In-Class Problems Week 1, Wed.

Problem 1.
Identify exactly where the bugs are in each of the following bogus proofs.[1]

(a) Bogus Claim: $1/8 > 1/4$.

Bogus proof.

$$3 > 2$$
$$3\log_{10}(1/2) > 2\log_{10}(1/2)$$
$$\log_{10}(1/2)^3 > \log_{10}(1/2)^2$$
$$(1/2)^3 > (1/2)^2,$$

and the claim now follows by the rules for multiplying fractions.   ∎

(b) *Bogus proof*: $1\cent = \$0.01 = (\$0.1)^2 = (10\cent)^2 = 100\cent = \$1.$ ∎

(c) Bogus Claim: If a and b are two equal real numbers, then $a = 0$.

Bogus proof.

$$
\begin{aligned}
a &= b \\
a^2 &= ab \\
a^2 - b^2 &= ab - b^2 \\
(a-b)(a+b) &= (a-b)b \\
a+b &= b \\
a &= 0.
\end{aligned}
$$

∎

[1]From Stueben, Michael and Diane Sandford. *Twenty Years Before the Blackboard*, Mathematical Association of America, ©1998.

Problem 2.
It's a fact that the Arithmetic Mean is at least as large the Geometric Mean, namely,

$$\frac{a+b}{2} \geq \sqrt{ab}$$

for all nonnegative real numbers a and b. But there's something objectionable about the following proof of this fact. What's the objection, and how would you fix it?

Bogus proof.

$$\frac{a+b}{2} \overset{?}{\geq} \sqrt{ab}, \qquad \text{so}$$

$$a+b \overset{?}{\geq} 2\sqrt{ab}, \qquad \text{so}$$

$$a^2 + 2ab + b^2 \overset{?}{\geq} 4ab, \qquad \text{so}$$

$$a^2 - 2ab + b^2 \overset{?}{\geq} 0, \qquad \text{so}$$

$$(a-b)^2 \geq 0 \qquad \text{which we know is true.}$$

The last statement is true because $a - b$ is a real number, and the square of a real number is never negative. This proves the claim. ∎

Problem 3.
Albert announces that he plans a surprise 6.042 quiz next week. His students wonder if the quiz could be next Friday. The students realize that it obviously cannot, because if it hadn't been given before Friday, everyone would know that there was only Friday left on which to give it, so it wouldn't be a surprise any more.

So the students ask whether Albert could give the surprise quiz Thursday? They observe that if the quiz wasn't given *before* Thursday, it would have to be given *on* the Thursday, since they already know it can't be given on Friday. But having figured that out, it wouldn't be a surprise if the quiz was on Thursday either. Similarly, the students reason that the quiz can't be on Wednesday, Tuesday, or Monday. Namely, it's impossible for Albert to give a surprise quiz next week. All the students now relax, having concluded that Albert must have been bluffing.

And since no one expects the quiz, that's why, when Albert gives it on Tuesday next week, it really is a surprise!

What do you think is wrong with the students' reasoning?

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