Mini-Quiz Apr. 7

Your name: ______________________________

- This quiz is **closed book**. Total time is 25 minutes.
- Write your solutions in the space provided. If you need more space, write on the back of the sheet containing the problem. Please keep your entire answer to a problem on that problem's page.
- GOOD LUCK!

DO NOT WRITE BELOW THIS LINE

Problem	Points	Grade	Grader
1	8		
2	6		
3	6		
Total	20		

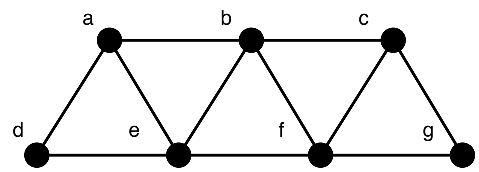
Problem 1 (8 points). (a) Use the Pulverizer to find $gcd(84, 108)$

(b) Find integers x, y with $0 \leq y < 84$ such that

$$x \cdot 84 + y \cdot 108 = \text{gcd}(84, 108).$$

(c) Find the multiplicative inverse of 84 modulo 108 in the range {1,...,107}. If no such inverse can be found, briefly explain why not.

Problem 2 (**6 points**). **(a)** For the planar embedding picture below, list all the discrete faces (simple cycles that define the region borders).



(b) Provide a drawing of a different planar embedding of the graph above. Also list all the faces of the embedding.

Problem 3 (6 points).

Definition. Consider a new recursive definition, MB_0, of the same set of "matching" brackets strings as MB (definition of MB is provided in the Appendix):

- **Base case:** $\lambda \in \text{MB}_0$.
- **Constructor cases:**

  (i) If s is in MB_0, then $[s]$ is in MB_0.

  (ii) If $s, t \in \text{MB}_0$, $s \neq \lambda$, and $t \neq \lambda$, then st is in MB_0.

(a) Suppose structural induction was being used to prove that $\text{MB}_0 \subseteq \text{MB}$. Circle the one predicate below that would fit the format for a structural induction hypothesis in such a proof.

- $P_0(n) ::= |s| \leq n$ IMPLIES $s \in \text{MB}$.
- $P_1(n) ::= |s| \leq n$ IMPLIES $s \in \text{MB}_0$.
- $P_2(s) ::= s \in \text{MB}$.
- $P_3(s) ::= s \in \text{MB}_0$.
- $P_4(s) ::= (s \in \text{MB}$ IMPLIES $s \in \text{MB}_0)$.

(b) The recursive definition MB_0 is *ambiguous*. Verify this by giving two different derivations for the string "[ ][ ][ ]" according to MB_0.

Appendix

Matched Brackets

Recursively define the set, MB, of strings of "matching" brackets as follows:

- **Base case:** $\lambda \in \text{MB}$.
- **Constructor case:** If $s, t \in \text{MB}$, then $[s]t \in \text{MB}$.