In-Class Problems Week 10, Fri.

Problem 1.
A certain company wants to have security for their computer systems. So they have given everyone a name and password. A length 10 word containing each of the characters:

a, d, e, f, i, l, o, p, r, s,

is called a *cword*. A password will be a cword which does not contain any of the subwords "fails", "failed", or "drop".

For example, the following two words are passwords:

adefiloprs, srpolifeda,

but the following three cwords are not:

a**drop**eflis, **failedrop**s, **drop**e**fails**.

(a) How many cwords contain the subword "drop"?

(b) How many cwords contain both "drop" and "fails"?

(c) Use the Inclusion-Exclusion Principle to find a simple formula for the number of passwords.

Problem 2.
Solve the following counting problems by defining an appropriate mapping (bijective or k-to-1) between a set whose size you know and the set in question.

(a) How many different ways are there to select a dozen donuts if four varieties are available?

(b) In how many ways can Mr. and Mrs. Grumperson distribute 13 identical pieces of coal to their two —no, three! —children for Christmas?

(c) How many solutions over the nonnegative integers are there to the inequality:

$$x_1 + x_2 + \ldots + x_{10} \leq 100$$

(d) We want to count step-by-step paths between points in the plane with integer coordinates. Ony two kinds of step are allowed: a right-step which increments the x coordinate, and an up-step which increments the y coordinate.

Creative Commons 2010, Prof. Albert R. Meyer.

(i) How many paths are there from $(0,0)$ to $(20,30)$?

(ii) How many paths are there from $(0,0)$ to $(20,30)$ that go through the point $(10,10)$?

(iii) How many paths are there from $(0,0)$ to $(20,30)$ that do *not* go through either of the points $(10,10)$ and $(15,20)$?

Hint: Let P be the set of paths from $(0,0)$ to $(20,30)$, N_1 be the paths in P that go through $(10,10)$ and N_2 be the paths in P that go through $(15,20)$.

Problem 3.
Here are the solutions to the next 10 problem parts, in no particular order.

$$n^m \qquad m^n \qquad \frac{n!}{(n-m)!} \qquad \binom{n+m}{m} \qquad \binom{n-1+m}{m} \qquad \binom{n-1+m}{n} \qquad 2^{mn}$$

(a) How many solutions over the natural numbers are there to the inequality $x_1 + x_2 + \cdots + x_n \leq m$? ________

(b) How many length m words can be formed from an n-letter alphabet, if no letter is used more than once? ________

(c) How many length m words can be formed from an n-letter alphabet, if letters can be reused? ________

(d) How many binary relations are there from set A to set B when $|A| = m$ and $|B| = n$? ________

(e) How many injections are there from set A to set B, where $|A| = m$ and $|B| = n \geq m$? ________

(f) How many ways are there to place a total of m distinguishable balls into n distinguishable urns, with some urns possibly empty or with several balls? ________

(g) How many ways are there to place a total of m indistinguishable balls into n distinguishable urns, with some urns possibly empty or with several balls? ________

(h) How many ways are there to put a total of m distinguishable balls into n distinguishable urns with at most one ball in each urn? ________

MIT OpenCourseWare
http://ocw.mit.edu

6.042J / 18.062J Mathematics for Computer Science
Spring 2010

For information about citing these materials or our Terms of Use, visit: http://ocw.mit.edu/terms.