In-Class Problems Week 4, Fri.

Problem 1.
The table below lists some prerequisite information for some subjects in the MIT Computer Science program (in 2006). This defines an indirect prerequisite relation, $\prec$, that is a strict partial order on these subjects.

$$18.01 \rightarrow 6.042 \qquad\qquad 18.01 \rightarrow 18.02$$
$$18.01 \rightarrow 18.03 \qquad\qquad 6.046 \rightarrow 6.840$$
$$8.01 \rightarrow 8.02 \qquad\qquad 6.001 \rightarrow 6.034$$
$$6.042 \rightarrow 6.046 \qquad\qquad 18.03, 8.02 \rightarrow 6.002$$
$$6.001, 6.002 \rightarrow 6.003 \qquad\qquad 6.001, 6.002 \rightarrow 6.004$$
$$6.004 \rightarrow 6.033 \qquad\qquad 6.033 \rightarrow 6.857$$

(a) Explain why exactly six terms are required to finish all these subjects, if you can take as many subjects as you want per term. Using a *greedy* subject selection strategy, you should take as many subjects as possible each term. Exhibit your complete class schedule each term using a greedy strategy.

(b) In the second term of the greedy schedule, you took five subjects including 18.03. Identify a set of five subjects not including 18.03 such that it would be possible to take them in any one term (using some nongreedy schedule). Can you figure out how many such sets there are?

(c) Exhibit a schedule for taking all the courses —but only one per term.

(d) Suppose that you want to take all of the subjects, but can handle only two per term. Exactly how many terms are required to graduate? Explain why.

(e) What if you could take three subjects per term?

Problem 2.
A pair of 6.042 TAs, Liz and Oscar, have decided to devote some of their spare time this term to establishing dominion over the entire galaxy. Recognizing this as an ambitious project, they worked out the following table of tasks on the back of Oscar's copy of the lecture notes.

1. **Devise a logo** and cool imperial theme music - 8 days.

2. **Build a fleet** of Hyperwarp Stardestroyers out of eating paraphernalia swiped from Lobdell - 18 days.

---

3. **Seize control** of the United Nations - 9 days, after task #1.

4. **Get shots** for Liz's cat, Tailspin - 11 days, after task #1.

5. **Open a Starbucks chain** for the army to get their caffeine - 10 days, after task #3.

6. **Train an army** of elite interstellar warriors by dragging people to see *The Phantom Menace* dozens of times - 4 days, after tasks #3, #4, and #5.

7. **Launch the fleet** of Stardestroyers, crush all sentient alien species, and establish a Galactic Empire - 6 days, after tasks #2 and #6.

8. **Defeat Microsoft** - 8 days, after tasks #2 and #6.

We picture this information in Figure 1 below by drawing a point for each task, and labelling it with the name and weight of the task. An edge between two points indicates that the task for the higher point must be completed before beginning the task for the lower one.
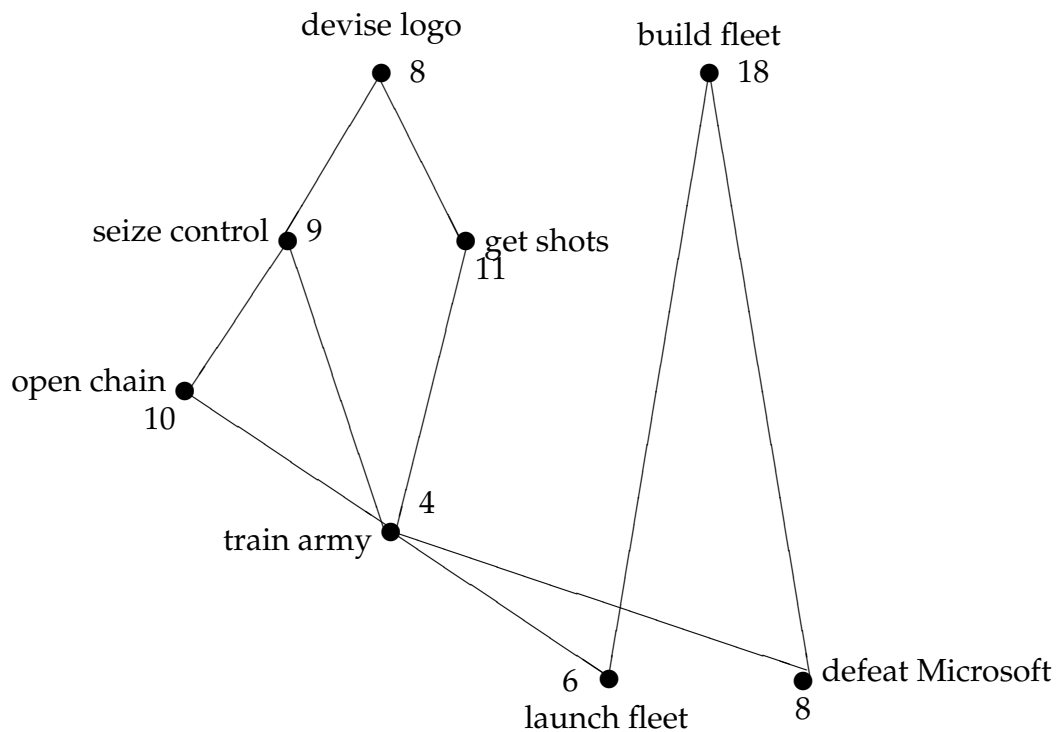

Figure 1: Graph representing the task precedence constraints.

(a) Give some valid order in which the tasks might be completed.

Liz and Oscar want to complete all these tasks in the shortest possible time. However, they have agreed on some constraining work rules.

- Only one person can be assigned to a particular task; they can not work together on a single task.

- Once a person is assigned to a task, that person must work exclusively on the assignment until it is completed. So, for example, Liz cannot work on building a fleet for a few days, run to get shots for Tailspin, and then return to building the fleet.

(b) Liz and Oscar want to know how long conquering the galaxy will take. Oscar suggests dividing the total number of days of work by the number of workers, which is two. What lower bound on the time to conquer the galaxy does this give, and why might the actual time required be greater?

(c) Liz proposes a different method for determining the duration of their project. He suggests looking at the duration of the "critical path", the most time-consuming sequence of tasks such that each depends on the one before. What lower bound does this give, and why might it also be too low?

(d) What is the minimum number of days that Liz and Oscar need to conquer the galaxy? No proof is required.

Problem 3. (a) What are the maxim*al* and minim*al* elements, if any, of the power set $\mathcal{P}(\{1, \ldots, n\})$, where n is a positive integer, under the *empty relation*?

(b) What are the maxim*al* and minim*al* elements, if any, of the set, $\mathbb{N}$, of all nonnegative integers under divisibility? Is there a minim*um* or maxim*um* element?

(c) What are the minimal and maximal elements, if any, of the set of integers greater than 1 under divisibility?

(d) Describe a partially ordered set that has no minimal or maximal elements.

(e) Describe a partially ordered set that has a *unique minimal* element, but no minimum element. *Hint:* It will have to be infinite.

6.042J / 18.062J Mathematics for Computer Science
Spring 2010