

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Mathematics for Computer Science

MIT 6.042J/18.062J

Partial Orders

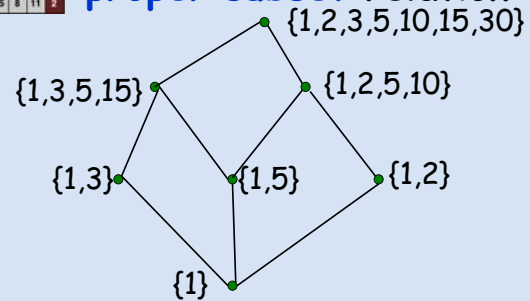


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lec4W.1

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proper subset relation



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lec4W.3

6	9	13	7
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proper subset relation

$A \subset B$ means
 B has everything
 that A has
 and more: $B \not\subset A$



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lec4W.4

6	9	13	7
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properties of \subset

$A \subset B$ implies $B \not\subset A$
 asymmetry



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6	9	13	7
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\subset is asymmetric

binary relation R on set A
 is asymmetric:

aRb implies NOT(bRa)
 for all $a, b \in A$



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6	9	13	7
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properties of \subset

$[A \subset B \text{ and } B \subset C]$
 implies $A \subset C$
 transitivity



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6	9	13	7
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⊂ is transitive

binary relation **R** on set **A**
is **transitive**:
aRb and **bRc** implies **aRc**
for all **a,b,c ∈ A**

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6	9	13	7
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strict partial orders

**transitive &
asymmetric**

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Subject Prerequisites



subject **c** is a **direct**
prerequisite for subject **d**

c → d

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Direct Prerequisites

18.01 → 6.042 → 6.046 → 6.840

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Indirect Prerequisites

18.01 → 6.042 → 6.046 → 6.840

18.01 is **indirect** prerequisite
of 6.042 and 6.840

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6	9	13	7
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Indirect Prerequisites

18.01 → 6.042 → 6.046 → 6.840

another **indirect** prereq

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Indirect Prerequisites

$18.01 \rightarrow 6.042 \rightarrow 6.046 \rightarrow 6.840$

3 more indirect prerequisites
 (\rightarrow is a special case of \rightarrow)

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6	9	13	7
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Indirect Prerequisites

If subjects c, d are mutual prereq's
 $c \rightarrow d$ and $d \rightarrow c$
 then no one can graduate!
 Comm. on Curricula ensures:
 if $c \rightarrow d$, then NOT($d \rightarrow c$)
asymmetry

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6	9	13	7
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Indirect Prerequisites

\rightarrow better be a strict partial order on MIT subjects

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partial order: properly divides

$\$ \&$ on $\{1, 2, 3, 5, 10, 15, 30\}$

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6	9	13	7
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same shape as \subset example

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6	9	13	7
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proper subset

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partial order: properly divides

$\$ \&$ on $\{1, 2, 3, 5, 10, 15, 30\}$

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same shape
as \subset example
isomorphic

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p.o. has same shape as \subset

Theorem: Every strict partial order is isomorphic to a collection of subsets partially ordered by \subset .

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6	9	13	7
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subsets from divides

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p.o. has same shape as \subset

proof: map each element, a ,
to the set of elements below it

$a \rightarrow$
 $\{b \in A \mid b R a \text{ OR } b = a\}$

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weak partial orders
same as a strict partial
order R , except that
 $a R a$ always holds
reflexivity

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weak partial orders

same as a strict partial order R , except that aRa always holds

examples:

- \subseteq is weak p.o. on sets
- \leq is weak p.o. on \mathbb{R}



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Reflexivity

relation R on set A is **reflexive** iff aRa for all $a \in A$



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antisymmetry

binary relation R on set A is **antisymmetric** iff it is asymmetric except for aRa case.



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weak partial orders

transitive
antisymmetric
& reflexive



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Team Problems

Problems 1-3



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