*Mathematics for Computer Science*
MIT 6.042J/18.062J

# Predicate Logic Quantifiers ∀,∃



## Predicates

**Propositions with variables**

**Example:**

$P(x,y) ::= [x + 2 = y]$



## Predicates

$P(x,y) ::= [x + 2 = y]$

$x = 1$ and $y = 3$: $P(1,3)$ is true

$x = 1$ and $y = 4$: $P(1,4)$ is false

NOT($P(1,4)$) is true



## Quantifiers

$\forall x$ For ALL x

$\exists y$ There EXISTS some y



## ∀ is like AND

Let s range over 6.042 staff

P(s) ::= [s is Pumped about 6.042]

$\forall s.\ P(s)$

same as

P(Stav) AND P(Rich) AND P(Megumi) AND…AND P(Oscar)



## ∃ is like OR

Let t range over 6.042 staff

B(t) ::= [t took 6.042 Before]

$\exists t.\ B(t)$

same as

B(Stav) OR B(Rich) OR B(Megumi) OR…OR B(Oscar)

## Existential Quantifier

Let x, y range over $\mathbb{N}$

$Q(y) ::= \exists x.\ x < y$

$Q(3)$ is T ([x<3] is T for x=1)
$Q(1)$ is T ([x<1] is T for x=0)
$Q(0)$ is F ([x<0] is not T for any x in $\mathbb{N}$)



## Universal Quantifier

x, y range over $\mathbb{N}$

$R(y) ::= \forall x.\ x < y$

$R(1)$ is F ([x<1] is F for x=5)
$R(8)$ is F ([x<8] is F for x=12)
$R(10^{100})$ is F ([x<$10^{100}$] is F for x=$10^{100}$)



## virus attack, I: ∀∃

~~$\forall v \in$ virus~~ . $\exists d \in$ defense.
d protects against v

For every virus, I have a defense:
against MYDOOM, use Defender
against ILOVEYOU, use Norton
against BABLAS, use Zonealarm…

∀∃ is expensive!



## virus attack, II: ∃∀

$\exists d \in$ defense . $\forall v \in$ virus.
d protects against v

That's what we want!

Example: d is MITviruscan, protects against *all* viruses



## Alternating Quantifiers

$G ::= \forall x \exists y.\ x < y$

x, y range over Domain of Discourse

| Domain | G is: |
|---|---|
| $\mathbb{N}$ | T |
| ints < 0 | F |
| reals < 0 | T |



## Reverse the Quantifiers

$H ::= \exists y \forall x.\ x \leq y$

| Domain | H is: |
|---|---|
| $\mathbb{N}$ | F |
| $\mathbb{Z}^-$ | T |
| $\mathbb{R}^-$ | F |


## Team Problems

# Problems 1 & 2

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