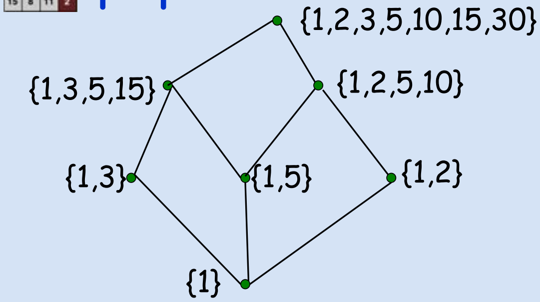
Mathematics for Computer Science

MIT 6.042J/18.062J

Partial Orders

proper subset relation



proper subset relation

$A \subset B$ means
B has everything
that A has
and more: $B \not\subset A$

properties of $\subset$

$A \subset B$ implies $B \not\subset A$

asymmetry

$\subset$ is **asymmetric**

binary relation R on set A is asymmetric:

aRb implies NOT(bRa)
for all $a,b \in A$

properties of $\subset$

[$A \subset B$ and $B \subset C$]
implies $A \subset C$

transitivity

⊂ is **transitive**

binary relation R on set A is transitive:
aRb and bRc implies aRc
for all a,b,c ∈ A

strict partial orders

transitive & asymmetric

Subject Prerequisites

subject **c** is a direct prerequisite for subject **d**

c ⟶ d

Direct Prerequisites

18.01 → 6.042 → 6.046 → 6.840

Indirect Prerequisites

18.01 → 6.042 → 6.046 → 6.840

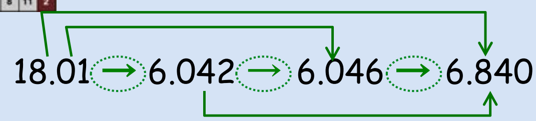
18.01 is indirect prerequisite of 6.042 and 6.840

Indirect Prerequisites

18.01 → 6.042 → 6.046 → 6.840

another indirect prereq

Indirect Prerequisites



3 more indirect prerequisites
(→ is a special case of →)

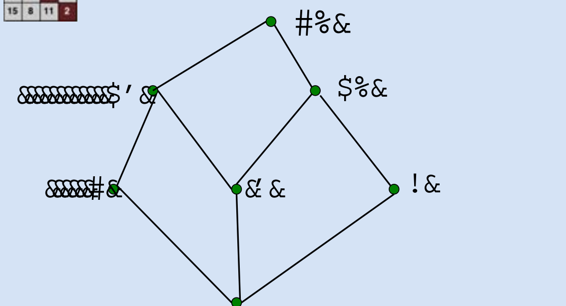
Indirect Prerequisites

If subjects c, d are *mutual* prereq's
c → d and d → c
then no one can graduate!
Comm. on Curricula ensures:
if c → d, then NOT(d → c)
asymmetry

Indirect Prerequisites

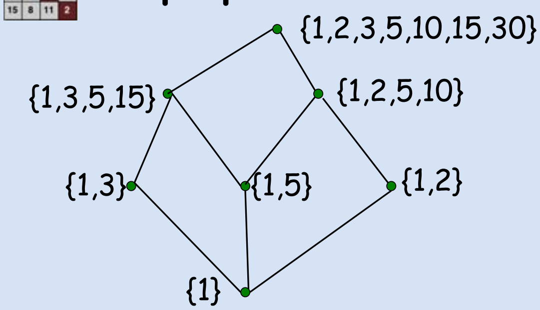
→ better be a strict partial order on MIT subjects

partial order: properly divides

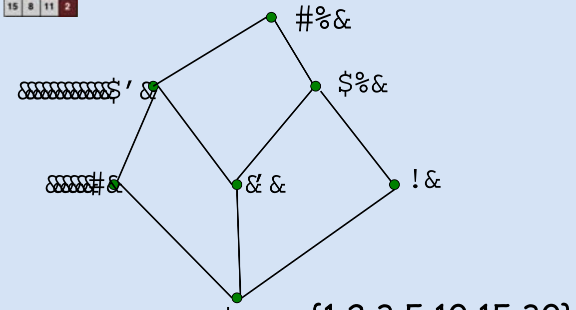


same shape
as ⊂ example

proper subset



partial order: properly divides



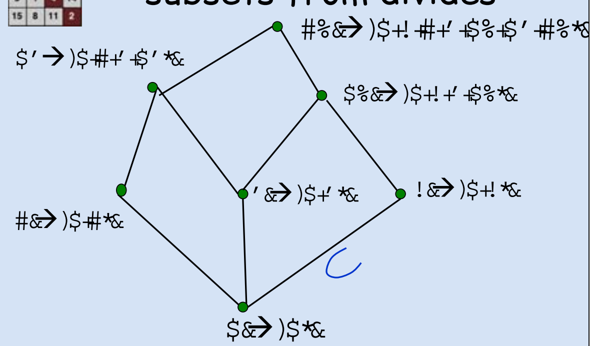
same shape as ⊂ example

isomorphic

p.o. has same shape as ⊂

Theorem: Every strict partial order is isomorphic to a collection of subsets partially ordered by ⊂.

subsets from divides



p.o. has same shape as ⊂

proof: map each element, a, to the set of elements below it

$$a \to \{b \in A \mid b R a \text{ OR } b = a\}$$

weak partial orders

same as a strict partial order R, except that

aRa always holds

reflexivity

weak partial orders

same as a strict partial order R, except that aRa always holds

examples:

- ⊆ is weak p.o. on sets
- ≤ is weak p.o. on ℝ

Reflexivity

relation R on set A is reflexive iff aRa for all a ∈ A

antisymmetry

binary relation R on set A is antisymmetric iff it is asymmetric except for aRa case.

weak partial orders

transitive
antisymmetric
& reflexive

Team Problems

Problems

1–3