# Mini-Quiz May 5

**Your name:**_____

- This quiz is **closed book**. Total time is 25 minutes.
- Write your solutions in the space provided. If you need more space, write on the back of the sheet containing the problem. Please keep your entire answer to a problem on that problem's page.
- GOOD LUCK!

**DO NOT WRITE BELOW THIS LINE**

| Problem | Points | Grade | Grader |
|---|---|---|---|
| 1 | 6 | | |
| 2 | 8 | | |
| 3 | 6 | | |
| Total | 20 | | |

**Problem 1 (6 points).**
You would like to give a bouquet for Mother's Day. You find an online service that will make bouquets of **lilies**, **roses** and **tulips**, subject to the following constraints:

- there must be at most 3 lilies,
- there can be any number of roses,
- there must be a multiple of four tulips.

Example: A bouquet of 4 tulips, 5 roses and no lilies satisfies the constraints.

Let $f_n$ be the number of possible bouquets with $n$ flowers that fit the service's constraints. Express $F(x)$, the generating function corresponding to $\langle f_0, f_1, f_2, \dots \rangle$, as a quotient of polynomials (or products of polynomials). You do not need to simplify this expression.

**Problem 2 (8 points).**
**[A Baseball Series]**
The New York Yankees and the Boston Red Sox are playing a two-out-of-three series. (In other words, they play until one team has won two games. Then that team is declared the overall winner and the series ends.) Assume that the Red Sox win each game with probability $2/3$, regardless of the outcomes of previous games.

Answer the questions below using the four step method. You can use the same tree diagram for all three problems.

**(a)** What is the probability that only 2 games are played?

**(b)** What is the probability that the winner of the series loses the first game?

**(c)** What is the probability that the Red Sox loses the series?

**Problem 3 (6 points).**
The following combinational identity is known as Pascal's Triangle:

$$\binom{n-1}{k-1} + \binom{n-1}{k} = \binom{n}{k}$$

**(a)** Give a *combinatorial proof* that for Pascal's Triangle.

**(b)** Verify this combinational proof by giving an algebraic proof of this same fact.

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