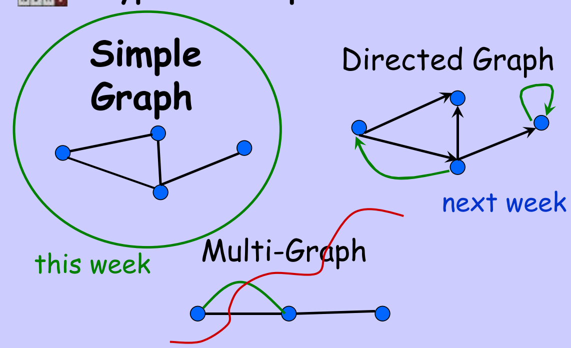
Mathematics for Computer Science

MIT 6.042J/18.062J

Simple Graphs

Degrees, Isomorphism, Paths

Types of Graphs



A simple graph:

Definition:

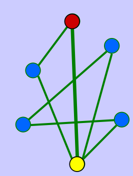
A simple graph G consists of

- a nonempty set, V, of vertices
- a set, E, of edges

(edge = set of 2 vertices)

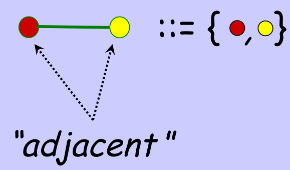
A Simple Graph

edge



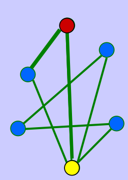
vertices, V

undirected edges, E



Vertex degree

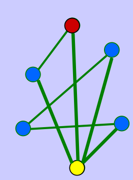
degree of a vertex is # of incident edges



deg(●) = 2

Vertex degree

degree of a vertex is # of *incident* edges

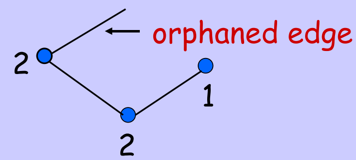


deg(●) = 4

Impossible Graph

Is there a graph with vertex degrees 2,2,1?

NO!



Handshaking Lemma

sum of degrees is twice # edges

$$2|E| = \sum_{v \in V} \deg(v)$$

Proof: Each edge contributes 2 to the sum on the right

Handshaking Lemma

sum of degrees is twice # edges

$$2|E| = \sum_{v \in V} \deg(v)$$

2+2+1 = odd, so impossible

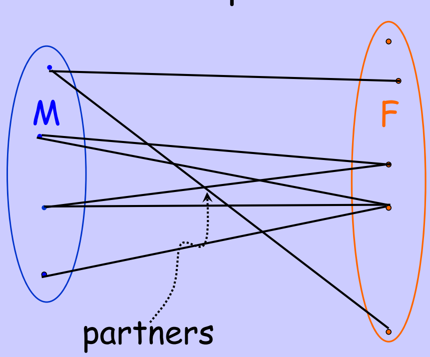
Sex in America: Men more Promiscuous?

Study claims:

Men average many more partners than women.

Graph theory shows this is nonsense

Sex Partner Graph



Counting pairs of partners

$$\sum_{m \in M} \deg(m) = |E| = \sum_{f \in F} \deg(f)$$

now divide by both sides by $|M|$

$$\underbrace{\frac{\sum_{m \in M} \deg(m)}{|M|}}_{\text{avg-deg}(M)} = \frac{|F|}{|M|} \cdot \underbrace{\frac{\sum_{f \in F} \deg(f)}{|F|}}_{\text{avg-deg}(F)}$$

Average number of partners

$\text{avg-deg}(M) = 1.035 \cdot \text{avg-deg}(F)$

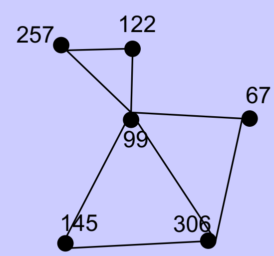
Averages differ solely by ratio of females to males.

No big difference

Nothing to do with promiscuity

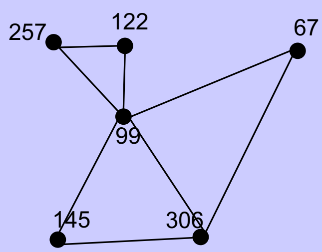
The Graph Abstraction

picture of a graph



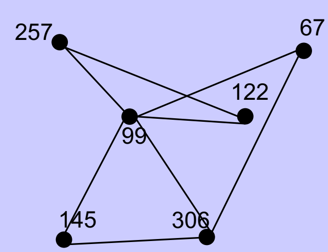
The Graph Abstraction

picture of same graph



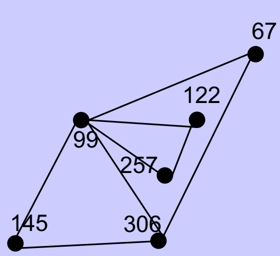
The Graph Abstraction

picture of same graph



The Graph Abstraction

picture of same graph



The Graph Abstraction

All that matters are the connections: graphs with the same connections are isomorphic

Isomorphism

two graphs are isomorphic

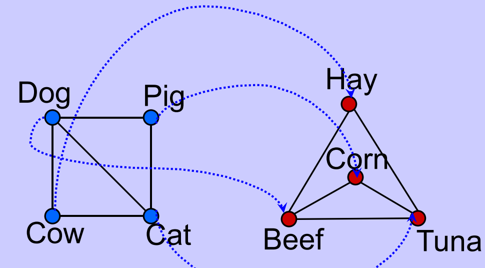
when there is an

edge-preserving

matching

of their vertices.

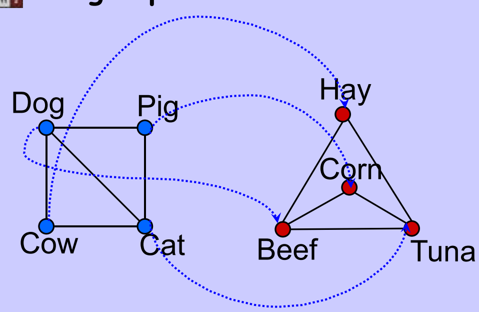
Are these isomorphic?



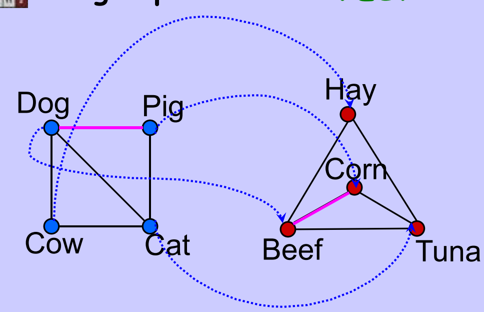
f(Dog) = Beef f(Cow) = Hay

f(Cat) = Tuna f(Pig) = Corn

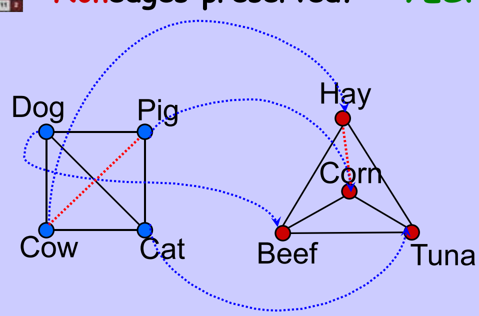
Edges preserved?



Edges preserved? YES!



Nonedges preserved? YES!



isomorphic!

Formal Def of Graph Isomorphism

G_1 isomorphic to G_2 means edge-preserving vertex matching:

$\exists$ bijection $f: V_1 \to V_2$ with

$u—v$ in E_1 iff $f(u)—f(v)$ in E_2

Nonisomorphism

degree 2   all degree 3

 lec 6W.36

Proving nonisomorphism

If some property *preserved by isomorphism* differs for two graphs, then they're not isomorphic:

- # of nodes,
- # of edges,
- degree distributions, ...

 lec 6W.37

Finding an isomorphism?

many possible mappings: large search

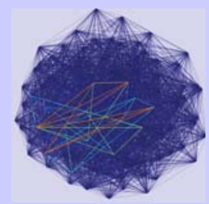
can use properties *preserved* by isomorphisms as a guide, for example:

- a deg 4 vertex adjacent to a deg 3
  can only match with
- a deg 4 vertex also adjacent to a deg 3

but even so...

 lec 6W.38

Are these two graphs isomorphic?

...nothing known is *sure* to be much faster than searching thru all bijections for an isomorphism

 lec 6W.39

Paths

Path: sequence of *adjacent* vertices

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 lec 6W.41

Paths

Path: sequence of *adjacent* vertices

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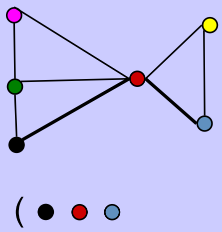
 lec 6W.42

Paths

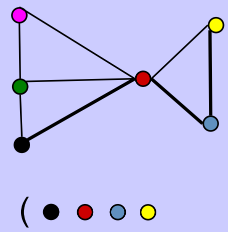
Path: *sequence of* *adjacent* vertices



Albert R Meyer, March 10, 2010 lec 6W.43

Paths

Path: *sequence of* *adjacent* vertices



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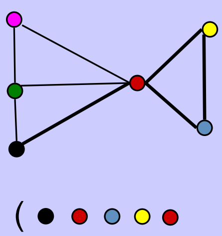
Paths

Path: *sequence of* *adjacent* vertices



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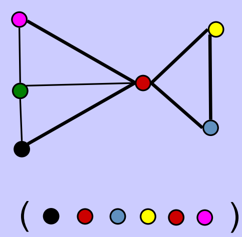
Paths

Path: *sequence of* *adjacent* vertices



Albert R Meyer, March 10, 2010 lec 6W.46

Connectedness

two *vertices* are connected iff there is a path from one to the other.

a *graph* is connected iff every two vertices are connected.

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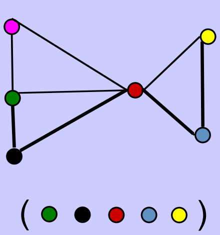
Simple Paths

Simple Path: all vertices different



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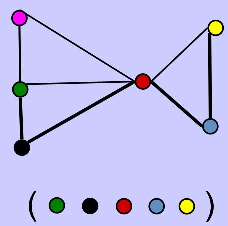
Simple Paths

Simple Path: (doesn't cross itself)

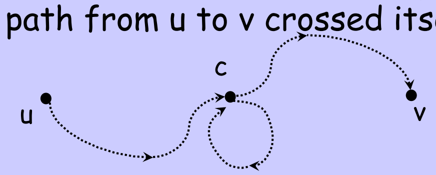


Paths & Simple Paths

Lemma:

The *shortest* path between two vertices is simple!

Proof: (by contradiction) suppose path from u to v crossed itself:

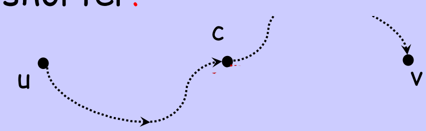


Paths & Simple Paths

Lemma:

The *shortest* path between two vertices is simple!

then path without c---c is shorter!



Team Problems

Problems 1—3