Mathematics for Computer Science

MIT 6.042J/18.062J

Counting

Counting in Gambling

What *fraction* of poker hands are "a pair of Jacks?"

Image by MIT OpenCourseWare.

(probability of a pair of Jacks)

Counting in Algorithms

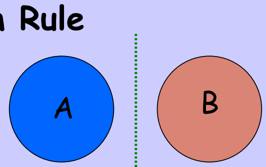
- # ops to update a data structure (# comparisons needed to sort n items)
- # steps in a computation (# multiplies to compute d^n)

Counting in Cryptography

possible passwords

possible keys

Sum Rule



If sets A and B are disjoint, then

$|A \cup B| = |A| + |B|$

The Sum Rule

- Class has 43 women, 54 men so total enrollment = 43 + 54 = 97
- 26 lower case letters, 26 upper case letters, and 10 digits, so # characters = 26+26+10 = 62

The Product Rule

If there are 4 boys and 3 girls, there are

$$4 \cdot 3 = 12$$

different boy/girl couples

Product Rule

If $|A| = m$ and $|B| = n$, then

$$|A \times B| = m \cdot n$$

$A = \{a, b, c, d\}$, $B = \{1, 2, 3\}$

$A \times B = \{(a,1),(a,2),(a,3),$
$(b,1),(b,2),(b,3),$
$(c,1),(c,2),(c,3),$
$(d,1),(d,2),(d,3)\}$

Product Rule: Counting Strings

\# length-4 binary strings

$= |B \times B \times B \times B|$

$= |B^4|$ where $B ::= \{0,1\}$

$= 2 \cdot 2 \cdot 2 \cdot 2 = 2^4$

Product Rule: Counting Strings

\# length n strings from an alphabet of size m is

$$m^n$$

Example: Counting Passwords

Password conditions:

- characters are digits & letters
- between 6 & 8 characters long
- starts with a letter
- case sensitive

Counting Passwords

$L ::= \{a,b,\ldots,z,A,B,\ldots,Z\}$

$D ::= \{0,1,\ldots\ldots,9\}$

$P_n ::=$ length n words starting w/letter

$= L \times (L \cup D)^{n-1}$

Counting Passwords

$|L \times (L \cup D)^{n-1}|$

$= |L|\cdot|(L \cup D)|^{n-1}$

$= |L|\cdot(|L| + |D|)^{n-1}$

$= 52\cdot(52+10)^{n-1}$

Counting Passwords

set of passwords:

$P ::= P_6 \cup P_7 \cup P_8$

$|P| = |P_6|+|P_7|+|P_8|$

$= 52\cdot(62^5+62^6+62^7)$

$\approx 19\cdot 10^{14}$

4-digit nums w/ $\geq$ one 7

cases by 1st occurrence of 7:

x: any digit o: any digit $\neq$ 7

7xxx or o7xx or oo7x or ooo7

$10^3 + 9\cdot 10^2 + 9^2\cdot 10 + 9^3$

$= 3439$

at least one 7: another way

|4-digit nums w/ $\geq$ one 7|

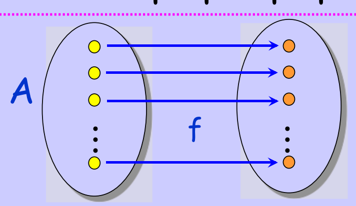
= |4-digit nums|

— |those w/ no 7|

$= 10^4 - 9^4 = 3439$

Mapping Rule: Bijections

If f is a bijection from A to B, then $|A| = |B|$



Counting Doughnut Selections

From 5 kinds of doughnuts select a dozen.

let A ::= all selections of 12 doughnuts

$\underbrace{00}_{\text{chocolate}}\ \underbrace{\text{(none)}}_{\text{lemon}}\ \underbrace{000000}_{\text{sugar}}\ \underbrace{00}_{\text{glazed}}\ \underbrace{00}_{\text{plain}}$

Counting Doughnut Selections

B::= 16-bit words with four 1's

$\underbrace{00}_{\text{chocolate}}\ 1\ \underbrace{\text{(none)}}_{\text{lemon}}\ 1\ \underbrace{000000}_{\text{sugar}}\ 1\ \underbrace{00}_{\text{glazed}}\ 1\ \underbrace{00}_{\text{plain}}$

Counting Doughnut Selections

B::= 16-bit words with four 1's

00 1 1000000 1 00100

$\underbrace{00}_{\text{chocolate}}\ \underbrace{\text{(none)}}_{\text{lemon}}\ \underbrace{000000}_{\text{sugar}}\ \underbrace{00}_{\text{glazed}}\ \underbrace{00}_{\text{plain}}$

Counting Doughnut Selections

B::= 16-bit words with four 1's

0011000000100100

a bijection: $|A| = |B|$

$\underbrace{00}_{\text{chocolate}}\ \underbrace{\text{(none)}}_{\text{lemon}}\ \underbrace{000000}_{\text{sugar}}\ \underbrace{00}_{\text{glazed}}\ \underbrace{00}_{\text{plain}}$

Team Problems

Problems

1—4

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