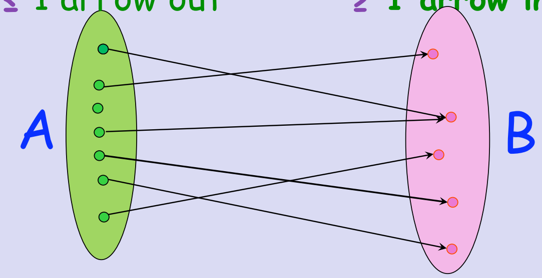
Mathematics for Computer Science

MIT 6.042J/18.062J

Cardinality
(the size of sets)

surjective & function

≤ 1 arrow out ≥ 1 arrow in



Mapping Rule (surj)

Surjective function from A to B implies

$$|A| \geq |B|$$

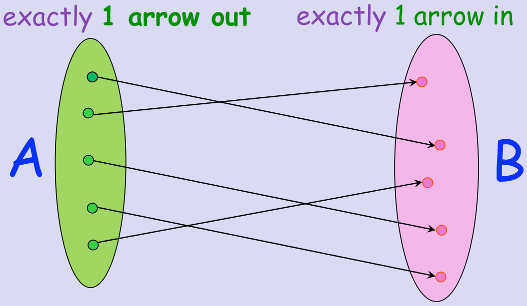
for finite A, B

Mapping Rule archery

proof: function: A→B implies |A| ≥ #arrows.
surjection implies
#arrows ≥|B|.

bijection archery

exactly 1 arrow out exactly 1 arrow in



Mapping Rule (bij)

A bijection from A to B implies

$$|A| = |B|$$

Same Size Infinite Sets?

$\{1, 2, 3, 4, \ldots\}$

and ↑ ↑ ↑ ↑

$\{0, 1, 2, 3, \ldots\}$

a bijection

Same Size Infinite Sets?

$\{1, 2, 3, 4, \ldots\}$

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the "same size"!

size of the power set

subsets of a finite set A?

$|\text{pow}(A)|$?

for $A = \{a, b, c\}$, $\text{pow}(A) =$

$\{ \emptyset, \{a\}, \{b\}, \{c\},$

$\{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\} \}$

pow(A) bijection to bit-strings

A: $\{a_0, a_1, a_2, a_3, a_4, \ldots, a_{n-1}\}$

subset: $\{a_0, \quad a_2, a_3, \quad \ldots, a_{n-1}\}$

string: 1 0 1 1 0 … 1

this defines a bijection, so

n-bit strings = $|\text{pow}(A)|$

pow(A) bijection to bin-strings

every computer scientist knows #n-bit strings, so

Corollary:

$$|\text{pow}(A)| = 2^{|A|}$$

pow(N) bijection to 1 bit-strings

infinite set $\mathbb{N} = \{0,1,2,\ldots\}$

subset: $\{0, \quad 2, 3, \quad , 5 \ \ldots \}$

string: 1 0 1 1 0 1 … }

a bijection from pow(N) to *infinite* bit-strings, $\{0,1\}^\omega$

Team Problems

Problems 1—4