

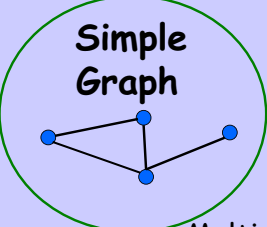

Mathematics for Computer Science
 MIT 6.042J/18.062J

Simple Graphs

 Degrees,
 Isomorphism,
 Paths

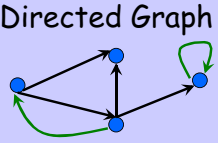

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Types of Graphs



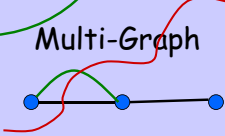
Simple Graph

this week





Directed Graph

next week



Multi-Graph


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A simple graph:


Definition:

A simple graph G consists of

- a nonempty set, V , of vertices
- a set, E , of edges


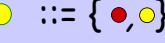
(edge = set of 2 vertices)


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A Simple Graph

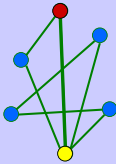
vertices, V


undirected edges, E



::= {

}

"adjacent"

edge

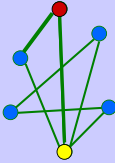




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Vertex degree

degree of a vertex is
 # of incident edges

$\deg(\bullet) = 2$

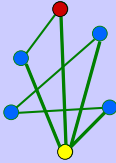




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Vertex degree

degree of a vertex is
 # of incident edges

$\deg(\bullet) = 4$




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Impossible Graph

Is there a graph with vertex degrees 2,2,1?

NO!

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Handshaking Lemma

sum of degrees is twice # edges

$$2|E| = \sum_{v \in V} \text{deg}(v)$$

Proof: Each edge contributes 2 to the sum on the right

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Handshaking Lemma

sum of degrees is twice # edges

$$2|E| = \sum_{v \in V} \text{deg}(v)$$

$2+2+1 = \text{odd}$, so impossible

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Sex in America: Men more Promiscuous?

Study claims:
Men average many more partners than women.

Graph theory shows **this is nonsense**

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Sex Partner Graph

partners

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Counting pairs of partners

$$\sum_{m \in M} \text{deg}(m) = |E| = \sum_{f \in F} \text{deg}(f)$$

now divide by both sides by $|M|$

$$\frac{\sum_{m \in M} \text{deg}(m)}{|M|} = \frac{|F|}{|M|} \cdot \frac{\sum_{f \in F} \text{deg}(f)}{|F|}$$

avg-deg(M) = (|F|/|M|) * avg-deg(F)

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Average number of partners

$$\text{avg-deg}(M) = 1.035 \cdot \text{avg-deg}(F)$$

Averages differ solely by
ratio of females to males.

No big difference
Nothing to do with promiscuity



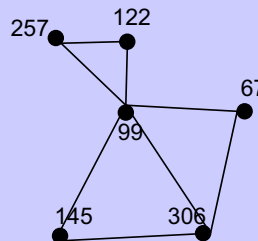
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The Graph Abstraction

picture of a graph



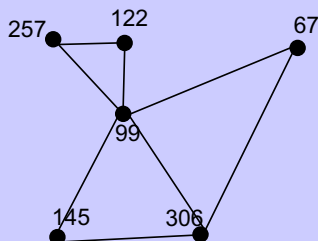
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The Graph Abstraction

picture of **same** graph



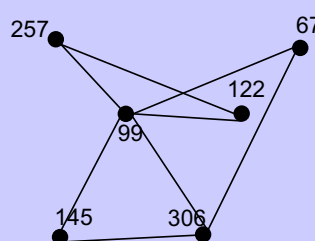
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The Graph Abstraction

picture of **same** graph



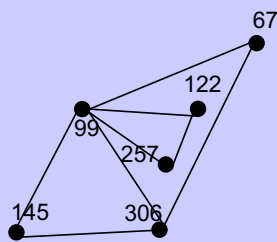
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The Graph Abstraction

picture of **same** graph



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The Graph Abstraction

All that matters
are the **connections**:
graphs with the
same connections
are **isomorphic**



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Isomorphism
 two graphs are **isomorphic**
 when there is an
edge-preserving
matching
 of their vertices.

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Are these isomorphic?

$f(\text{Dog}) = \text{Beef}$ $f(\text{Cow}) = \text{Hay}$
 $f(\text{Cat}) = \text{Tuna}$ $f(\text{Pig}) = \text{Corn}$

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Edges preserved?

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Edges preserved? YES!

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Nonedges preserved? YES!

isomorphic!

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Formal Def of Graph Isomorphism

G_1 **isomorphic** to G_2 means
 edge-preserving vertex matching:

\exists bijection $f: V_1 \rightarrow V_2$ with
 $u-v$ in E_1 iff $f(u)-f(v)$ in E_2

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Nonisomorphism

degree 2 all degree 3

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Proving nonisomorphism

If some property *preserved by isomorphism* **differs** for two graphs, then they're **not** isomorphic:

- # of nodes,
- # of edges,
- degree distributions,

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Finding an isomorphism?

many possible mappings: **large search**
 can use properties *preserved by isomorphisms* as a guide, for example:

- a **deg 4 vertex adjacent to a deg 3** can only match with
- a **deg 4 vertex also adjacent to a deg 3**

but even so...

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Are these two graphs isomorphic?

...nothing known is *sure* to be much faster than searching thru all bijections for an isomorphism

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Paths

Path: sequence of *adjacent* vertices

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Paths

Path: sequence of *adjacent* vertices

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Paths

Path: sequence of *adjacent* vertices

(● ● ●)

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Paths

Path: sequence of *adjacent* vertices

(● ● ● ●)

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Paths

Path: sequence of *adjacent* vertices

(● ● ● ● ●)

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Paths

Path: sequence of *adjacent* vertices

(● ● ● ● ● ●)

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Connectedness

two *vertices* are **connected** iff there is a path from one to the other.

a *graph* is **connected** iff every two vertices are connected.


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Simple Paths

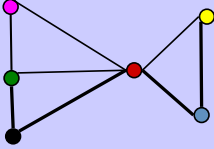
Simple Path: all vertices different

(● ● ● ● ●)


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 **Simple Paths**

Simple Path: (doesn't cross itself)

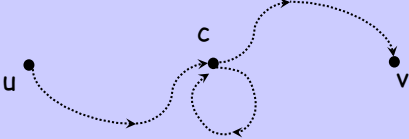



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 **Paths & Simple Paths**

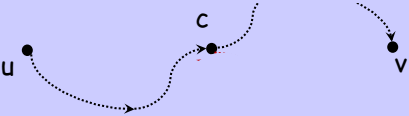
Lemma:
The shortest path between two vertices is simple!
Proof: (by contradiction) suppose path from u to v crossed itself:





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 **Paths & Simple Paths**

Lemma:
The shortest path between two vertices is simple!
 then path without $c \text{---} c$ is shorter!




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 **Team Problems**

Problems

1—3

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