Mathematics for Computer Science

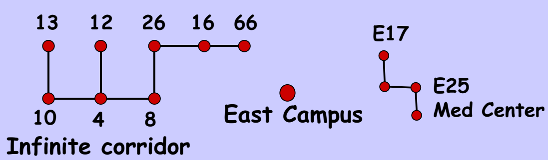
MIT 6.042J/18.062J

Graph Connectivity Trees

Connected Components

Every graph consists of separate connected pieces (subgraphs) called connected components

Connected Components



3 connected components

the more connected components, the more “broken up" the graph is.

Connected Components

The connected component of vertex v ::=

$$\{w \mid v \text{ and } w \text{ are connected}\}$$

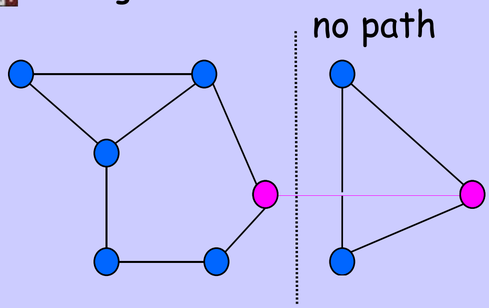
Connected Components

So a graph is connected iff it has only 1 connected component

Edge Connectedness

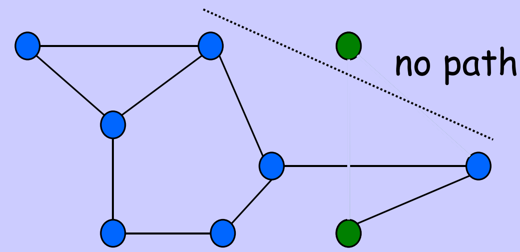
Def: vertices v, w are k-edge connected if they remain connected whenever fewer than k edges are deleted.

k-edge Connectedness



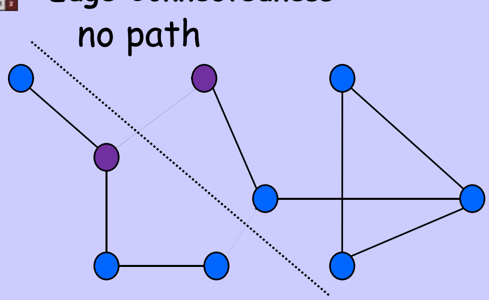
1-edge connected

Edge Connectedness



2-edge connected

Edge Connectedness



3-edge connected

k-edge Connectedness

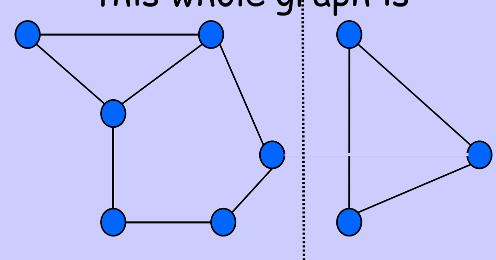
Def: A whole *graph* is k-edge connected iff every two vertices are k-edge connected.

Edge Connectedness

Connectivity measures fault tolerance of a network: how many connections can fail without cutting off communication?

k-edge Connectedness

this whole graph is

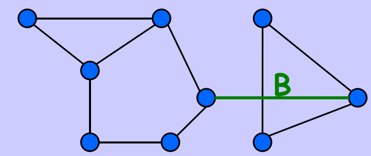


1-edge connected

Cut Edges

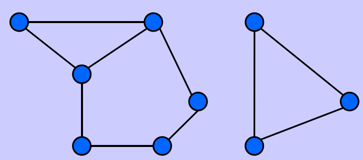
An edge is a cut edge if removing it from the graph disconnects two vertices.

Cut Edges



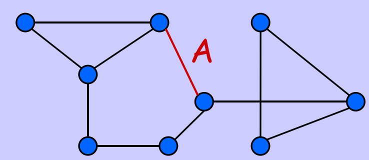
B is a cut edge

Cut Edges



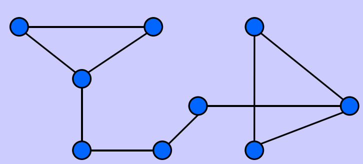
deleting B gives two components

Cut Edges



A is *not* a cut edge

Cut Edges



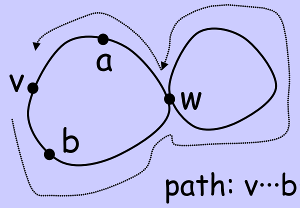
still connected with edge A deleted

Cut Edges

So a connected graph is 2-edge connected iff it has no cut edge.

Cycles

A cycle is a path that begins and ends with same vertex

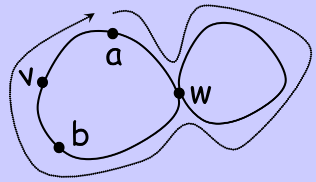


path: v···b···w···w···a···v

also: a···v···b···w···w···a

Cycles

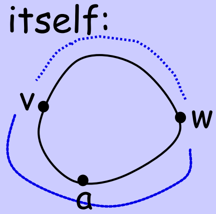
A cycle is a path that begins and ends with same vertex



also: a ···w ···w ···b ···v ···a

Simple Cycles

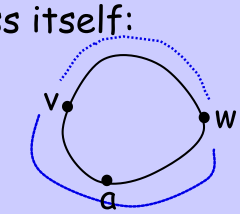
A simple cycle is a cycle of length > 2 that doesn't cross itself:



path: v···a···w···v

Simple Cycles

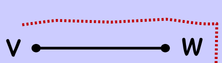
A simple cycle is a cycle of length > 2 that doesn't cross itself:



path: v···a···w···v also: w···a···v···w

Simple Cycles

length > 2 implies that going back & forth over an edge is not a simple cycle



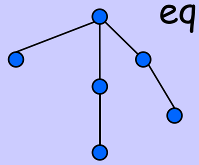
Cut Edges and Cycles

Lemma: An edge is a not a cut edge iff it is on a simple cycle.

Trees

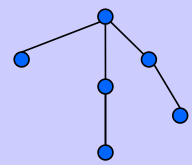
A tree is a connected graph with no simple cycles.

equivalently:

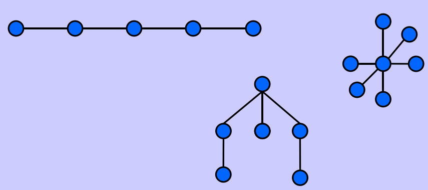


Trees

A tree is a connected graph with every edge a cut edge.



More Trees



Other Tree Definitions

- graph with a unique *simple* path between any 2 vertices
- connected graph with n vertices and n–1 edges
- an edge-maximal acyclic graph

Team Problems

Problems 1 & 2

6.042J / 18.062J Mathematics for Computer Science
Spring 2010