
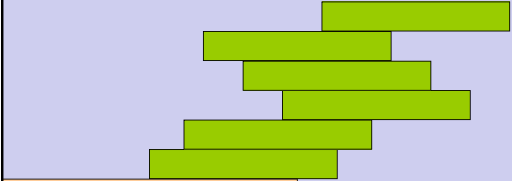

Mathematics for Computer Science
 MIT 6.042J/18.062J

Harmonic Sum Integral Method


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Book Stacking

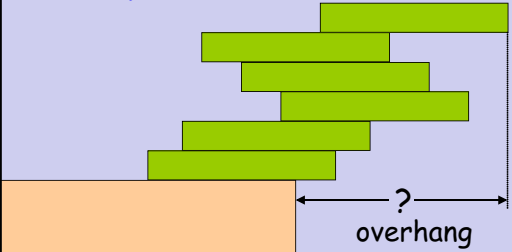


table

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Book Stacking

How far out?

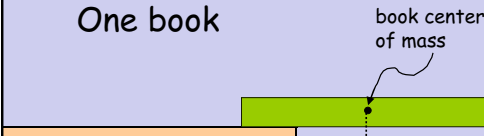


overhang

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Book Stacking

One book

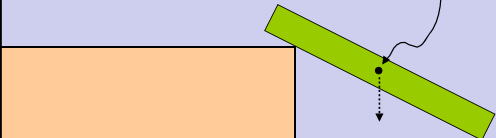


book center of mass

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Book Stacking

One book

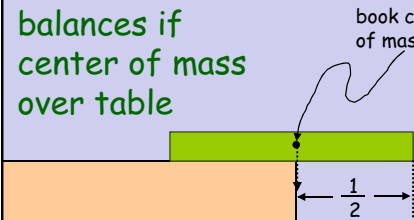


book center of mass

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Book Stacking

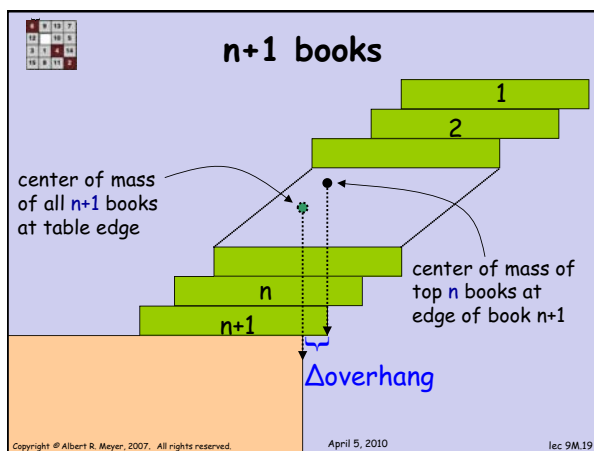
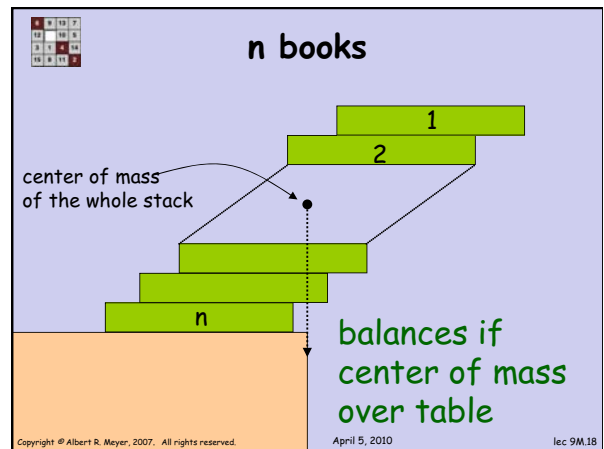
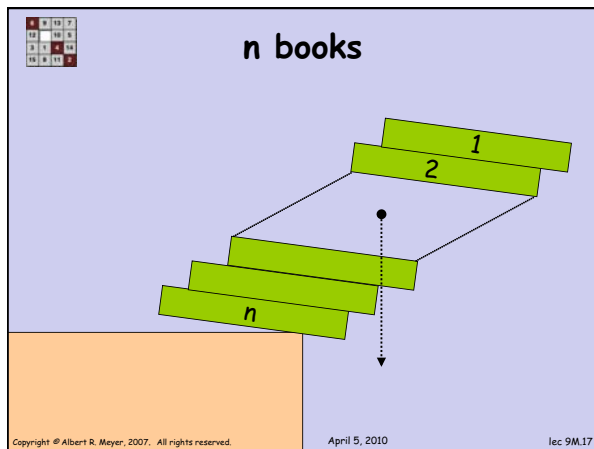
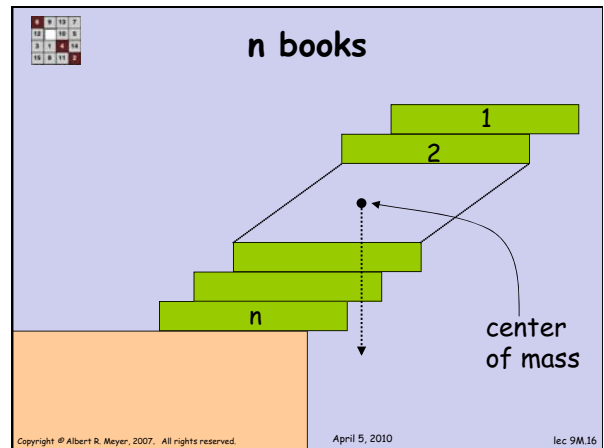
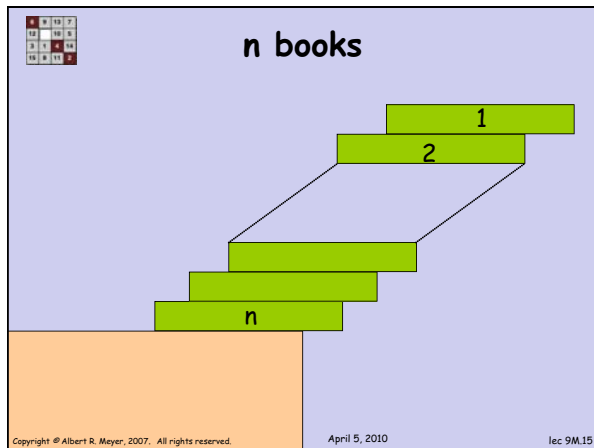
balances if center of mass over table



book center of mass

$\frac{1}{2}$

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Δ -overhang ::= horizontal distance from n -book to $(n+1)$ -book centers of mass

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Δ -overhang

$$\Delta = \frac{1/2}{n+1} = \frac{1}{2(n+1)}$$

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$n+1$ books

center of mass of all $n+1$ books

center of mass of top n books

$$\frac{1}{2(n+1)}$$

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Book stacking summary

B_n ::= overhang of n books

$B_1 = 1/2$

$B_{n+1} = B_n + \frac{1}{2(n+1)}$

$B_n = \frac{1}{2} \left(1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n} \right)$

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Harmonic Sums

$H_n ::= 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n}$

n^{th} Harmonic number

$B_n = H_n/2$

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Integral estimate for H_n

$\frac{1}{x+1}$

1, $\frac{1}{2}$, $\frac{1}{3}$

0 1 2 3 4 5 6 7 8

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Integral estimate for H_n


$H_n = \text{area of rectangles}$


$> \text{area under } 1/(x+1) =$


$\int_0^n \frac{1}{x+1} dx = \int_1^{n+1} \frac{1}{x} dx = \ln(n+1)$


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Book stacking
 for overhang 3, need $B_n \geq 3$
 $H_n \geq 6$
 integral bound: $\ln(n+1) \geq 6$
 so ok with $n \geq \lceil e^6 - 1 \rceil = 403$ books
 actually calculate H_n :
 227 books are enough.

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Book stacking
 $H_n \rightarrow \infty$ as $n \rightarrow \infty$,
 so overhang can be
 as big as desired!


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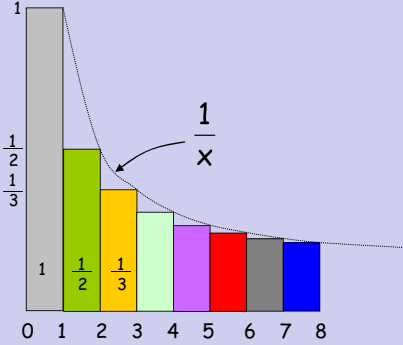

CD cases over the edge




43 cases high --top 4 cases completely
 off the table --1.8 or 1.9 case-lengths

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Upper bound for H_n





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Upper bound for H_n

$$H_n < 1 + \int_1^n \frac{1}{x} dx$$


$$= 1 + \ln(n)$$

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Asymptotic bound for H_n

$$\ln(n+1) < H_n < 1 + \ln(n)$$

$$H_n \sim \ln(n)$$

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Asymptotic Equivalence

Def: $f(n) \sim g(n)$

$$\lim_{n \rightarrow \infty} \left(\frac{f(n)}{g(n)} \right) = 1$$



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Asymptotic Equivalence \sim

Example: $(n^2 + n) \sim n^2$

pf:

$$\lim_{n \rightarrow \infty} \frac{n^2 + n}{n^2} = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n} \right) = 1$$



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Team Problems

Problems

1–3



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