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# Predicate Logic

## Quantifiers $\forall, \exists$



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# Predicates

## Propositions with variables

Example:

$$P(x,y) ::= [x + 2 = y]$$



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# Predicates

$$P(x,y) ::= [x + 2 = y]$$

$x = 1$  and  $y = 3$ :  $P(1,3)$  is true

$x = 1$  and  $y = 4$ :  $P(1,4)$  is false  
 $\text{NOT}(P(1,4))$  is true



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# Quantifiers

$\forall x$  For ALL  $x$

$\exists y$  There EXISTS some  $y$



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$\forall$  is like AND

Let  $s$  range over 6.042 staff  
 $P(s) ::= [s \text{ is Pumped about 6.042}]$

$$\forall s. P(s)$$

same as

$P(\text{Stav})$  AND  $P(\text{Rich})$  AND  
 $P(\text{Megumi})$  AND...AND  $P(\text{Oscar})$



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$\exists$  is like OR

Let  $t$  range over 6.042 staff  
 $B(t) ::= [t \text{ took 6.042 Before}]$

$$\exists t. B(t)$$

same as

$B(\text{Stav})$  OR  $B(\text{Rich})$  OR  
 $B(\text{Megumi})$  OR...OR  $B(\text{Oscar})$



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## Existential Quantifier

Let  $x, y$  range over  $\mathbb{N}$

$$Q(y) ::= \exists x. x < y$$

$Q(3)$  is **T** ( $[x < 3]$  is **T** for  $x=1$ )

$Q(1)$  is **T** ( $[x < 1]$  is **T** for  $x=0$ )

$Q(0)$  is **F** ( $[x < 0]$  is **not T**  
for any  $x$  in  $\mathbb{N}$ )



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## Universal Quantifier

$x, y$  range over  $\mathbb{N}$

$$R(y) ::= \forall x. x < y$$

$R(1)$  is **F** ( $[x < 1]$  is **F** for  $x=5$ )

$R(8)$  is **F** ( $[x < 8]$  is **F** for  $x=12$ )

$R(10^{100})$  is **F**  
( $[x < 10^{100}]$  is **F** for  $x=10^{100}$ )



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## virus attack, I: $\forall \exists$

~~$\forall v \in \text{virus} . \exists d \in \text{defense}.$~~

~~$d$  protects against  $v$~~

For every virus, I have a defense:

against **MYDOOM**, use **Defender**

against **ILOVEYOU**, use **Norton**

against **BABLAS**, use **Zonealarm...**

$\forall \exists$  is **expensive!**



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## virus attack, II: $\exists \forall$

$\exists d \in \text{defense} . \forall v \in \text{virus}.$

$d$  protects against  $v$

**That's what we want!**

Example:  $d$  is **MITviruscan**,  
protects against *all* viruses



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## Alternating Quantifiers

$$G ::= \forall x \exists y. x < y$$

$x, y$  range over **Domain of Discourse**

Domain

$\mathbb{N}$

ints  $< 0$

reals  $< 0$

G is:

**T**

**F**

**T**



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## Reverse the Quantifiers

$$H ::= \exists y \forall x. x \leq y$$

Domain

$\mathbb{N}$

$\mathbb{Z}^-$

$\mathbb{R}^-$

H is:

**F**

**T**

**F**



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6	9	13	7
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## Team Problems

# Problems 1 & 2



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