In-Class Problems Week 8, Wed.

Problem 1. (a) Use the Pulverizer to find the inverse of 13 modulo 23 in the range $\{1, \ldots, 22\}$.

(b) Use Fermat's theorem to find the inverse of 13 modulo 23 in the range $\{1, \ldots, 22\}$.

Problem 2. (a) Why is a number written in decimal evenly divisible by 9 if and only if the sum of its digits is a multiple of 9? *Hint:* $10 \equiv 1 \pmod 9$.

(b) Take a big number, such as 37273761261. Sum the digits, where every other one is negated:

$$3 + (-7) + 2 + (-7) + 3 + (-7) + 6 + (-1) + 2 + (-6) + 1 = -11$$

Explain why the original number is a multiple of 11 if and only if this sum is a multiple of 11.

Problem 3.
The following properties of equivalence mod n follow directly from its definition and simple properties of divisibility. See if you can prove them without looking up the proofs in the text.

(a) If $a \equiv b \pmod n$, then $ac \equiv bc \pmod n$.

(b) If $a \equiv b \pmod n$ and $b \equiv c \pmod n$, then $a \equiv c \pmod n$.

(c) If $a \equiv b \pmod n$ and $c \equiv d \pmod n$, then $ac \equiv bd \pmod n$.

(d) $\text{rem}(a, n) \equiv a \pmod n$.

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