Mathematics for Computer Science

MIT 6.042J/18.062J

Partial Orders & Scheduling

Some Course 6 Prerequisites

18.01 → 6.042
18.01 → 18.02
18.01 → 18.03
6.001 → 6.034
6.042 → 6.046

8.02 → 6.002
18.03, 6.002 → 6.004
6.001, 6.004 → 6.033
6.033 → 6.857
6.046 → 6.840

subjects with no prereq's

18.01 8.02 6.001

$\langle nothing \rangle \rightarrow d$

"d is a Freshman subject"

minimal elements

d is minimal:
nothing else is smaller

$\forall c \neq d.\ \text{NOT}(cRd)$

Minimal elements

d is minimal:
nothing else is smaller

$\forall c \neq d.\ \text{NOT}(c \leq d)$

d is minimum:
smaller than everything else

$\forall c \neq d.\ d \leq c$

Constructing a Term Schedule

18.01 → 6.042
18.01 → 18.02
18.01 → 18.03
6.001 → 6.034
6.042 → 6.046

8.02 → 6.002
18.03, 6.002 → 6.004
6.001, 6.004 → 6.033
6.033 → 6.857
6.046 → 6.840

identify minimal elements

Constructing a Term Schedule

18.01 8.02 6.001

start schedule with them

Constructing a Term Schedule

18.01 → 6.042
18.01 → 18.02
18.01 → 18.03
6.001 → 6.034
6.042 → 6.046

8.02 → 6.002
18.03, 6.002 → 6.004
6.001, 6.004 → 6.033
6.046 → 6.840

remove minimal elements

Constructing a Term Schedule

→ 6.042
→ 18.02
→ 18.03
→ 6.034
6.042 → 6.046

→ 6.002
18.03, 6.002 → 6.004
6.004 → 6.033
6.033 → 6.857
6.046 → 6.840

remove minimal elements

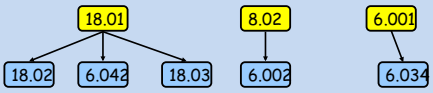
Constructing a Term Schedule

→ 6.042
→ 18.02
→ 18.03
→ 6.034
6.042 → 6.046

→ 6.002
18.03, 6.002 → 6.004
6.004 → 6.033
6.033 → 6.857
6.046 → 6.840

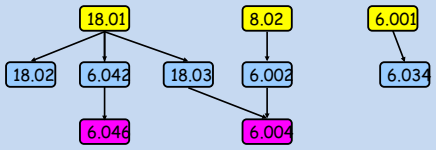
identify new minimal elements

Constructing a Term Schedule



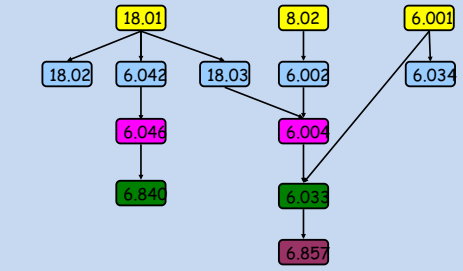
schedule them next

Constructing a Term Schedule



continue in this way…

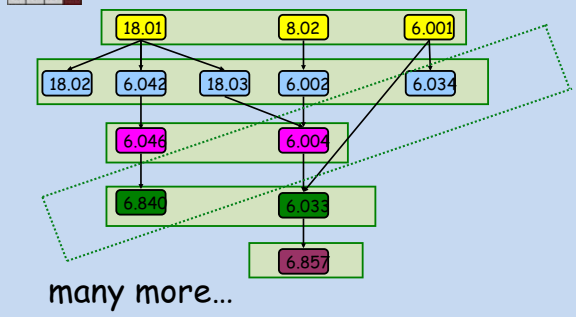
complete term schedule



an antichain

Set of subjects with no prereqs among them:
so can be taken in *any order*.
(said to be *incomparable*)

some antichains

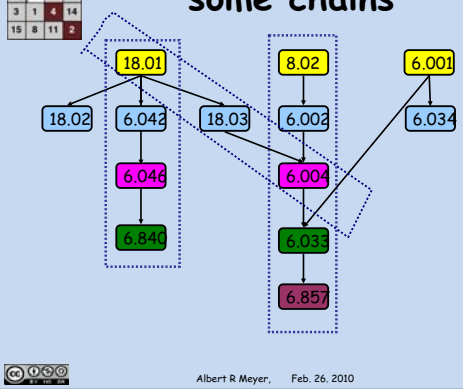


many more…

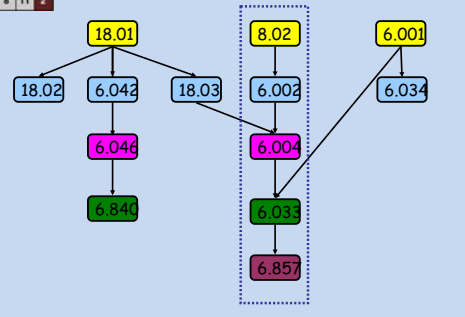
a chain

Set of successive prereqs:
must be taken in order.
(subjects are *comparable*)

some chains



maximum length chain



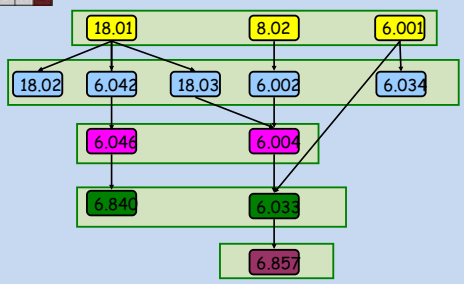
how many terms to graduate?

5 terms are **necessary** to graduate --because max chain length is 5

and 5 are **sufficient**

--if you can take unlimited subjects per term...

...sufficient



parallel processing time

min # terms to graduate:

min parallel time = max chain size

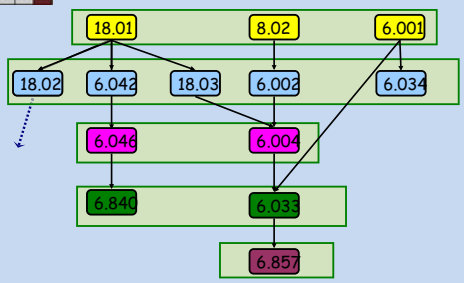
max term load:

processors for min time

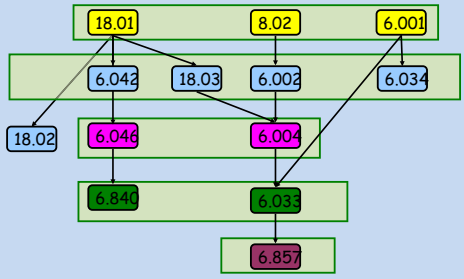
≤ max antichain size

5 in this case

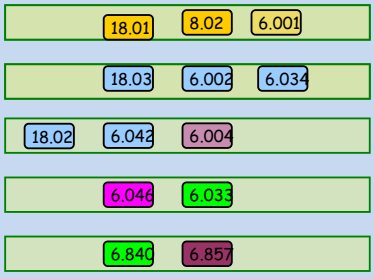
reduce the term load



max 4 subjects per term



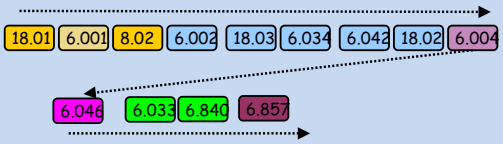
3 Subjects per Term Possible



a leisurely schedule

Graduate taking only 1 subject/term?

Sure,



a *topological sort*

For min time: $\geq$ 3-subject term

13 subjects

max chain size = 5

so load of *some* term must be

$$\geq \left\lceil 13/5 \right\rceil = 3$$

Dilworth's Lemma

Prereq's among n subjects has

- a chain of size $> t$
- or antichain of size $\geq \frac{n}{t}$

for all $1 \leq t \leq n$.

Height/Birthday Partial Order

Two students are related to each other iff one is shorter and younger than the other.

$(s_1, a_1) \preccurlyeq (s_2, a_2)$

iff $(s_1 \leq s_2)$ and $(a_1 \leq a_2)$

(the product p.o.)

Dilworth Demo

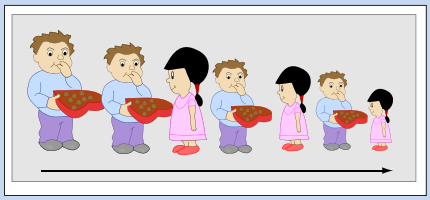


Image by MIT OpenCourseWare.

older

(a $\preccurlyeq$-antichain)

Team Problems

Problems 1–3

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