

## In-Class Problems Week 10, Mon.

### Problem 1.

Solve the following problems using the pigeonhole principle. For each problem, try to identify the *pigeons*, the *pigeonholes*, and a *rule* assigning each pigeon to a pigeonhole.

(a) Every MIT ID number starts with a 9 (we think). Suppose that each of the 75 students in 6.042 sums the nine digits of his or her ID number. Explain why two people must arrive at the same sum.

(b) In every set of 100 integers, there exist two whose difference is a multiple of 37.

(c) For any five points inside a unit square (not on the boundary), there are two points at distance less than  $1/\sqrt{2}$ .

(d) Show that if  $n + 1$  numbers are selected from  $\{1, 2, 3, \dots, 2n\}$ , two must be consecutive, that is, equal to  $k$  and  $k + 1$  for some  $k$ .

### Problem 2.

Answer the following questions using the Generalized Product Rule.

(a) Next week, I'm going to get really fit! On day 1, I'll exercise for 5 minutes. On each subsequent day, I'll exercise 0, 1, 2, or 3 minutes more than the previous day. For example, the number of minutes that I exercise on the seven days of next week might be 5, 6, 9, 9, 9, 11, 12. How many such sequences are possible?

(b) An  $r$ -permutation of a set is a sequence of  $r$  distinct elements of that set. For example, here are all the 2-permutations of  $\{a, b, c, d\}$ :

$(a, b)$	$(a, c)$	$(a, d)$
$(b, a)$	$(b, c)$	$(b, d)$
$(c, a)$	$(c, b)$	$(c, d)$
$(d, a)$	$(d, b)$	$(d, c)$

How many  $r$ -permutations of an  $n$ -element set are there? Express your answer using factorial notation.

(c) How many  $n \times n$  matrices are there with *distinct* entries drawn from  $\{1, \dots, p\}$ , where  $p \geq n^2$ ?

**Problem 3.**

Your 6.006 tutorial has 12 students, who are supposed to break up into 4 groups of 3 students each. Your TA has observed that the students waste too much time trying to form balanced groups, so he decided to pre-assign students to groups and email the group assignments to his students.

(a) Your TA has a list of the 12 students in front of him, so he divides the list into consecutive groups of 3. For example, if the list is ABCDEFGHIJKL, the TA would define a sequence of four groups to be  $(\{A, B, C\}, \{D, E, F\}, \{G, H, I\}, \{J, K, L\})$ . This way of forming groups defines a mapping from a list of twelve students to a sequence of four groups. This is a  $k$ -to-1 mapping for what  $k$ ?

(b) A group assignment specifies which students are in the same group, but not any order in which the groups should be listed. If we map a sequence of 4 groups,

$$(\{A, B, C\}, \{D, E, F\}, \{G, H, I\}, \{J, K, L\}),$$

into a group assignment

$$\{\{A, B, C\}, \{D, E, F\}, \{G, H, I\}, \{J, K, L\}\},$$

this mapping is  $j$ -to-1 for what  $j$ ?

(c) How many group assignments are possible?

(d) In how many ways can  $3n$  students be broken up into  $n$  groups of 3?

**Problem 4.**

A pizza house is having a promotional sale. Their commercial reads:

We offer 9 different toppings for your pizza! Buy 3 large pizzas at the regular price, and you can get each one with as many different toppings as you wish, absolutely free. That's 22, 369, 621 different ways to choose your pizzas!

The ad writer was a former Harvard student who had evaluated the formula  $(2^9)^3/3!$  on his calculator and gotten close to 22, 369, 621. Unfortunately,  $(2^9)^3/3!$  is obviously not an integer, so clearly something is wrong. What mistaken reasoning might have led the ad writer to this formula? Explain how to fix the mistake and get a correct formula.

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