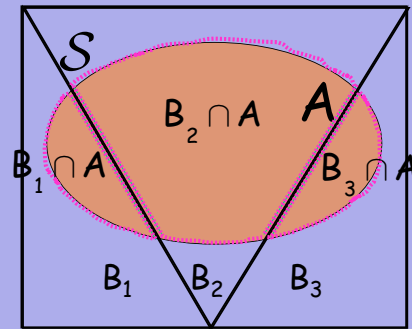# Conditional Probability & Independence

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Law of Total Probability

Law of Total Probability

$$A = (B_1 \cap A) \cup (B_2 \cap A) \cup (B_3 \cap A)$$

$$Pr\{A\} = Pr\{B_1 \cap A\} + Pr\{B_2 \cap A\} + Pr\{B_3 \cap A\}$$

Conditional Probability: A Fair Die

$$Pr\{roll\ 1\} = \frac{|\{1\}|}{|\{1,2,3,4,5,6\}|} = \frac{1}{6}$$

"knowledge" changes probabilities:

Pr{roll 1 knowing rolled odd}

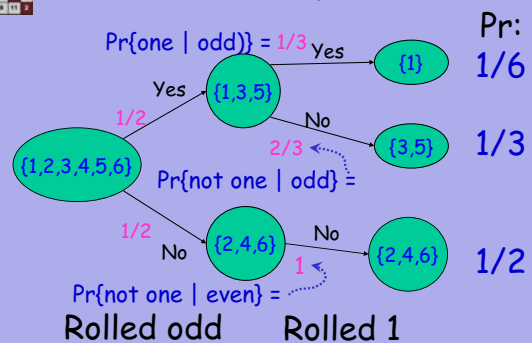$$= \frac{|\{1\}|}{|\{1,3,5\}|} = \frac{1}{3}$$

Conditional Probability

$Pr\{A|B\}$ is the probability of event *A, given* that event *B* has occurred:

$$Pr\{A \mid B\} ::= \frac{Pr\{A \cap B\}}{Pr\{B\}}$$

Conditional Probability: A Fair Die

1

Product Rule

$$Pr\{A \cap B\} = Pr\{A \mid B\} \cdot Pr\{B\}$$

Law of Total Probability

If $\mathcal{S}$ is disjoint union of $B_0, B_1,\ldots$

$$Pr\{A\} = \sum_{i \geq 0} Pr\{A \cap B_i\}$$
$$= \sum_{i \geq 0} Pr\{A \mid B_i\} \cdot Pr\{B_i\}$$

Conditional Probability: Monty Hall

Pr{ prize at 1 | picked 1 &
     goat at 2} $= \dfrac{1}{2}$ Really!

[picked 1 & goat at 2] =
  { (1,1,2), (1,1,3), (3,1,2)}

Conditional Probability: Monty Hall

Pr{ prize at 1 | picked 1 &
     goat at 2} $= \dfrac{1}{2}$

[picked 1 & goat at 2] =
  { (1,1,2), (1,1,3), (3,1,2)}
    Pr=1/18 Pr=1/18 Pr=1/9

Conditional Probability: Monty Hall

Seems that the contestant may
as well stick, since the probability
is 1/2 *given what he knows* when
he chooses.
But wait, contestant knows more
than goat at door 2: he knows
     Carol opened door 2!

Conditional Probability: Monty Hall

Pr{ prize at 1 | picked 1 &
     Carol opens 2} = 1/3
[picked 1 & Carol opens 2] =
     { (1,1,2),(3,1,2) }
     Pr=1/18 Pr=1/9

$$\frac{1/18}{1/18 + 1/9} =$$

Independence

Definition 1:

Events A and B are independent iff

$$Pr\{A\} = Pr\{A \mid B\}.$$

Definition 2:

Events A and B are independent iff

$$Pr\{A\} \cdot Pr\{B\} = Pr\{A \cap B\}.$$

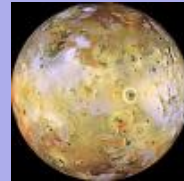
Independent Events?

B: Baby born at Mass General Hospital between 1:00AM and 1:01AM.

F: Jupiter's moon IO is full.

Independent Events?

Does event B (baby born) have anything to do with event F (IO is full)?

Babies & Full Moons

My sweet Aunt Daisy believed in Astrology. She thought celestial events could influence babies.

We might say "nonsense," there's no effect.

But Daisy might be right (for wrong reasons)

C:\42\pub\jup-radio_070115.htm

** INFORMATION FOR AMATEUR RADIO ASTRONOMERS ** JUPITER DECAMETRIC EMISSIONS ** JUPITER EPHEMERIS 01 Jul 1994, 0000UTC, Julian Day: 2449534.5, GMT Sidereal Time: 18h35m17s ….

C:\42\pub\jup-radio_070115.htm

SUMMARY: Jupiter's HF emissions are …heard on earth when Jupiter's magnetic field "sweeps" the earth every 9h55m27s and at other times when Io's geometric position influences activity.

Babies & Full Moons

influence of IO's magnetic field changes with phases! --might affect radios in ambulances, for example

Babies & Full Moons

So independence of B and F is actually unclear.

Deciding whether to treat them as independent is a matter of experiment, not Mathematics.

Mutual Independence

events $A_1, A_2, \ldots, A_n$ are

mutually independent

iff $\Pr\{A_{i_1} \cap A_{i_2} \cap \cdots \cap A_{i_k}\} =$

$\Pr\{A_{i_1}\} \cdot \Pr\{A_{i_2}\} \cdots \Pr\{A_{i_k}\}$

for all A_{i_j} $\left[\begin{array}{l} 2^n\text{-}(n+1) \text{ equations} \\ \text{to check!} \end{array}\right]$

Mutual Independence

Events $E_1, E_2, \ldots$ are

k-way independent

iff every subset of k of them is mutually independent

Team Problems

Problems
1 – 3

6.042J / 18.062J Mathematics for Computer Science
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