
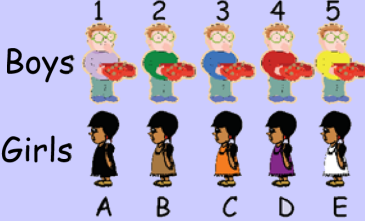

Mathematics for Computer Science
 MIT 6.042J/18.062J


Stable Matching

Albert R Meyer, March 8, 2010 lec 6M.1


Stable Marriage
 A Marriage Problem



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Stable Marriage
 Preferences


<p>Boys</p> <p>1 : CBEAD</p> <p>2 : ABECD</p> <p>3 : DCBAE</p> <p>4 : ACDBE</p> <p>5 : ABDEC</p>	<p>Girls</p> <p>A : 35214</p> <p>B : 52143</p> <p>C : 43512</p> <p>D : 12345</p> <p>E : 23415</p>
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Stable Marriage
 Preferences

<p>1 : CBEAD</p> <p>2 : ABECD</p> <p>3 : DCBAE</p> <p>4 : ACDBE</p> <p>5 : ABDEC</p>	<p>Try "greedy" strategy for boys</p>
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Stable Marriage

Preferences Marry **Boy 1** with **Girl C**
 (his 1st choice)

<p>1 : CBEAD</p> <p>2 : ABECD</p> <p>3 : DCBAE</p> <p>4 : ACDBE</p> <p>5 : ABDEC</p>	 <p>1 C</p>
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Stable Marriage

Preferences

<p>2 : ABED</p> <p>3 : DBAE</p> <p>4 : ADBE</p> <p>5 : ABDE</p>

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Stable Marriage

Preferences

2 : ABED
 3 : DBAE
 4 : ADBE
 5 : ABDE

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Stable Marriage

Next:
 Preferences Marry Boy 2 with Girl A
 (his remaining 1st choice)

~~2 : ABED~~
 3 : DBAE
 4 : ~~ADBE~~
 5 : ~~ABDE~~

2 A

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Stable Marriage

Final "boy greedy" marriages

1 C
 2 A
 3 D
 4 B
 5 E

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Stable Marriage

Trouble!

1 C
 4 B

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Stable Marriage

Boy 4 likes Girl C better than his wife.

1 C
 4 B

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Stable Marriage

and vice-versa

1 C
 4 B

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Stable Marriage
a **rogue** couple

1 C
4 B

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Stable Marriage

Stable Marriage
Problem:

Marry everyone without
any rogue couples!

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Stable Marriage

More than a puzzle:
College Admissions
(original Gale & Shapley paper, 1962)
Matching Hospitals & Residents.
Matching Dance Partners.

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Stable Marriage

Photograph of dancing students removed due to copyright restrictions.

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The Mating Ritual
(day by day)

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Mating Ritual

Morning: boy serenades favorite girl


Billy Bob Brad Angelina

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Mating Ritual

Morning: boy serenades favorite girl
 Afternoon: girl **rejects** all but favorite

if you're not Brad
take a hike!




Billy Bob Brad Angelina

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Mating Ritual

Morning: boy serenades favorite girl
 Afternoon: girl rejects all but favorite
 Evening: rejected boy writes off girl



Angelina Billy Bob

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Mating Ritual

Stop when no girl rejects.
 Each girl marries her
 favorite suitor (if any).

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Mating Ritual

Termination:
 there exists a Wedding Day.

Partial Correctness:
 • everyone is married.
 • marriages are stable.

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Stable Marriage: termination

total # remaining names
 on boy's lists:
 strictly decreasing
 & \mathbb{N} -valued
 So \exists Wedding Day

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Mating Ritual: girls improve

Lemma:
 A girl's favorite tomorrow
 will be at least as desirable
 as today's.
 ...because today's favorite will
 stay until she rejects him for
 someone better.

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Mating Ritual: girls improve

Lemma:

A girl's favorite tomorrow will be at least as desirable as today's.

(*favorite* (G) is weakly increasing for each G)



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Mating Ritual: boys get worse

Lemma:

A boy's 1st love tomorrow will be no more desirable than today's.

...because boys work straight down their lists.



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Mating Ritual: boys get worse

Lemma:

A boy's 1st love tomorrow will be no more desirable than today's.

(*serenading* (B) is weakly decreasing for each B)



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Mating Ritual: invariant

If G is not on B 's list, then she has a better current favorite.

Proof: When G rejected B she had a better suitor, and *favorite* (G) is weakly increasing.



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On Wedding Day

- Each girl has ≤ 1 suitors (by def of wedding day)
- Each boy is married, or has no girls on his list



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Mating Ritual: everyone marries

Everyone is married on wedding day

Proof: by contradiction.

If B is not married, his list is empty.

By *Invariant*, all girls have favorites better than B -- so they do have a favorite. That is, all girls are married, so all boys are married.



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Mating Ritual: stable marriages

Marriages are Stable:

Bob won't be in rogue couple with case 1: a girl G on his final list, since he's already married to the best of them.



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Mating Ritual: stable marriages

Marriages are Stable:

Bob won't be in rogue couple with case 2: a girl G not on his list, since by invariant, G likes her spouse better than Bob.



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Mating Ritual

Who does better, boys or girls?

Girls' suitors get better, and boy's sweethearts get worse, so girls do better?

No!



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Boy Optimal

Mating Ritual is Optimal for all Boys at once. Pessimal for all Girls.



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Stable Marriage

More questions, rich theory

- other stable marriages possible? - can be many
- do better by lying? boys -No! girls -Yes!



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Team Problems

Problems
1-4



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