

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Mathematics for Computer Science
MIT 6.042J/18.062J

Partial Orders & Scheduling

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6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Some Course 6 Prerequisites

18.01 → 6.042

18.01 → 18.02

18.01 → 18.03

6.001 → 6.034

6.042 → 6.046

8.02 → 6.002

18.03, 6.002 → 6.004

6.001, 6.004 → 6.033

6.033 → 6.857

6.046 → 6.840

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6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

subjects with no prereq's

18.01
8.02
6.001

<nothing> → d

"d is a Freshman subject"

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6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

minimal elements

d is minimal:

nothing else is smaller

$\forall c \neq d. \text{NOT}(cRd)$

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6	9	13	7
12		10	5
3	1	4	14
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Minimal elements

d is minimal:

nothing else is smaller

$\forall c \neq d. \text{NOT}(c \leq d)$

d is minimum:

smaller than everything else

$\forall c \neq d. d \leq c$

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6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Constructing a Term Schedule

18.01

 → 6.042

18.01 → 18.02

18.01 → 18.03

6.001

 → 6.034

6.042 → 6.046

8.02

 → 6.002

18.03, 6.002 → 6.004

6.001, 6.004 → 6.033

6.033 → 6.857

6.046 → 6.840

identify minimal elements

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Constructing a Term Schedule

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

18.01 8.02 6.001

start schedule with them

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Constructing a Term Schedule

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

~~18.01~~ → 6.042 ~~8.02~~ → 6.002
~~18.01~~ → 18.02 18.03, ~~6.002~~ → 6.004
~~18.01~~ → 18.03 ~~6.001~~, 6.004 → 6.033
~~6.001~~ → 6.034 6.046 → 6.840
 6.042 → 6.046

remove minimal elements

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Constructing a Term Schedule

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

→ 6.042 → 6.002
 → 18.02 18.03, 6.002 → 6.004
 → 18.03 6.004 → 6.033
 → 6.034 6.033 → 6.857
 6.042 → 6.046 6.046 → 6.840

remove minimal elements

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Constructing a Term Schedule

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

→ 6.042 → 6.002
 → 18.02 18.03, 6.002 → 6.004
 → 18.03 6.004 → 6.033
 → 6.034 6.033 → 6.857
 6.042 → 6.046 6.046 → 6.840

identify new minimal elements

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Constructing a Term Schedule

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

18.01
8.02
6.001

18.01

↓

18.02
6.042
18.03

8.02

↓

6.002

6.001

↓

6.034

schedule them next

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Constructing a Term Schedule

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

18.01
8.02
6.001

18.01

↓

18.02
6.042
18.03

6.046

8.02

↓

6.002

6.004

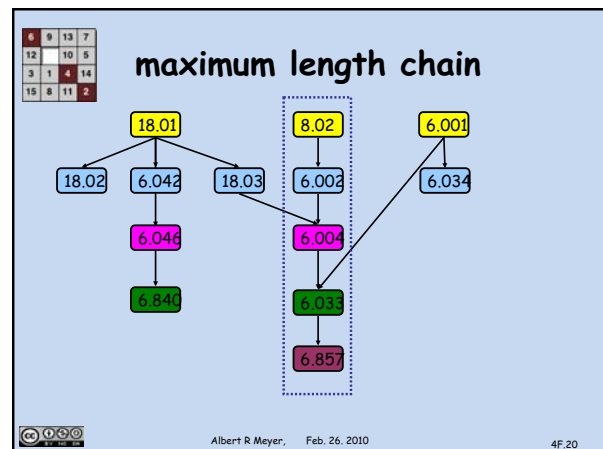
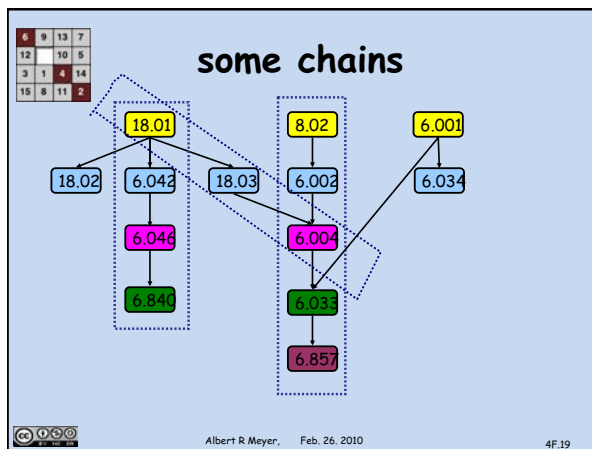
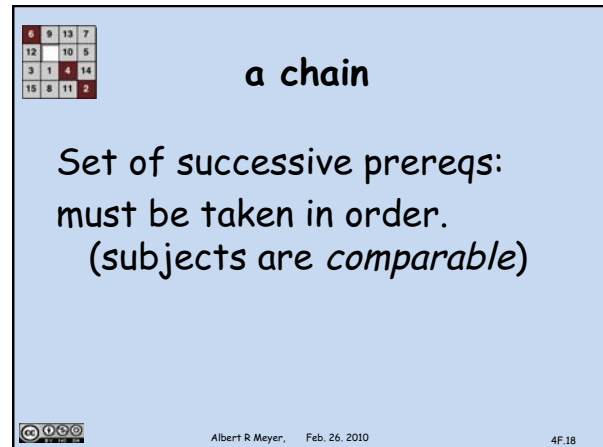
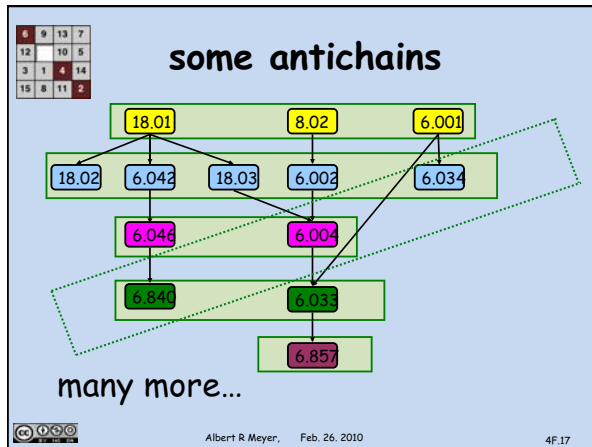
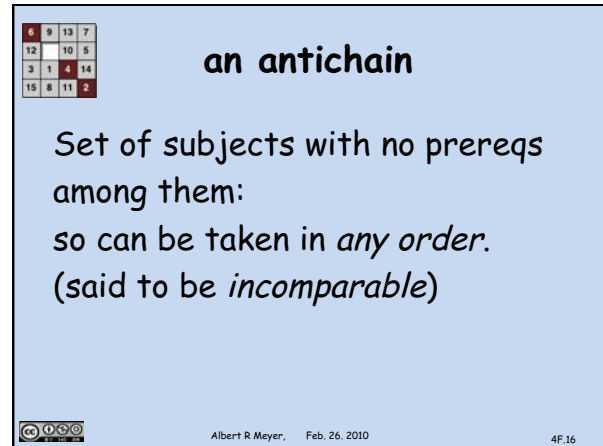
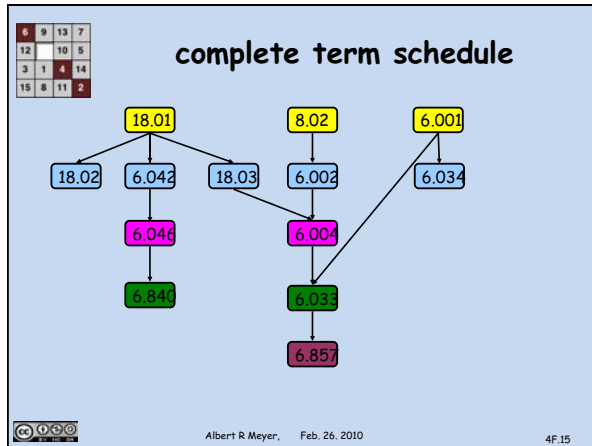
6.001

↓

6.034

continue in this way...

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6	9	13	7
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how many terms to graduate?

5 terms are **necessary** to graduate --because max chain length is 5
and 5 are **sufficient**
--if you can take unlimited subjects per term...

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6	9	13	7
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...sufficient

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parallel processing time

min # terms to graduate:

$\text{min parallel time} = \text{max chain size}$

max term load:

$\# \text{ processors for min time} \leq \text{max antichain size}$

5 in this case

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6	9	13	7
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reduce the term load

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max 4 subjects per term

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6	9	13	7
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3 Subjects per Term Possible

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6	9	13	7
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a leisurely schedule

Graduate taking only 1 subject/term?
Sure,

a topological sort

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6	9	13	7
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For min time: ≥ 3 -subject term

13 subjects

max chain size = **5**

so load of *some* term must be

$$\geq \left\lceil 13/5 \right\rceil = 3$$

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Dilworth's Lemma

Prereq's among n subjects has

- a chain of size $> t$
- or antichain of size $\geq \frac{n}{t}$

for all $1 \leq t \leq n$.

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Height/Birthday Partial Order

Two students are related to each other iff one is shorter and younger than the other.

$$(s_1, a_1) \preceq (s_2, a_2)$$

iff $(s_1 \leq s_2)$ and $(a_1 \leq a_2)$
(the product p.o.)

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Dilworth Demo

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older
(a \preceq -antichain)

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6	9	13	7
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Team Problems

Problems 1-3

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Spring 2010

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