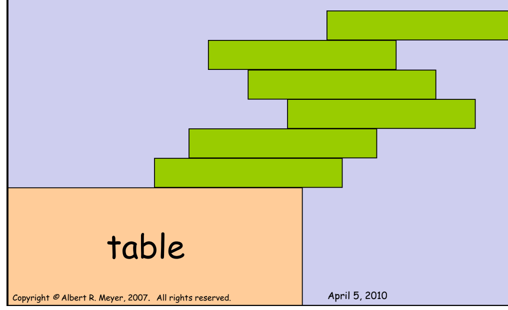
Mathematics for Computer Science
MIT 6.042J/18.062J

Harmonic Sum Integral Method

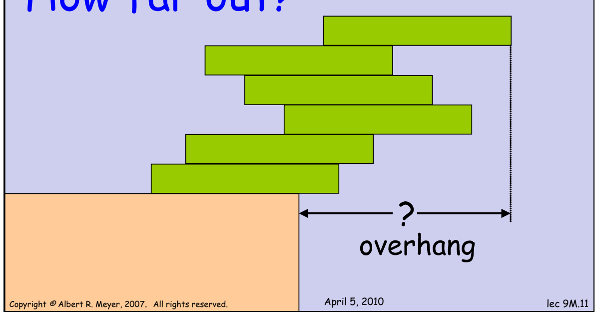
Albert R Meyer, April 5, 2010

Book Stacking



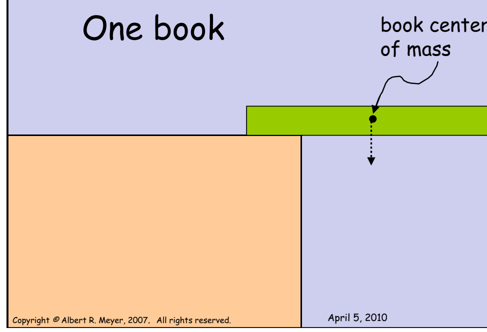
Book Stacking

How far out?



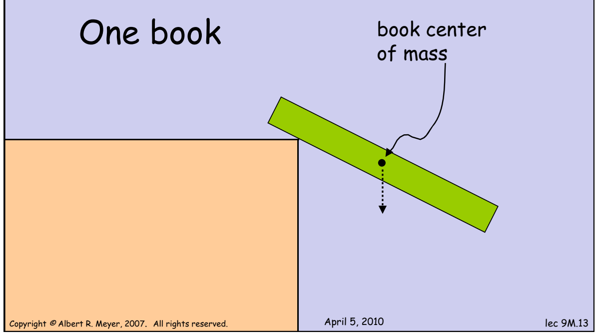
Book Stacking

One book



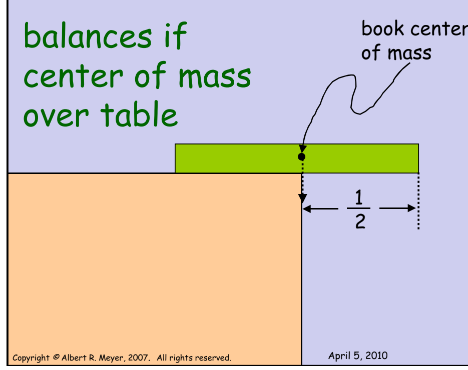
Book Stacking

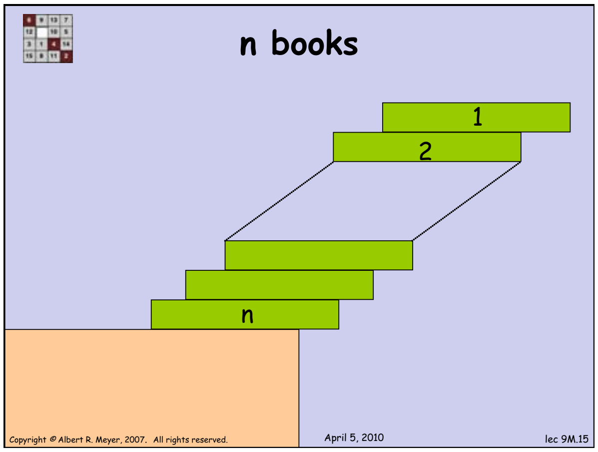
One book

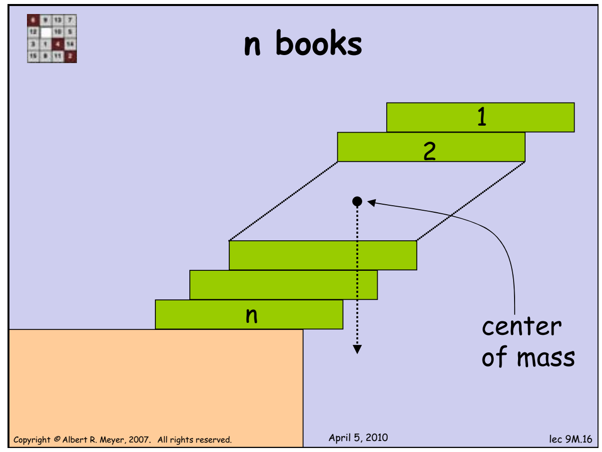


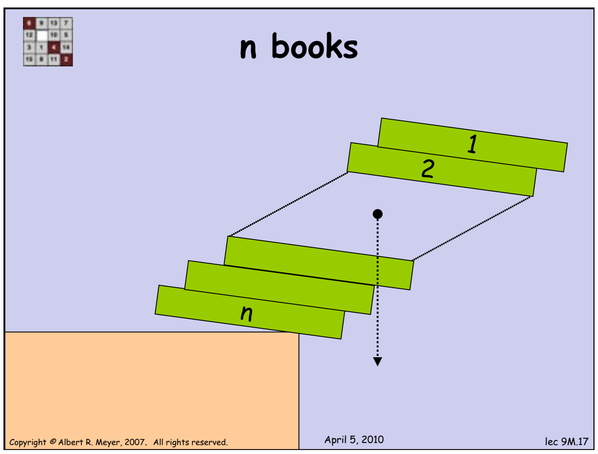
Book Stacking

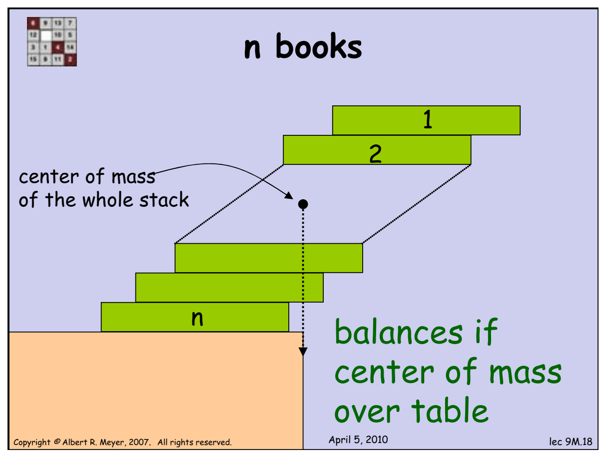
balances if
center of mass
over table

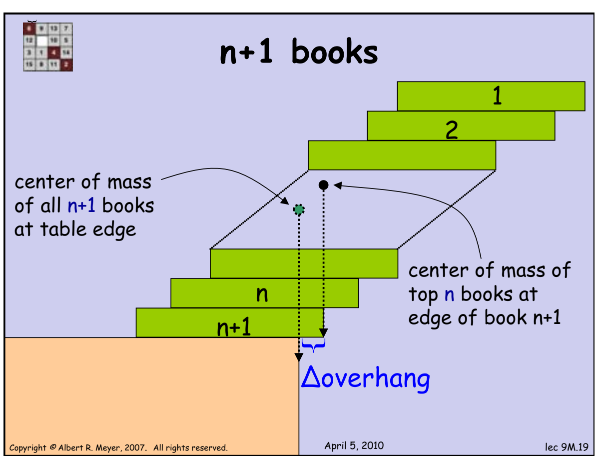








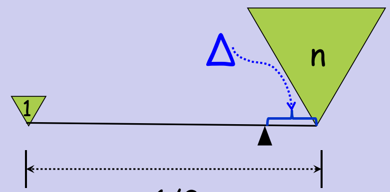




Δ-overhang ::=
horizontal distance from
n-book to (n+1)-book
centers of mass

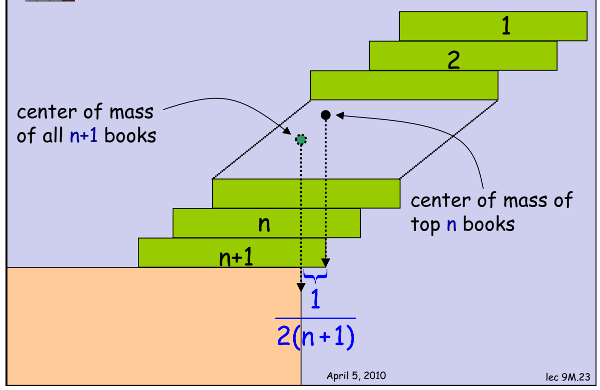
Albert R Meyer, April 5, 2010 lec 9M.20

Δ-overhang



$$\Delta = \frac{1/2}{n+1} = \frac{1}{2(n+1)}$$

n+1 books



Book stacking summary

B_n ::= overhang of n books

$B_1 = 1/2$

$$B_{n+1} = B_n + \frac{1}{2(n+1)}$$

$$B_n = \frac{1}{2}\left(1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}\right)$$

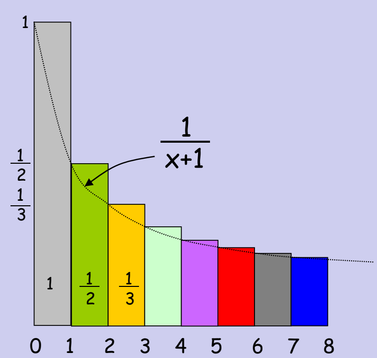
Harmonic Sums

$$H_n ::= 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}$$

n^{th} Harmonic number

$$B_n = H_n/2$$

Integral estimate for H_n



Integral estimate for H_n

H_n = area of rectangles

> area under $1/(x+1)$ =

$$\int_0^n \frac{1}{x+1}\,dx = \int_1^{n+1} \frac{1}{x}\,dx = \ln(n+1)$$

Book stacking

for overhang 3, need $B_n \geq 3$

$H_n \geq 6$

integral bound: $\ln(n+1) \geq 6$

so ok with $n \geq \lceil e^6 - 1 \rceil = 403$ books

actually calculate H_n:

227 books are enough.

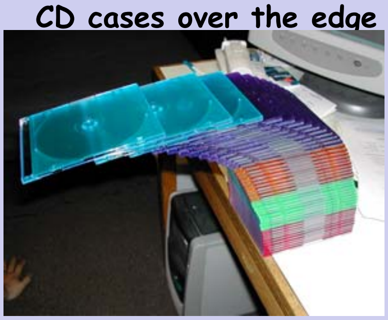
Book stacking

$H_n \to \infty$ as $n \to \infty$,

so overhang can be

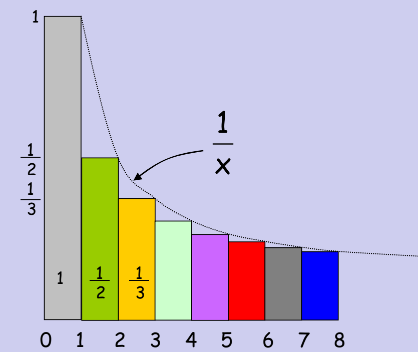
as big as desired!

CD cases over the edge



43 cases high --top 4 cases completely off the table --1.8 or 1.9 case-lengths

Upper bound for H_n



Upper bound for H_n

$$H_n < 1 + \int_1^n \frac{1}{x}\,dx$$

$$= 1 + \ln(n)$$

Asymptotic bound for H_n

$$\ln(n+1) < H_n < 1 + \ln(n)$$

$$H_n \sim \ln(n)$$

Asymptotic Equivalence

Def: $f(n) \sim g(n)$

$$\lim_{n\to\infty}\left(\frac{f(n)}{g(n)}\right) = 1$$

Asymptotic Equivalence ~

Example: $(n^2 + n) \sim n^2$

pf:

$$\lim_{n\to\infty}\frac{n^2+n}{n^2} = \lim_{n\to\infty}\left(1+\frac{1}{n}\right) = 1$$

Team Problems

Problems

1—3

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