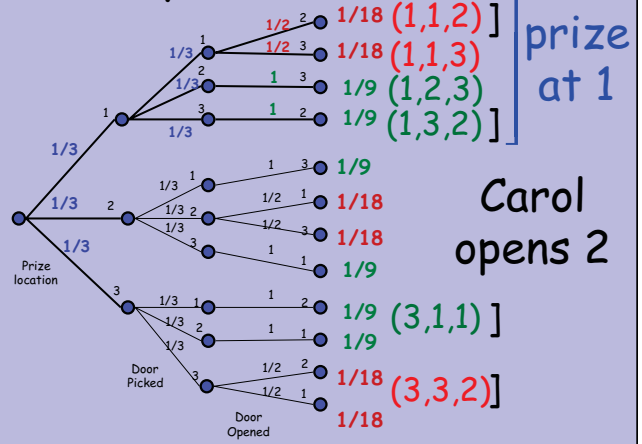
$$\Pr[\text{prize at 1} \mid \text{goat at 2}] = \frac{1}{2}$$

Really!

Monty Hall Probabilities



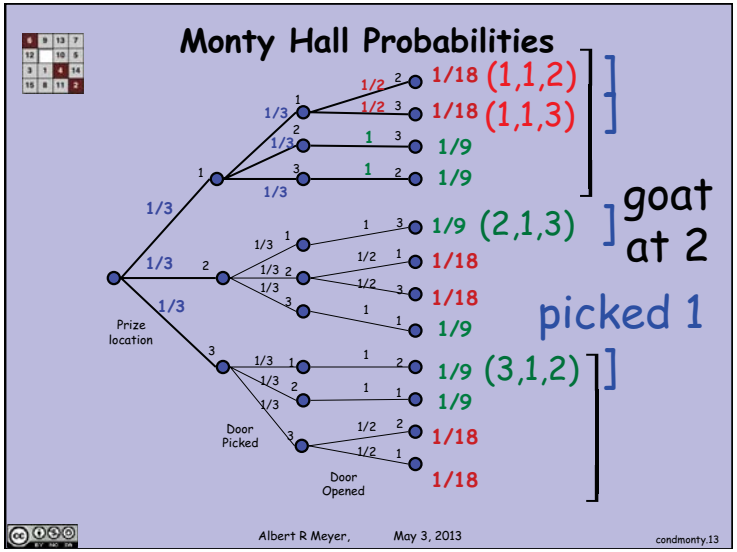
Monty Hall Probabilities

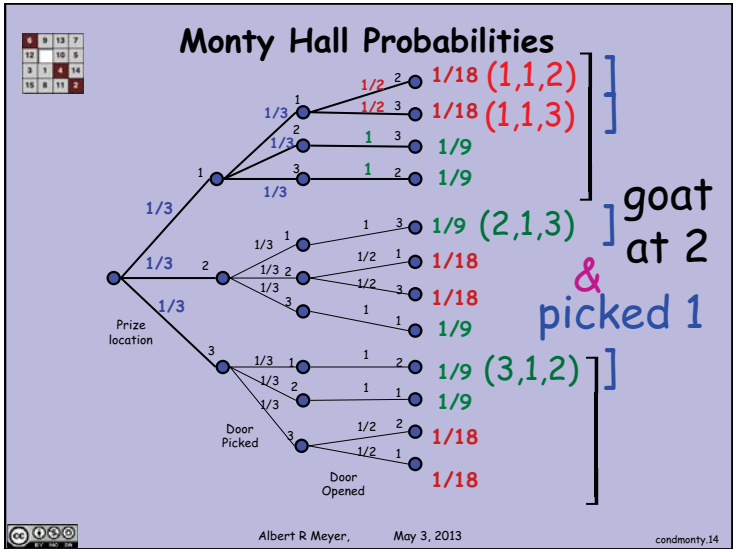


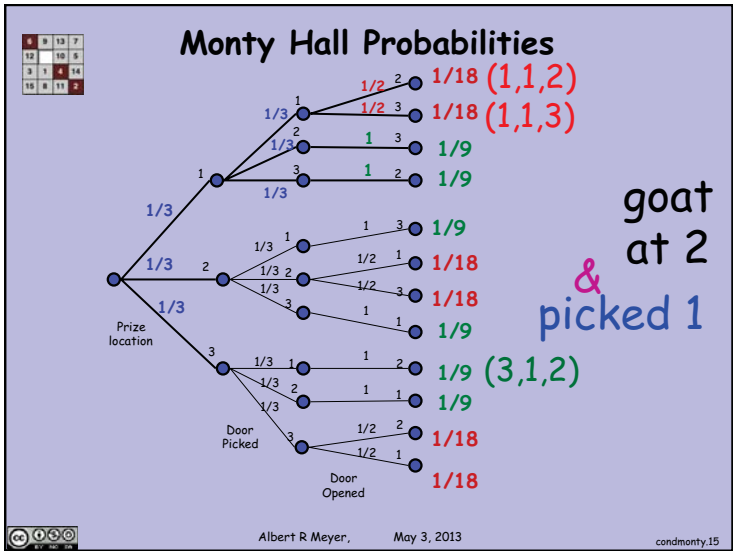
Conditional Probability: Monty Hall

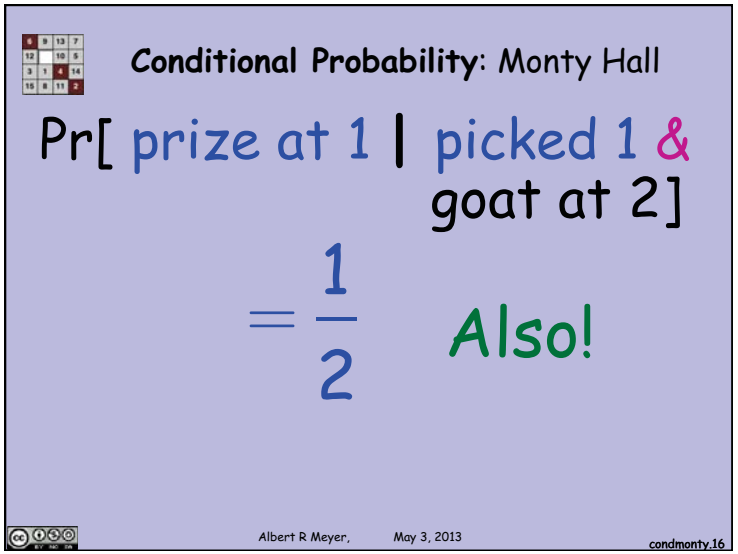
$$\Pr[\text{prize at 1} \mid \text{Carol opens 2}] = \frac{1}{2}$$

Likewise!


Monty Hall Probabilities
1/18 (1,1,2)
1/18 (1,1,3)
1/9
1/9
1/9 (2,1,3)
1/18
1/18
1/9
1/9 (3,1,2)
1/9
1/18
1/18
goat at 2
picked 1
Prize location
Door Picked
Door Opened
Albert R Meyer, May 3, 2013
condmonty.13


Monty Hall Probabilities
1/18 (1,1,2)
1/18 (1,1,3)
1/9
1/9
1/9 (2,1,3)
1/18
1/18
1/9
1/9 (3,1,2)
1/9
1/18
1/18
goat at 2
&
picked 1
Prize location
Door Picked
Door Opened
Albert R Meyer, May 3, 2013
condmonty.14


Monty Hall Probabilities
1/18 (1,1,2)
1/18 (1,1,3)
1/9
1/9
1/9
1/18
1/18
1/9
1/9 (3,1,2)
1/9
1/18
1/18
goat at 2
&
picked 1
Prize location
Door Picked
Door Opened
Albert R Meyer, May 3, 2013
condmonty.15


Conditional Probability: Monty Hall
Pr[prize at 1 | picked 1 & goat at 2]
= 1/2
Also!
Albert R Meyer, May 3, 2013
condmonty.16

Stick or Switch?

Seems the contestant may as well **stick**, since the probability is 1/2 given what he knows when he chooses. Wait! contestant knows more than what door he picked & where a goat is, he knows

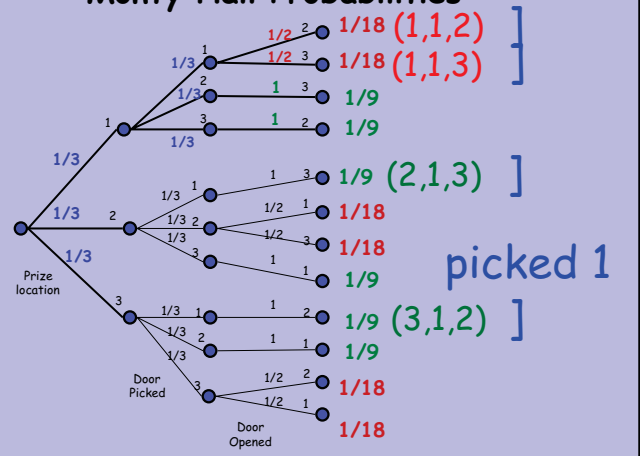
what door Carol opened!

Conditional Probability: Monty Hall

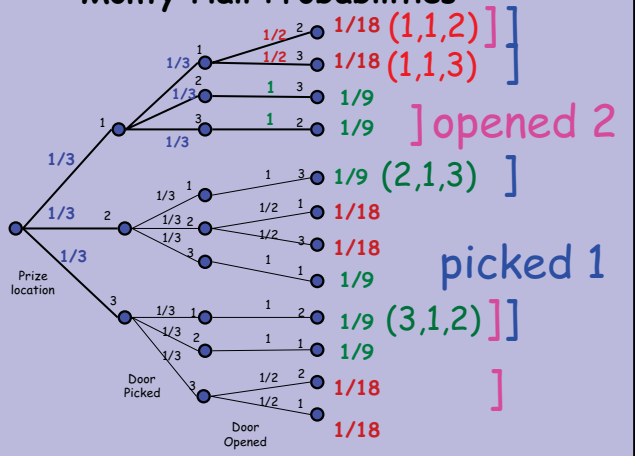
So until now, we have been conditioning on the wrong events —a common blunder.

Using the correct one:

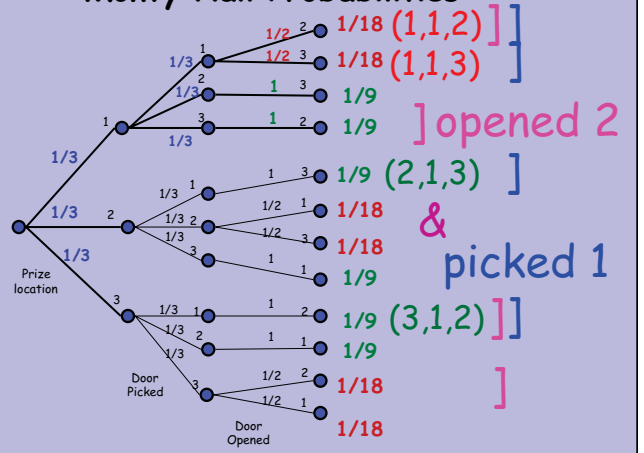
Monty Hall Probabilities



Monty Hall Probabilities



Monty Hall Probabilities



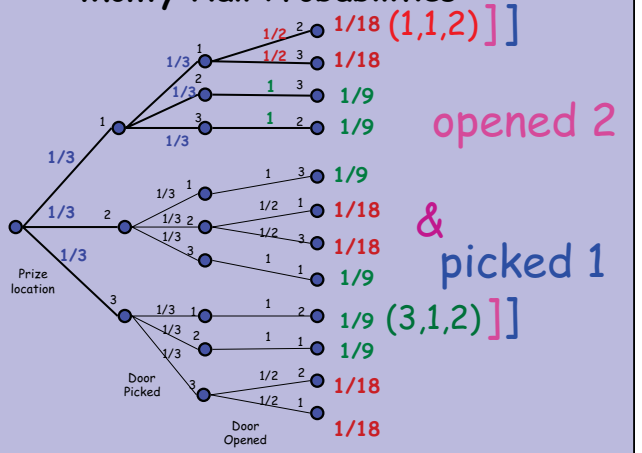
] opened 2]

&

picked 1]]

]

Monty Hall Probabilities

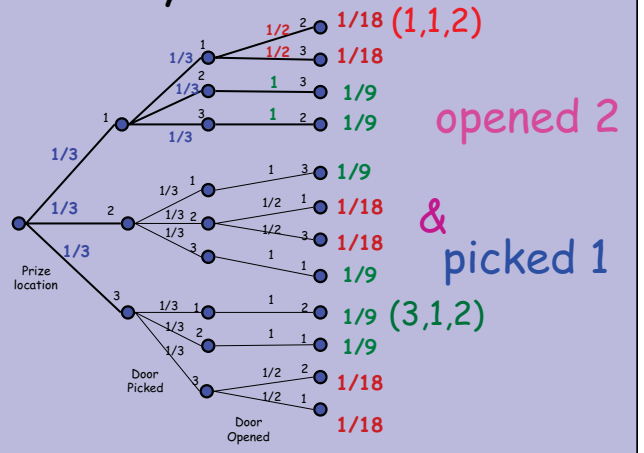


opened 2

&

picked 1]]

Monty Hall Probabilities



opened 2

&

picked 1

Conditional Probability: Monty Hall

prize at 1

[picked 1 & opened 2] =

{(1,1,2), (3,1,2) }

Conditional Probability: Monty Hall

$$\Pr[\text{prize at 1} \mid \text{picked 1 \& opened 2}]$$

[picked 1 & opened 2] =

{(1,1,2) (3,1,2) }

Pr=1/18 Pr=1/9

Conditional Probability: Monty Hall

$$\Pr[\text{prize at 1} \mid \text{picked 1 \& opened 2}]$$

$$= \frac{1/18}{1/18 + 1/9}$$

Conditional Probability: Monty Hall

$$\Pr[\text{prize at 1} \mid \text{picked 1 \& opened 2}]$$

$$= \frac{1/18}{1/18 + 1/9} = \frac{1}{3}$$

$$= \Pr[\text{sticking wins}]$$

Stick or Switch?

$$\Pr[\text{prize at 1} \mid \text{picked 1 \& opened 2}]$$

$$= \frac{1/18}{1/18 + 1/9} = \frac{1}{3}$$

$$= \Pr[\text{sticking wins}]$$

Switch!

By conditioning on everything the contestant knows, we've finally confirmed what we learned earlier:

$$\Pr[\text{switching wins}] = \frac{2}{3}$$

The 4 Step Method

It's easy to how so many smart people get confused by Monty Hall.

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It's easy to how so many smart people get confused by Monty Hall. Finding the right event to condition on can be tricky.

The 4 Step Method

It's easy to how so many smart people get confused by Monty Hall. Finding the right event to condition on can be tricky. The 4 step method is a good fall back approach.

6.042J / 18.062J Mathematics for Computer Science
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