

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Cardinality (the size of sets)

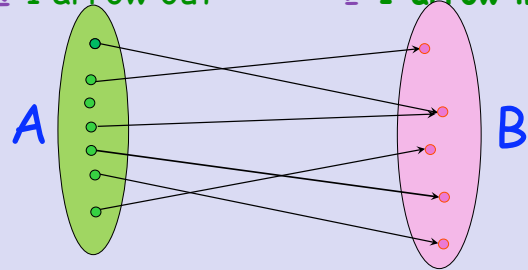


6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

surjective & function

≤ 1 arrow out

≥ 1 arrow in



6	9	13	7
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15	8	11	2

Mapping Rule (surj)

Surjective function
from A to B implies
 $|A| \geq |B|$
for finite A, B



6	9	13	7
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Mapping Rule archery

proof: function: $A \rightarrow B$
implies $|A| \geq \# \text{arrows}$.
surjection implies
 $\# \text{arrows} \geq |B|$.

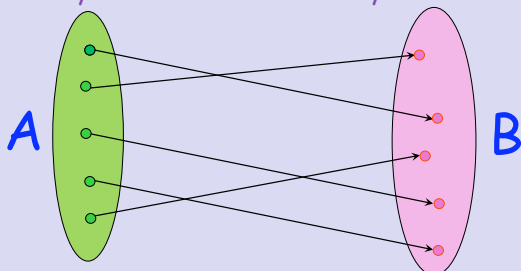


6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

bijection archery

exactly 1 arrow out

exactly 1 arrow in



6	9	13	7
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3	1	4	14
15	8	11	2

Mapping Rule (bij)

A bijection from
 A to B implies
 $|A| = |B|$



6	9	13	7
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3	1	4	14
15	8	11	2

Same Size Infinite Sets?

$\{1, 2, 3, 4, \dots\}$

and $\uparrow \uparrow \uparrow \uparrow$

$\{0, 1, 2, 3, \dots\}$

a bijection

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6	9	13	7
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Same Size Infinite Sets?

$\{1, 2, 3, 4, \dots\}$

and $\uparrow \uparrow \uparrow \uparrow$

$\{0, 1, 2, 3, \dots\}$

the "same size"!

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6	9	13	7
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size of the power set

subsets of a finite set A ?

$|\text{pow}(A)|$?

for $A = \{a, b, c\}$, $\text{pow}(A) =$

$\{\emptyset, \{a\}, \{b\}, \{c\},$
 $\{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$

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6	9	13	7
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$\text{pow}(A)$ bijection to bit-strings

$A: \{a_0, a_1, a_2, a_3, a_4, \dots, a_{n-1}\}$

subset: $\{a_0, a_2, a_3, \dots, a_{n-1}\}$

string: 1 0 1 1 0 ... 1

this defines a bijection, so

$\# \text{ n-bit strings} = |\text{pow}(A)|$

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6	9	13	7
12		10	5
3	1	4	14
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$\text{pow}(A)$ bijection to bin-strings

every computer scientist knows #n-bit strings, so

Corollary:

$|\text{pow}(A)| = 2^{|A|}$

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6	9	13	7
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3	1	4	14
15	8	11	2

$\text{pow}(N)$ bijection to 1 bit-strings

infinite set $N = \{0, 1, 2, \dots\}$

subset: $\{0, 2, 3, 5, \dots\}$

string: 1 0 1 1 0 1 ... }

a bijection from $\text{pow}(N)$ to infinite bit-strings, $\{0,1\}^\omega$

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6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Team Problems

Problems

1-4



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