

6	9	13	7
12		10	5
3	1	4	14
15	8	11	2

Mathematics for Computer Science
MIT 6.042J/18.062J

Proof by Contradiction Proof by Cases



Albert R. Meyer

February 5, 2010

lec 1F.1

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Proof by Contradiction

Is $\sqrt[3]{1332} \leq 11$?

If so, $1332 \leq 1331$

That's **not true**, so

$\sqrt[3]{1332} > 11$



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6	9	13	7
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Proof by Contradiction

If an assertion implies something **false**, then the assertion itself must be **false**!



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6	9	13	7
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Proof by Contradiction

Theorem: $\sqrt{2}$ is irrational.

- Suppose $\sqrt{2}$ was **rational**
- So have n, d integers **without common prime factors** such that

$$\sqrt{2} = \frac{n}{d}$$
- We will show that n & d are **both even**. This **contradicts no common factor**.



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Proof by Contradiction

Theorem: $\sqrt{2}$ is irrational.

so can assume

$$\sqrt{2} = \frac{n}{d}$$

$$\sqrt{2}d = n$$

$$2d^2 = n^2$$

So n is **even**

$$n = 2k$$

$$n^2 = 4k^2$$

$$2d^2 = 4k^2$$

$$d^2 = 2k^2$$

So d is **even**



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Quickie

Proof assumes that **if n^2 is even, then n is even**.

Why is this true?



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Proof by Cases



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Java Logical Expression

if ((x>0) || (x <= 0 && y>100))
 OR : AND
 (more code)

better: if ((x>0) || y>100)
 :
 (more code)



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Case 1: $x > 0$

true
 if ((x>0) || (x <= 0 && y>100))
 OR AND

true
 if ((x>0) || y>100)
 OR
 so both are true



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Case 2: $x \leq 0$

false
 if ((x>0) || (x <= 0 && y>100))
 OR AND

false
 if ((x>0) || y>100)
 OR



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Case 2: $x \leq 0$

true
 if (x <= 0 && y>100)
 AND

if (y>100)



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Case 2: $x \leq 0$

if (y>100)

if (y>100)

so both still the same



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Proof by Cases

Reasoning by cases can break a complicated problem into easier subproblems.
Some philosophers* think reasoning this way is worrisome.

*intuitionists



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\$1,000,000 Question

Is $P = NP$?



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lec 2M.28

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\$1,000,000 Question

The answer is on my desk!
(Proof by Cases)



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lec 1F.30

6	9	13	7
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Team Problems

Problems
1–4



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